2014 Leaving Cert Ordinary Level
Official Sample Paper 2

Section A	Concepts and Skills	150 marks

Question 1 **(25 marks)**

(a) State the *fundamental principle of counting*.

The fundamental principle of counting states that if one event has a outcomes and another event has b outcomes, then there are $a \times b$ different outcomes for the two events together.

(b) How many different ways are there to arrange five distinct objects in a row?

The number of ways of arranging 5 different objects in a row is given by

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

(c) Peter is arranging books on a shelf. He has five novels and three poetry books. He wants to keep the five novels together and the three poetry books together. In how many different ways can he arrange the books?

Since all novels have to be together and the poetry books have to be together, we can think of this as being a block of novels and a block of poetry books. There are $2! = 2$ different ways of arranging these blocks. Similarly, there are $5! = 120$ different ways of arranging the novels. Finally, there are $3! = 6$ different ways of arranging the poetry books. Thus, the total number of ways of arranging all 8 books is $2! \times 5! \times 3! = 1440$.

Question 2 **(25 marks)**

A biased die is used in a game. The probabilities of getting the six different numbers on the die are shown in the table below.

Number	1	2	3	4	5	6
Probability	0.25	0.25	0.15	0.15	0.1	0.1

(a) Find the expected value of the random variable X, where X is the number thrown.

Given a random variable X with outcomes $x_1, x_2, \ldots, x_n$, the expected value of X, $E(X)$ is given by

$$E(X) = x_1\mathbb{P}(x_1) + x_2\mathbb{P}(x_2) + \cdots x_n\mathbb{P}(x_n)$$

So, for the biased dice, we have

$$E(X) = 1(0.25) + 2(0.25) + 3(0.15) + 4(0.15) + 5(0.1) + 6(0.1) = 2.9$$

(b) There is a game at a funfair. It costs €3 to play the game. The player rolls a die once and wins back the number of euro shown on the die. The sentence below describes the difference between using the above biased die and using a fair (unbiased) die when playing this game. By doing the calculations required, complete the sentence.

"If you play the game many times with a fair die, you will win an average of ______ per game, but if you play with the biased die you will lose an average of ______ per game."

For an unbiased die, the probability of getting any number is $\frac{1}{6}$. Hence, the expected value is

$$E(X_F) = 1\left(\tfrac{1}{6}\right) + 2\left(\tfrac{1}{6}\right) + 3\left(\tfrac{1}{6}\right) + 4\left(\tfrac{1}{6}\right) + 5\left(\tfrac{1}{6}\right) + 6\left(\tfrac{1}{6}\right) = 3.5$$

The average gain for a fair die is $3.5 - 3 = 50$c and the average gain for a biased die is $2.9 - 3 = -10$c so the sentence reads:

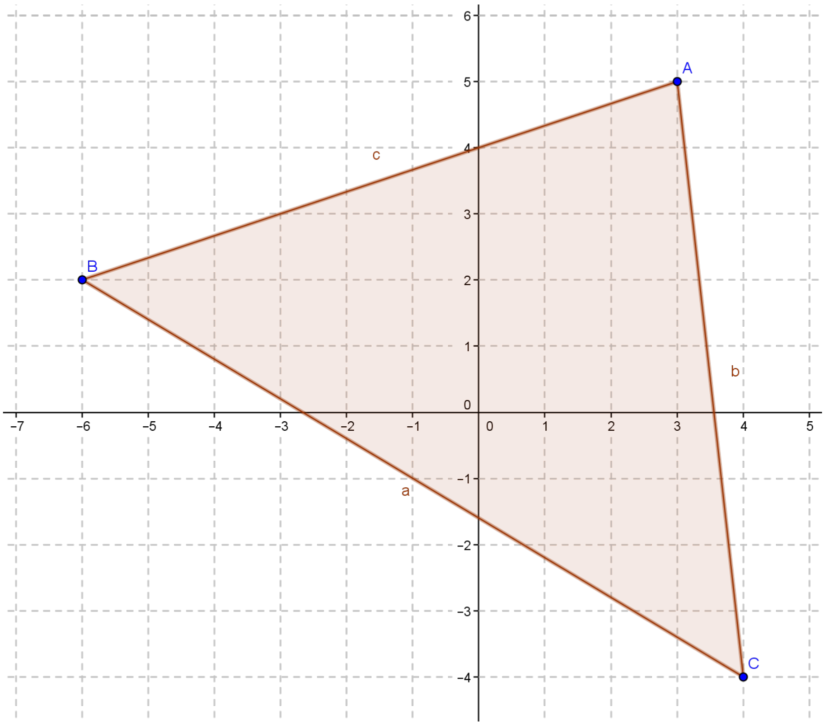
"If you play the game many times with a fair die, you will win an average of 50c per game. If you play the game with a biased die, you will lose an average of 10c per game."

Question 3 **(25 marks)**

The points A, B, and C have co-ordinates as follows:

$$A(3,5)$$
$$B(-6,2)$$
$$C(4,-4)$$

(a) Plot A, B, and C on the diagram.



MODEL ANSWER BY thematstutor.ie ONLINE SUPPORT SYSTEM FOR PROJECT MATHS

(b) Find the equation of the line AB.

