2012 Leaving Cert Ordinary Level
Official Sample Paper 2

| Section A	Concepts and Skills	125 marks

Question 1 (25 marks)

The size, mean and standard deviation of four sets of data A,B,C and D are given in this table:

	A	B	C	D
size (N)	1000	100	100	10
mean (μ)	10	100	1000	100
standard deviation (σ)	20	30	20	10

Complete the sentences below by inserting the relevant letter in each space:

(a) The set that contains more numbers than any other is **A** and the set that contains fewer numbers than any other is **D**.

(b) On average, the data in set **C** are the biggest numbers and the data in set **A** are the smallest numbers.

(c) The data in set **B** are more spread out than the data in the other sets.

(d) The set that must contain some negative numbers is set **A**. (Not strictly true but this is the best answer out of the 4 possibilities).

(e) If the four sets are combined, the median is most likely to be a value in set **A**.
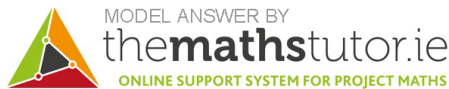
Question 2 (25 marks)

The 2006 census shows that the number of males living in Ireland is about the same as the number of females.

(a) If a person is selected at random, write down the probability that the person is a male.

$\frac{1}{2}$ since there are two outcomes and both are equally likely (equal probability).
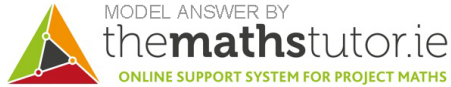
(b) Four people are chosen at random. We are interested in whether they are male or female.

    (i) Complete the sample space below showing the sixteen equally likely outcomes.

MMMM	FFFF	FMMF	MFFM
MMMF	MMFM	MFMM	FMMM
MMFF	MFMF	FMFM	FFMM
MFFF	FMFF	FFMF	FFFM
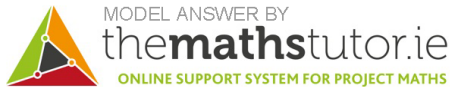
    (ii) Hence or otherwise, complete the table of probabilities below.

| four males	three males; one female	two males; two females	one male; three females	four females
$\frac{1}{16}$	$\frac{4}{16} = \frac{1}{4}$	$\frac{6}{16} = \frac{3}{8}$	$\frac{4}{16} = \frac{1}{4}$	$\frac{1}{16}$

(c) A person states the following: "If you pick four people at random, it's **more likely than not** that you'll get two males and two females."
Is this statement correct? Justify your answer using the answer(s) to part **(b)**.

Answer: No the statement is not correct.

Justification: Although this is the most likely outcome based on the table above, it is still more likely <u>not</u> to happen. Since the probability of two males and two females is $\frac{3}{8}$, the probability of any other outcome is $\frac{5}{8}$ which is bigger and thus more likely.
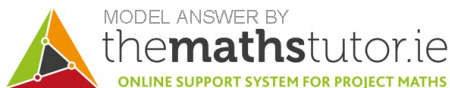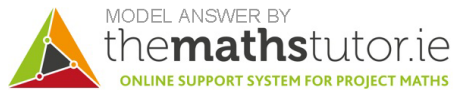
Question 3 (25 marks)
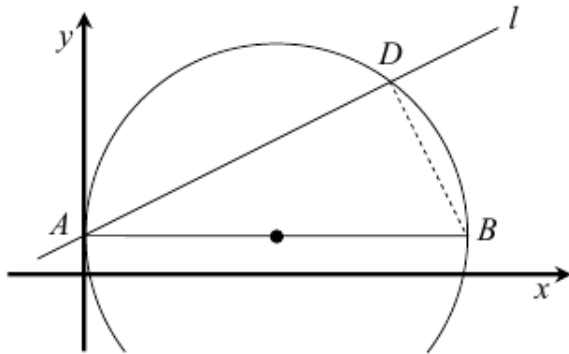
(a) The point A has co-ordinates $(0,1)$.

The line l passes through A and has slope $\dfrac{1}{2}$

Find the equation of l.

The line l passes through the point $(x_1, y_1) = (0, 1)$ and has slope $m = \dfrac{1}{2}$. Use the equation of a line formula $y - y_1 = m(x - x_1)$ to get

$$
\begin{aligned}
y - 1 &= \tfrac{1}{2}(x - 0) \\
2y - 2 &= x \\
-x + 2y - 2 &= 0
\end{aligned}
$$

(b) $[AB]$ is the diameter of a circle, where B is the point $(10, 1)$.