The slope of the line passing through A and B is

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-2}{3-(-6)} = \frac{3}{9} = \frac{1}{3}$$

Thus, the line passing through A and B is given by

$$\begin{aligned} y - y_1 = m(x - x_1) \quad &\Leftrightarrow \quad y - 2 = \frac{1}{3}(x-(-6)) \\ &\Leftrightarrow \quad 3y - 6 = x + 6 \\ &\Leftrightarrow \quad 3y - x = 12 \end{aligned}$$

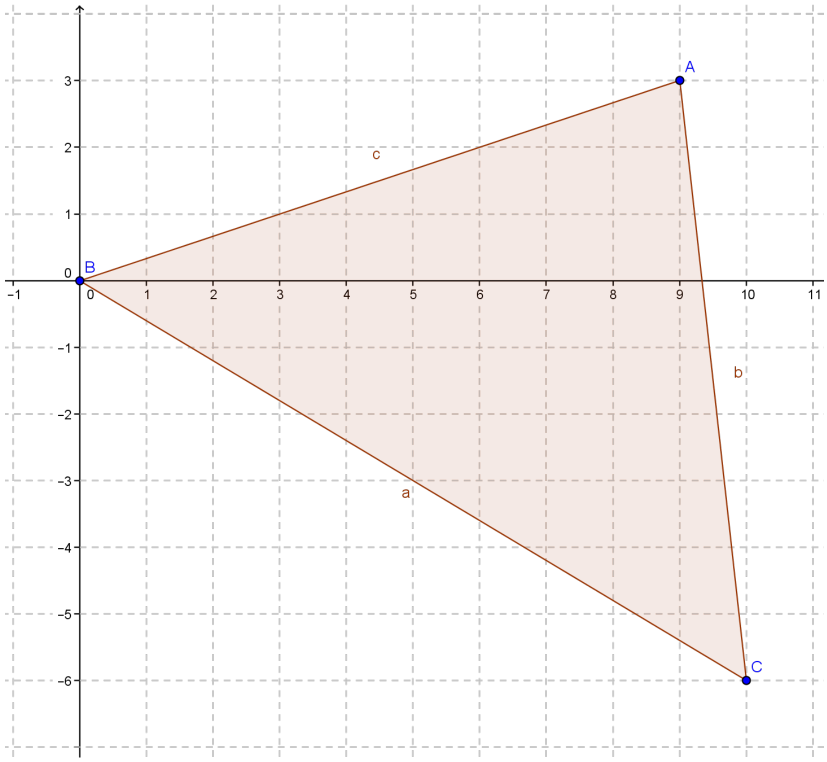
MODEL ANSWER BY thematstutor.ie ONLINE SUPPORT SYSTEM FOR PROJECT MATHS

(c) Find the area of the triangle ABC.

We wish to translate each of the points so that one of them is on the origin. We will move B to the origin. To do this, we subtract -6 from each x-coordinate and subtract 2 from each y-coordinate. This will leave the area of the triangle unchanged. Thus, we have

$$A' = A - B = (3,5) - (-6,2) = (9,3)$$
$$B' = B - B = (-6,2) - (-6,2) = (0,0)$$
$$C' = C - B = (4,-4) - (-6,2) = (10,-6)$$

Now, our points look like this:



The formula for the area of a triangle with one corner at the origin and the other two at (x_1, y_1) and (x_2, y_2) is $A = \frac{1}{2}|x_1y_2 - x_2y_1|$, so the area of $A'B'C'$, i.e. the area of ABC is

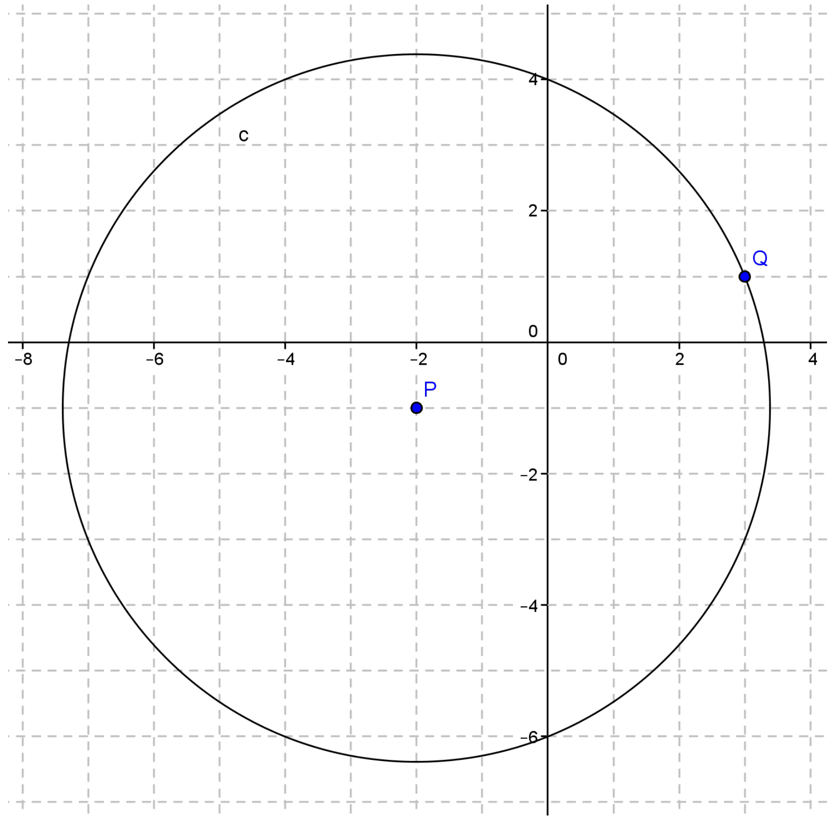
$$A = \frac{1}{2}|(9)(-6) - (10)(3)| = \frac{1}{2}|-54-30| = \frac{1}{2}|-84| = 42 \text{ units}^2$$

MODEL ANSWER BY
themathstutor.ie
ONLINE SUPPORT SYSTEM FOR PROJECT MATHS

Question 4 **(25 marks)**

The circle c has centre $P(-2,-1)$ and passes through the point $Q(3,1)$.

(a) Show c, P, and Q on a co-ordinate diagram.



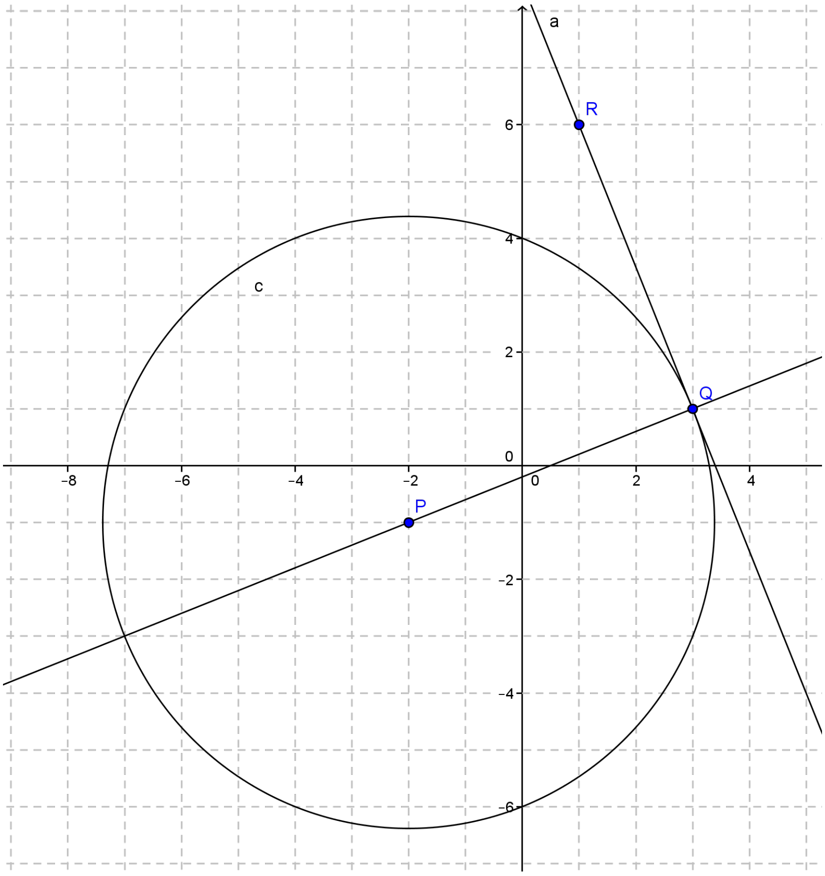
(b) Find the radius of c and hence write down its equation.

Since c passes through the point Q, the distance from P to Q must be the radius of the circle. This distance is given by

$$\begin{aligned} r &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(3 - (-2))^2 + (1 - (-1))^2} \\ &= \sqrt{(5)^2 + (2)^2} \\ &= \sqrt{25 + 4} \\ &= \sqrt{29} \end{aligned}$$

(c) R is the point $(1, 6)$. By finding the slopes of PQ and QR, show that QR is a tangent to c.

We'll construct the lines PQ and QR for reference:



If QR is tangent to the circle at the point Q, then QR will be perpendicular to the line PQ, and the product of the slopes m_{QR} and m_{PQ} will be equal to -1. The slopes are given by

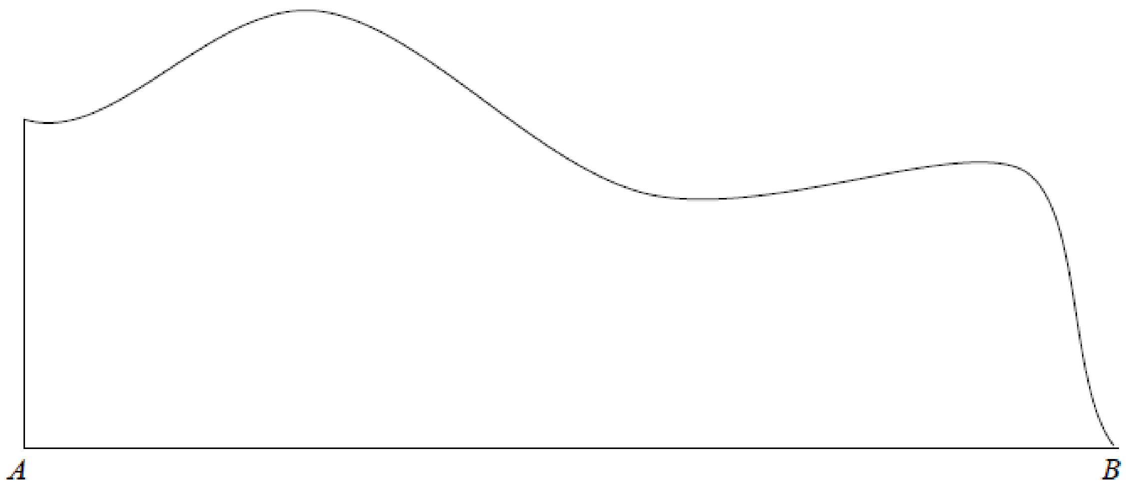
$$m_{QR} = \frac{3-1}{1-6} = -\frac{2}{5} \qquad \text{and} \qquad m_{PQ} = \frac{3-(-2)}{1-(-1)} = \frac{5}{2}$$

Thus $m_{PQ} \times m_{QR} = \left(\frac{5}{2}\right) \times \left(-\frac{2}{5}\right) = -1$ as required.