Find the centre and radius of the circle and hence write down its equation.

We have $A = (0,1) = (x_1, y_1)$ and $B = (10,1) = (x_2, y_2)$. Use the midpoint formula to find the centre of the circle

$$(midpoint\ AB) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0 + 10}{2}, \frac{1 + 1}{2}\right) = (5,1)$$

so the centre of the circle $(h, k) = (5, 1)$.

The radius of the circle is half the diameter. Use the distance formula to find the diameter (distance between A and B)

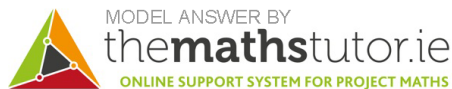$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(10 - 0)^2 + (1 - 1)^2} = \sqrt{100 + 0} = 10$$

which means the radius of the circle $r = 5$.

The equation of the circle is given by

$$\begin{aligned}
(x - h)^2 + (y - k)^2 &= r^2 \\
(x - 5)^2 + (y - 1)^2 &= 5^2 \\
(x - 5)^2 + (y - 1)^2 &= 25
\end{aligned}$$

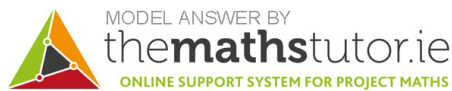**(c)** The line l crosses the circle at the points A and D.
Write down the slope of DB and explain how you know that this is the slope.

The line DB is perpendicular to AD because AB is a diameter and D is a point on the circle (Theorem 19 Corollary 2). If two lines are perpendicular their slopes multiply to give -1, i.e.

$$\begin{aligned}
(\text{slope } AD)(\text{slope } DB) &= -1 \\
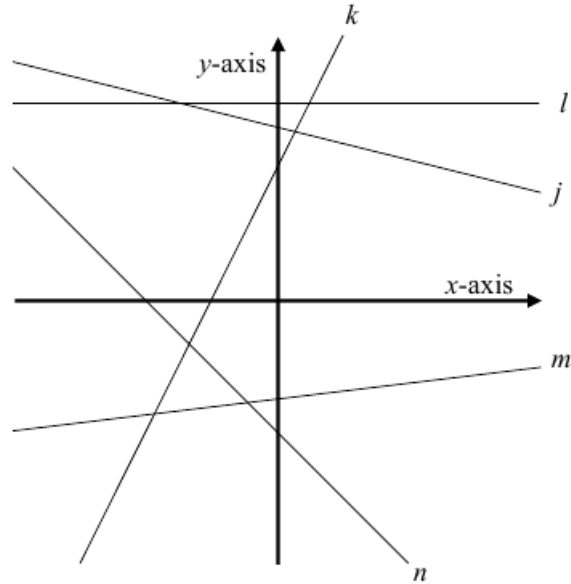\frac{1}{2}(\text{slope } DB) &= -1 \\
\text{slope } DB &= -2
\end{aligned}$$

Question 4

Five line j, k, l, m, and n in the co-ordinate plane are shown in the diagram.

(a) The slopes of the five lines are in the table below.
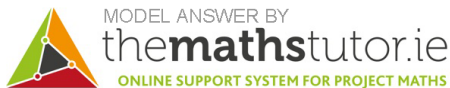
Complete the table, matching the lines to their slopes.

slope	line
2	k
$\dfrac{1}{8}$	m
0	l
$-\dfrac{1}{4}$	j
-1	n

(b) The diagram shows four circles of equal radius.
The circles are touching as shown.
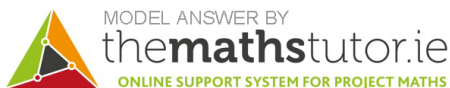
The equation of c_1 is $x^2 + y^2 = 9$.

(i) Write down the radius of c_1.

By comparing the equation of c_1 with the general equation of a circle $(x-h)^2 + (y-k)^2 = r^2$
we see that $r^2 = 9$ which means $r = 3$.

(ii) Write down the co-ordinates of the centre of c_3.
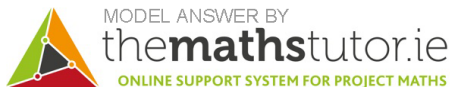
(6,6)

(iii) Write the equation of c_3.