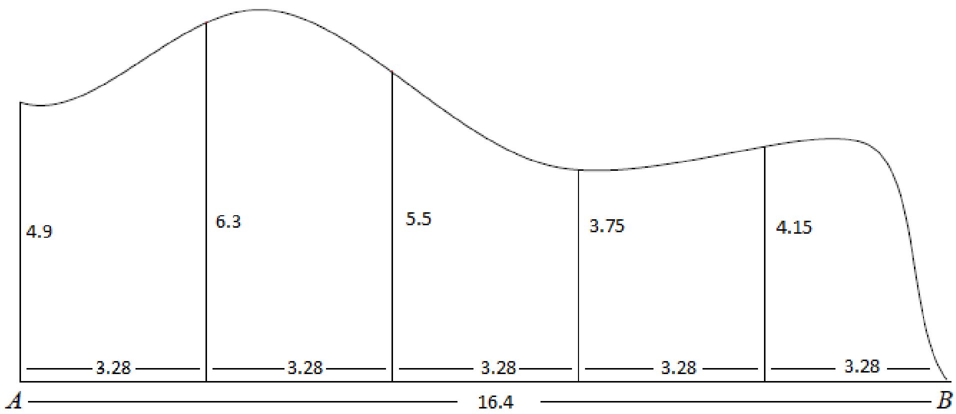
Question 5 **(25 marks)**

The diagram below shows a shape with two straight edges and one irregular edge. By dividing the edge [AB] into five equal intervals, use the trapezoidal rule to estimate the area of the shape.

Record your constructions and measurements on the diagram. Give your answer correct to the nearest cm^2



The line AB is approximately 16.4cm, so each interval will be $\frac{16.4}{5} = 3.28$ cm long. We then measure the height of the shape at each of these points:



Note that the height of the graph at the endpoint B is 0. From the Formulae and Tables book, the Trapezoidal Rule gives the approximate area as:

$$\frac{h}{2}\left[y_1 + y_n + 2(y_2 + y_3 + \cdots + y_{n-1})\right]$$

where h is the width of the subintervals. Thus, the area of our shape is given by

$$\begin{aligned} \text{Area} &= \frac{3.28}{2}\left[4.9 + 0 + 2(6.3 + 5.5 + 3.75 + 4.15)\right] \\ &= 1.6\,[44.3] \\ &= 72.652 \end{aligned}$$

which is equal to 73 cm^2 correct to the nearest cm^2.

Question 6A **(25 marks)**

(a) Explain what is meant by the *converse* of a theorem.

The converse of a theorem is a statement which swaps what is assumed and what is to be proved in a theorem. For example, given a theorem which states "If A is true, then B is true", the converse would be "If B is true, then A is true".

(b) There are some geometric statements that are true, but have converses that are false. Give one such geometric statement, and state also the (false) converse.

Statement: If a shape is a square, then the sides which intersect do so at a 90° angle.

Converse: Given a shape, if the sides which intersect do so at a 90° angle, then the shape is a square.

Note that the converse is false because the shape could be a rectangle, which need not be a square.

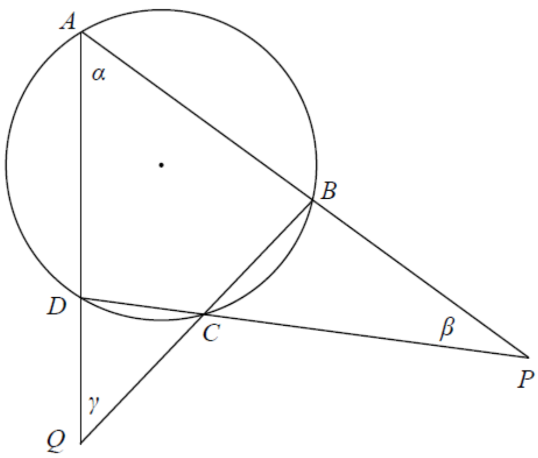
Question 6B **(25 marks)**

ABCD is a cyclic quadrilateral.

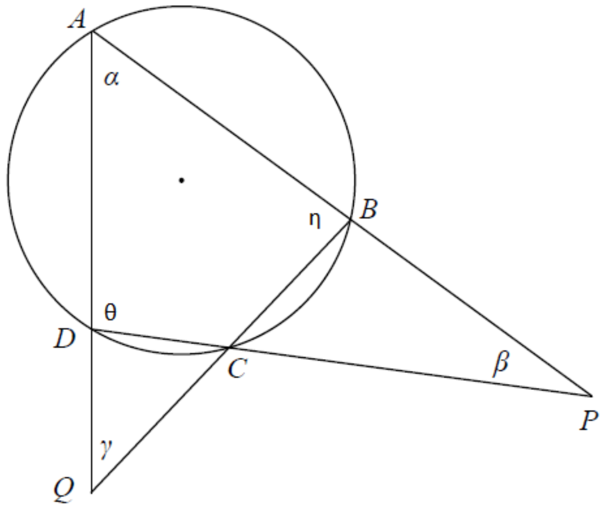
The opposite sides, when extended meet at P and Q, as shown.

The angles α, β, and γ are as shown.

Prove that $\beta + \gamma = 180° - 2\alpha$.



We will need to consider the angles $\angle ADP$ and $\angle ABQ$ within the shapes in the above diagram, so we will label them as follows:



Since APD is a triangle, the sum of its angles must be $180°$, so $\alpha+\beta+\theta=180$. Similarly, since AQB is a triangle, $\alpha+\gamma+\eta=180°$. We will add these equations to one another:

$$(\alpha+\beta+\theta)+(\alpha+\gamma+\eta)=180+180 \qquad \Leftrightarrow \qquad 2\alpha+\beta+\gamma+\theta+\eta=360$$

But recall that $ABCD$ is a cyclic quadrilateral, so we know that $\theta+\eta=180°$. Substituting this into our above equation gives us

$$2\alpha+\beta+\gamma+180=360 \qquad \Leftrightarrow \qquad \beta+\gamma=180-2\alpha$$

as required.

Section B Contexts and Applications 150 marks

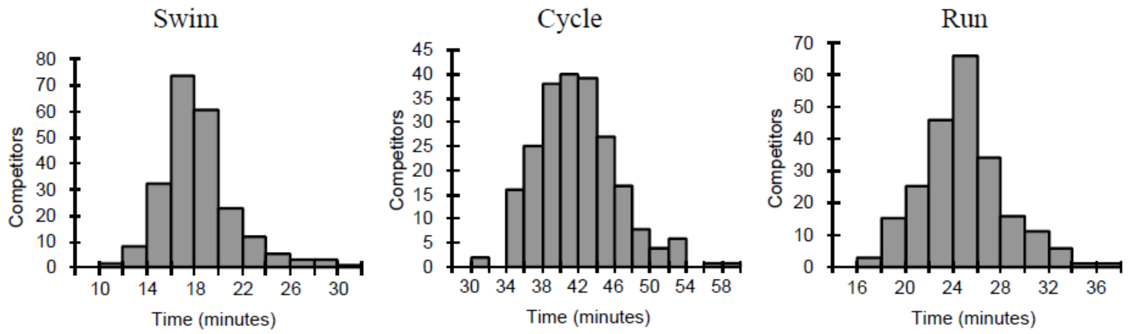
Question 7 **(75 marks)**

The King of the Hill triathlon race in Kinsale consists of a 750 metre swim, followed by a 20 kilometre cycle, followed by a 5 kilometre run.

The questions below are based on data from 224 athletes who completed this triathlon in 2010.

Máire is analysing data from the race, using statistical software. She has a data file with each competitor's time for each part of the race, along with various other details of the competitors.

Máire produces histograms of the times for the three events. Here are the three histograms.



(a) Use the histograms to complete the following sentences:

(i) The event that, on average, takes longest to complete is the **Cycle**.

Looking at where the largest number of competitors lie for a given event will tell us the estimated average. For the Swim, the average is somewhere in the 14 – 20 minute range. For the Cycle, the average is somewhere in the 36 – 46 minute range. For the Run, the average is somewhere in the 22 – 28 minute range.

(ii) In all three histograms, the times are grouped into intervals of **2** minutes.

We can determine this by examining the x-axis of each histogram.

(iii) The time of the fastest person in the swim was between | 10 | and | 12 | .

Reading off the histogram, the first non-zero column lies in this interval.

MODEL ANSWER BY themathstutor.ie ONLINE SUPPORT SYSTEM FOR PROJECT MATHS

(iv) The median time for the Run is | in the 24 to 26 minute interval | .

Given that there are 224 athletes, the median is the value such that 112 athletes had a slower Run time than this value and 112 athletes had a faster Run time. We will estimate the number of entries in each time range for the Run to determine where this median should lie.

Time Interval	Athletes	Cumulative
16 – 18	2	2
18 – 20	15	17
20 – 22	25	42
22 – 24	45	87
24 – 26	65	152
26 – 28	35	187
28 – 30	15	202
30 – 32	12	214
32 – 34	8	222
34 – 36	1	223
36 – 38	1	224

Note that there are 87 athletes who had a Run time of 24 minutes or fewer, but there are 152 athletes who had a Run time of 26 minutes or less. This means that the median Run time is between 24 and 26 minutes.

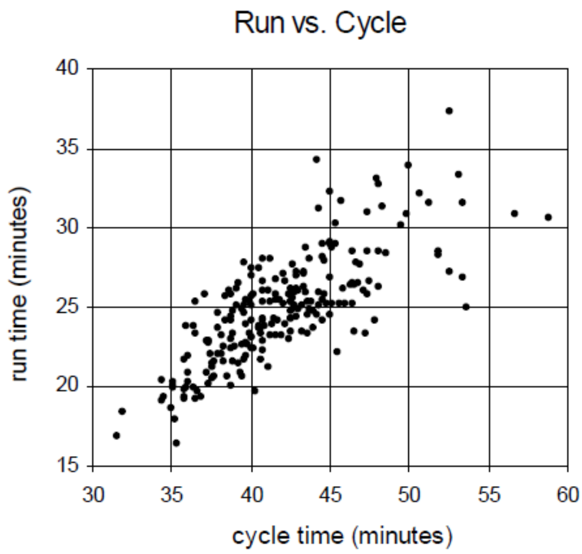
MODEL ANSWER BY themathstutor.ie ONLINE SUPPORT SYSTEM FOR PROJECT MATHS

(v) The event in which the times are most spread out is the | Cycle | .

We want to find the event which has the largest time difference between the fastest and slowest competitors. For the Swim, the difference is $30-10=20$ minutes. For the Cycle, the difference is $58-30=28$ minutes. For the Run, the difference is $36-16=20$ minutes.

MODEL ANSWER BY themathstutor.ie ONLINE SUPPORT SYSTEM FOR PROJECT MATHS

(b) Máire is interested in the relationship between the athletes' performance in the run and in the cycle. She produces the following scatter diagram.



(i) The correlation coefficient between the times for these two events is one of the numbers below. Write the letter corresponding to the correct answer in the box.

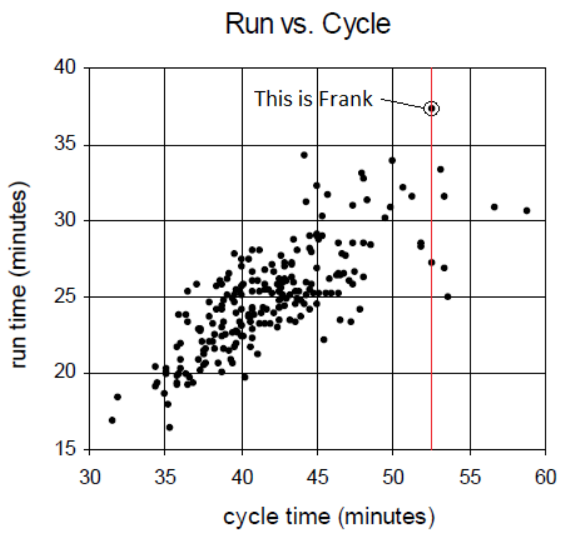
A 0.95
B 0.77
C 0.13
D −0.13
E −0.77
F −0.95

The correlation coefficient is **B**, or 0.77. Note that in the graph, the data seems to slope in a positive direction. Thus, the correlation coefficient must be positive, and so we must exclude **D**, **E** and **F**.