The equation of a circle with centre (h,k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$ which
means the equation of c_3 is

$$(x-6)^2 + (y-6)^2 = 3^2$$
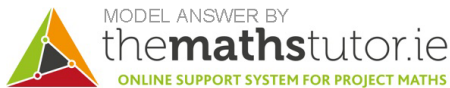$$(x-6)^2 + (y-6)^2 = 9$$

Question 5 (25 marks)

Answer **either** 5A **or** 5B.

Question 5A

**(a)** Explain what is meant by the *converse* of a theorem.

Explanation:

The converse of a theorem is a statement which swaps what is assumed and what is to be
proved in a theorem. For example, given a theorem which states "If A is true, then B is
true", the converse would be "If B is true, then A is true".

(b) There are some geometric statements that are true, but have converses that are false. Give
one such geometric statement, and state also the (false) converse.

Statement: If a shape is a square, then the sides which intersect do so at a 90° angle.

Converse: Given a shape, if the sides which intersect do so at a 90° angle, then the shape is a square. (This is false as it may not have 4 equal side lengths, so it may not be a square).
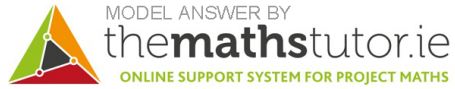
Question 6 **(75 marks)**

(a) The students in a 2012 Leaving Certificate class decided to investigate their heights. They measured the height of each student, in centimetres, and the results were as follows:

$$173 \quad 167 \quad 180 \quad 168 \quad 180 \quad 175$$

$$171 \quad 161 \quad 164 \quad 187 \quad 176 \quad 160$$

$$170 \quad 171 \quad 167 \quad 178 \quad 174 \quad 149$$

$$157 \quad 161 \quad 176 \quad 166 \quad 167 \quad 172$$

(i) Construct a stem and leaf plot of the above data.

| Stem	Leaf
14	9
15	7
16	0,1,1,4,6,7,7,7,8
17	0,1,1,2,3,4,5,6,6,8
18	0,0,7

Key: $14|9 = 149$ cm

(ii) Describe the distribution of the data, by making **one** statement about **each** of the three characteristics indicated below.

shape of distribution: Roughly bell-shaped. The data is normally distributed.

location of data (central tendency/average): The median of the data is the middle value. Since there are 24 observations here, the median is the average of the 12th and 13th values i.e. median $= \dfrac{170 + 171}{2} = 170.5$ cm

spread of the data (dispersion): The range of the data is the difference between the largest value and the smallest value. So the range of this data set is $187 - 149 = 38$ cm
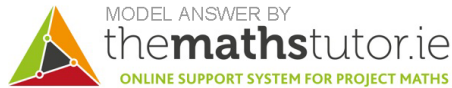
(iii) State **one** additional piece of information that you would need in order to decide whether these students are unusually tall.

We would need the mean/median for the larger population (all Leaving Cert students in Ireland) to decide whether this small sample contains students who are unusually tall.
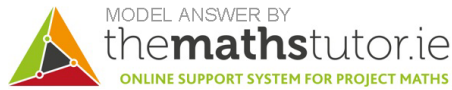
(b) The students decide to look at the heights of the males and the females in the class separately. The heights are given below:

Males			Females		
173	180	174	167	161	160
175	178	176	157	164	172
180	171	170	168	149	161
187	176	166	167	167	171

(i) Construct a back-to-back stem and leaf plot of the above data.

| Leaf	Stem	Leaf
	14	9
	15	7
6	16	0,1,1,4,7,7,7,8
8,6,6,5,4,3,1,0	17	1,2
7,0,0	18	

Key $7|14| = 147$ cm and $|15|2 = 152$ cm
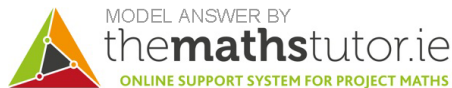
(ii) State **one difference** ad **one similarity** between the two distributions.

Difference: The median height of males is higher than females. The median for males is 175.5cm while the median for females is 165.5cm.

Similarity: The spread of the data is similar as most of the data on the male side is on the 170cm row while most of the data on the female side is on the 160cm row. Overall, both distributions are approximately bell-shaped.
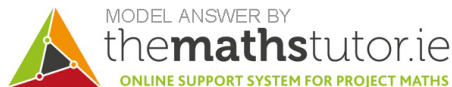
(c) The heights in 2011 of Irish males born in 1992 are normally distributed with a mean 178.8cm and standard deviation 7.9cm.