If the data were close to being a straight line, then the correlation coefficient would be close to 1. If there was no linear pattern to the data, then the correlation coefficient would be close to 0. The data is not exactly in a straight line, but there is some linear shape to it. Thus, since **A** (0.95) is too high and **C** (0.13) is far too low, the correct value must be 0.77.

(ii) Frank was the slowest person in the run. How many people took longer to complete the cycle than Frank did?

Looking at the graph, we consider the data point with the largest Run time. This happens when the Run is approximately 37.5 minutes and the Cycle is approximately 52.5 minutes:



We then count the number of data points which have a larger Cycle time, which is 6 people.

MODEL ANSWER BY thematheststutor.ie ONLINE SUPPORT SYSTEM FOR PROJECT MATHS

(c) Máire knows already that the male athletes tend to be slightly faster than the female athletes. She also knows that athletes can get slower as they get older. She thinks that male athletes in their forties might be about the same as female athletes in their thirties. She decides to draw a back-to-back stem-and-leaf diagram of the times of these two groups for the swim. There were 28 females in their thirties, and 32 males in their forties. Here is the diagram:

Female, 30 – 39 years		Male, 40 – 49 years
4	13	
	14	9
1 0	15	1 3 4 5 6
9 8 8 7 3 2 2	16	3 4 6 7 7 8
6 4 3 2	17	6 7 7
1	18	0 1 3 8 9
9 6 3 1 0 0	19	0 0 1 2 3 4
	20	3 9 9
3 3 2	21	2 2
4	22	
	23	0
8	24	
	25	
5	26	
	27	
	28	
7	29	

Key: | 14 | 9 means 14·9 minutes.

(i) Describe what differences, if any, there are between the two distributions above.

The range of the female group is $[13.4, 29.7]$ and the range of the Male group is $[14.9, 23.0]$, so the length of the range of the Female group is double that of the Male group. This could have been due to an outlier case, but there are several high values in the Female group. This implies that the times for the Female group have a wider distribution than the times of the Male group.

As well as this, the Male group seems to have a symmetric distribution around the 17 minute section, whereas the Female group seems to be skewed in favour of quicker times, peaking at the 16 minute section.

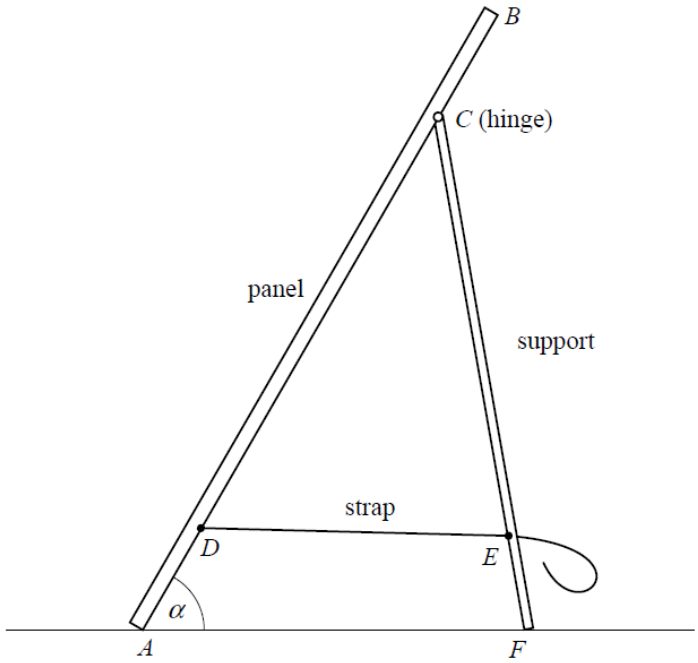
(ii) Máire drew the diagram because she thought that these two groups would be about the same. Do you think that the diagram would cause Máire to confirm her belief or change it? Give reasons for your answer.

Máire should change her belief that the two groups have the same behaviour. The distribution of times is quite difference for the two groups, so there is no reason to believe that they share any significant similarities.

Question 8 **(45 marks)**

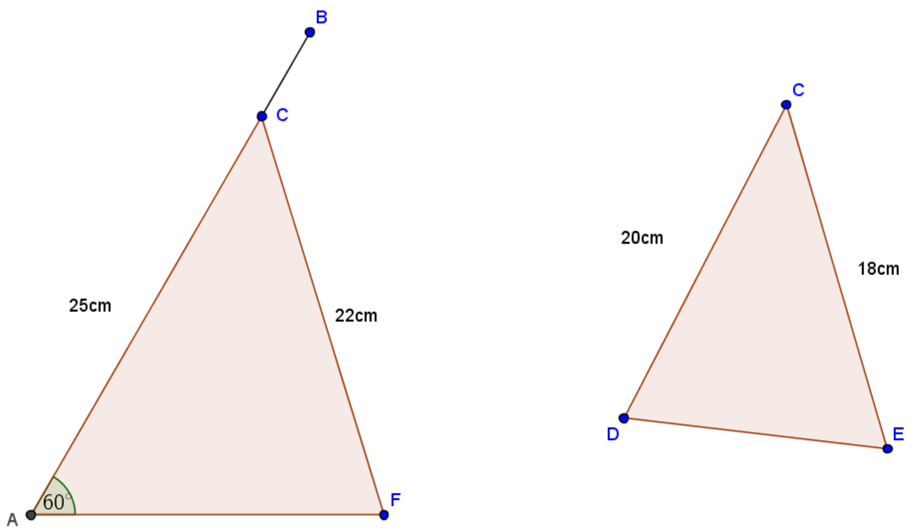
A stand is being used to prop up a portable solar panel. It consists of a support that is hinged to the panel near the top, and an adjustable strap joining the panel to the support near the bottom. By adjusting the length of the strap, the angle between the panel and the ground can be changed. The dimensions are as follows:

$|AB| = 30$ cm
$|AD| = |CB| = 5$ cm
$|CF| = 22$ cm
$|EF| = 4$ cm



We want to find out how long the strap has to be in order to make the angle α between the panel and the ground equal to 60°.

(a) Two diagrams are given below – one showing triangle CAF and the other showing triangle CDE. Use the measurements given above to record on the two diagrams below the lengths of two of the sides in each triangle.



$|AB| = |AC| + |CB|$. Using the measurements given in the question, we have $30 = |AC| + 5$, so $|AC| = 25$. The question tells us that $|CF| = 22$ cm, so that completes the two sides for the triangle on the left.

$|AC| = |AD| + |DC|$ and so $|DC| = 25 - 5 = 20$ cm. $|CF| = |CE| + |EF|$, which reduces to $|CE| = 22 - 4 = 18$ cm. This completes the two sides for the triangle on the right.

MODEL ANSWER BY themathstutor.ie ONLINE SUPPORT SYSTEM FOR PROJECT MATHS

(b) Taking $\alpha = 60°$, as shown, use the triangle CAF to find $\angle CFA$, correct to one decimal place.

The sine rule tells us that in a triangle, if the angle α is opposite a side of length a and the angle β is opposite a side of length b, then $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$. In our case we, have:

$$\frac{\sin(\angle CFA)}{25} = \frac{\sin(60°)}{22} \qquad \Leftrightarrow \qquad \sin(\angle CFA) = \frac{25\sin(60°)}{22} = \frac{25\sqrt{3}}{44}$$

since $\sin(60°) = \frac{\sqrt{3}}{2}$. This means that $\angle CFA = \sin^{-1}\left(\frac{25\sqrt{3}}{44}\right) = 79.8$ correct to one decimal place.

MODEL ANSWER BY themathstutor.ie ONLINE SUPPORT SYSTEM FOR PROJECT MATHS

(c) Hence find $\angle ACF$, correct to one decimal place.

Since all three angles in a triangle have to sum to 180°, we know that $180 = 60 + 79.8 + \angle ACF$, and so $\angle ACF = 40.2°$ correct to one decimal place.

MODEL ANSWER BY thematsstutor.ie ONLINE SUPPORT SYSTEM FOR PROJECT MATHS

(d) Use triangle CDE to find DE, the length of the strap, correct to one decimal place.

Firstly, we note that since D lies on the line AC and E lies on the line CF, the $\angle ACF$ and $\angle DCE$ must be the same. We now know an angle and two sides of a triangle and wish to find the third side. The cosine rule tell us that if we know two sides, a and b, of a triangle and we know the angle γ formed by these sides, then the third side c has to satisfy $c^2 = a^2 + b^2 - 2ab\cos\gamma$. In our case, we have

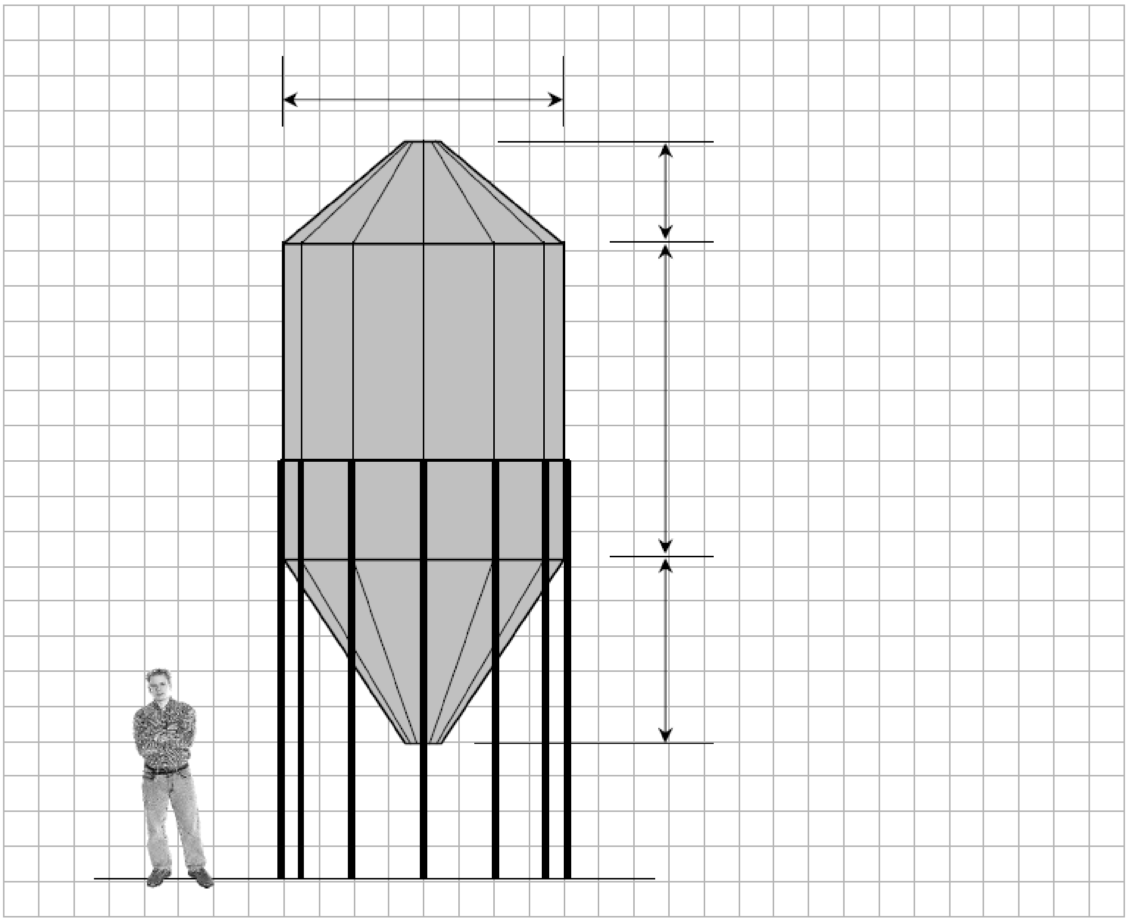
$$\begin{aligned} |DE|^2 &= |CD|^2 + |DE|^2 - 2\,(|CD|)\,(|DE)\cos(\angle DCE) \\ &= (20)^2 + (18)^2 - 2(20)(18)\cos(40.1) \\ &= 400 + 324 - (720)(0.764921) \\ &= 173.25688 \end{aligned}$$