(i) Use the *empirical rule* to complete the following sentence:

"95% of nineteen-year-old Irish men are between **163cm** and **194.6cm** in height."

We can say this because the empirical rule states that 95% of the population lies within 2 standard deviations of the mean.

$178.8 - 2(7.9) = 163$ and $178.8 + 2(7.9) = 194.6$.
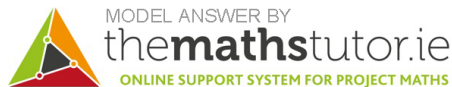
(ii) Use the empirical rule to make one other statement about the heights of nineteen-year-old Irish men.

The empirical rule also says that 68% of the population lies within one standard deviation of the mean. So 68% of nineteen-year-old Irish men are between 170.9cm and 186.7cm.
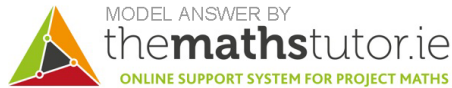
(d) The male students in part **(b)** are a *sample*. The males in part **(c)** are a *population*. Is this a suitable population to compare the sample to? Give a reason for your answer.

No this is not a suitable comparison. The students in the Leaving Cert class are not necessarily all born in 1992, but the population in part **(c)** is based only on those born in 1992.
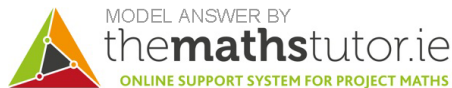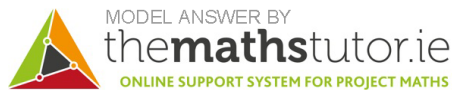
(e) Would you say that the males in the class are taller, smaller, or about the same as the population? Use the data to justify your answer.

The mean height of the males in the Leaving Cert class is

$$\frac{173+180+174+175+178+176+180+171+170+187+176+166}{12} = 175.5\text{cm}$$

whereas the mean height of the entire population is 178.8cm, so on average the Leaving Cert class students are **smaller**.

(f) An economics students wants to find out whether the length of time people spend in education affects the income they earn. The student carries out a small study. She asks twelve adults to state their annual income and the number of years they spent in full-time education. The data are given in the table below, and a partially completed scatter plot is given.

| Years of education	Income /€1,000
11	28
12	30
13	35
13	43
14	55
15	38
16	45
16	38
17	55
17	60
17	30
19	58
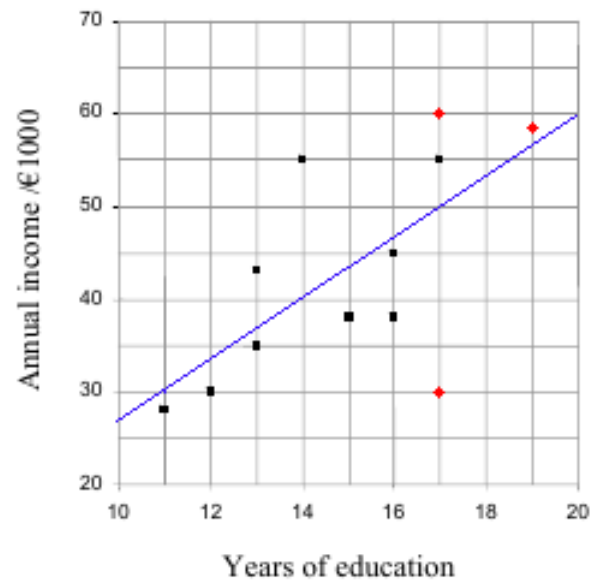
(i) The last three rows of the data have not been included on the scatter plot. Insert them now.

See red points inserted in the scatter plot above.
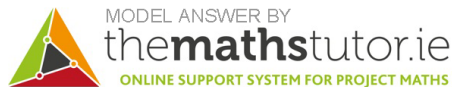
(ii) What can you conclude from the scatter plot?

We can conclude that there is a moderate positive correlation between number of years in education and annual income.
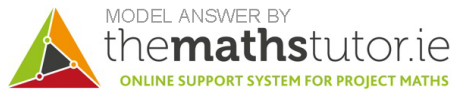
(iii) The student collected the data using a telephone survey. Numbers were randomly chosen from the Dublin area telephone directory. The calls were made in the evenings, between 7 and 9pm. If there was no answer, or if the person who answered did not agree to participate, then another number was chosen at random.
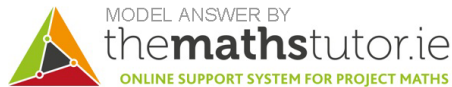
Give **one** possible problem that might make the results of the investigation unreliable. State clearly why the issue you mention could cause a problem.