Thus, $|DE| = \sqrt{173.25688} = 13.2$ cm correct to one decimal place.

MODEL ANSWER BY thematsstutor.ie ONLINE SUPPORT SYSTEM FOR PROJECT MATHS

Question 9 **(30 marks)**

The diagram below is a scale drawing of a hopper tank used to store grain. An estimate is needed of the capacity (volume) of the tank. The figure of the man standing beside the tank allows the scale of the drawing to be estimated.

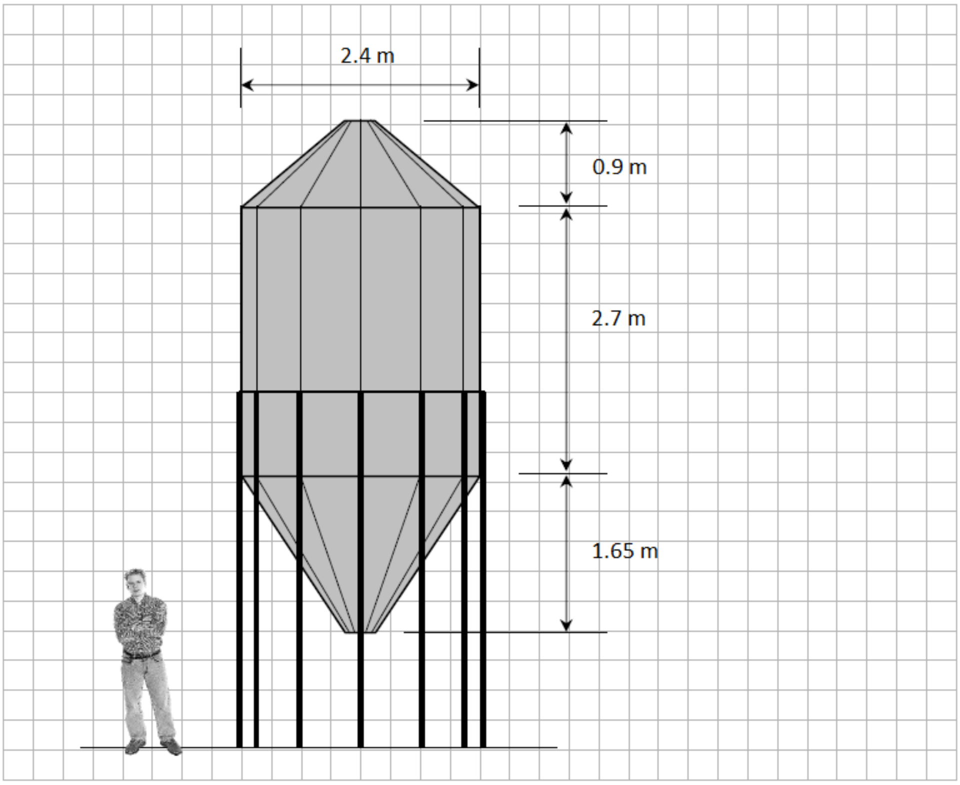


(a) Give an estimate, in metres, of the height of an average adult man.

I estimate that the average adult man is 1.8 metres tall.

(b) Using your answer to part **(a)**, estimate the dimensions of the hopper tank. Write your answers in the spaces provided on the diagram.

Given that the average adult man is 1.8 m tall and that in the above figure the man is 6 squares tall, this means that each square in the grid represents $\frac{1.8}{6} = 0.3$ m in height or width. Thus, we can approximate the dimensions of the tank by counting the number of squares. We will round each calculation to the nearest half-square



MODEL ANSWER BY thematthstutor.ie ONLINE SUPPORT SYSTEM FOR PROJECT MATHS

(c) Taking the tank to be a cylinder with a cone above and below, find an estimate for the capacity of the tank, in cubic metres.

Firstly, note that the volume of a cone is given by $\frac{1}{3}\pi r^2 h$ and the volume of a cylinder is given by $\pi r^2 h$. If the figure is a cross section through the centre of the tank, then the width of the tank (2.4 m) will be the diameter of the cylinder, and the diameter of the base of cones. Thus, $r = 1.2$ cm.

Now, the volume of the cylinder is $\pi(1.2)^2(2.7) = 3.888\pi$ m^3. The volume of the upper cone is $\frac{1}{3}\pi(1.2)^2(0.9) = 0.432\pi$ m^3. Finally, the volume of the lower cone is $\frac{1}{3}\pi(1.2)^2(1.65) = 0.792\pi$ m^3. Combining these three sections together, the volume of the hopper tank is 5.112π m^3 or 16.06 m^3 correct to two decimal places.