Question 7 (50 marks)

Gráinne has been out on a river in a kayak and has stopped at a point on one side of the river. However, she wants to get out on the other side. Looking across, she can only see two possible places to get out. On eis a bit up the river from where she is now, and one is farther down the river. Because of the current, she can go faster towards the point down the river than the one up the river.

The situation is shown in the diagram below. The banks of the river are parallel. Gráinne's position is marked *G*. The places where she can get out are marked *A* and *B*. The angles are as shown. The distance from *B* to *A* is 72 metres.

If she travels in a straight line to *A*, Gráinne can go at 0.9m/s and if she travels in a straight line to *B* she can go at 3.2m/s.
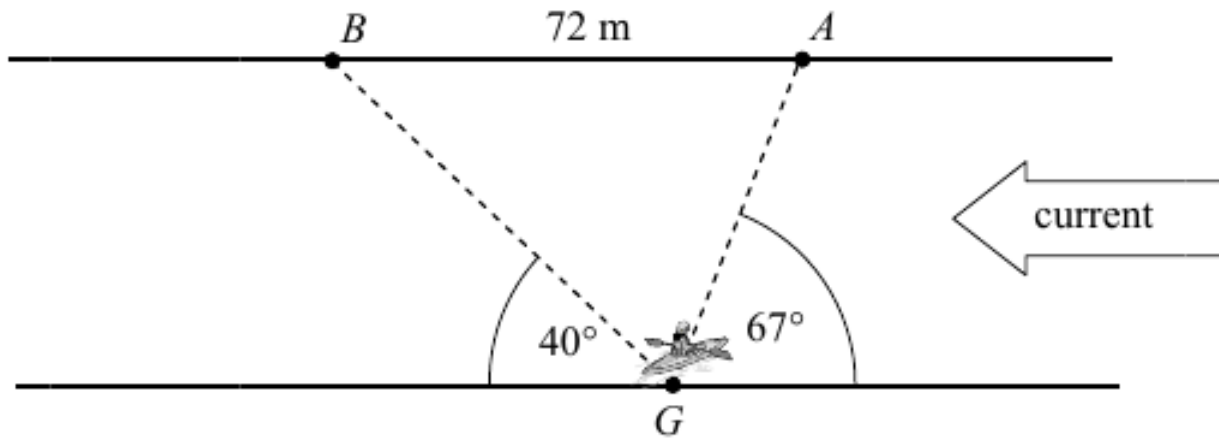
(a) Find the distances from *G* to *A* and from *G* to *B*.

Distance from G to A:

Using the alternate angles theorem we also see that $\angle GBA = 40°$ and $\angle BAG = 67°$. The three angles in the triangle $\triangle AGB$ add up to $180°$. So

$$
\begin{aligned}
\angle AGB + \angle GBA + \angle BAG &= 180° \\
\angle AGB + 40° + 67° &= 180° \\
\angle AGB &= 180° - 40° - 67° \\
\angle AGB &= 73°
\end{aligned}
$$

Now use the sine rule to get

$$
\begin{aligned}
\frac{|AG|}{\sin \angle GBA} &= \frac{|AB|}{\sin \angle AGB} \\
\frac{|AG|}{\sin 40°} &= \frac{72}{\sin 73°} \\
|AG| &= \frac{(\sin 40°)(72)}{\sin 73°} \\
|AG| &= \frac{(0.64278761)(72)}{0.956304756} \\
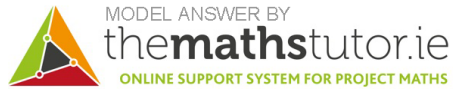|AG| &= 48.4 \text{ metres}
\end{aligned}
$$

correct to one decimal place.

Distance from G to B:

Using the sine rule again

$$\frac{|BG|}{\sin \angle BAG} = \frac{|AB|}{\sin \angle AGB}$$

$$\frac{|BG|}{\sin 67°} = \frac{72}{\sin 73°}$$

$$|BG| = \frac{(\sin 67°)(72)}{\sin 73°}$$

$$|BG| = \frac{(0.920504853)(72)}{0.956304756}$$

$$|BG| = 69.3 \text{ metres}$$
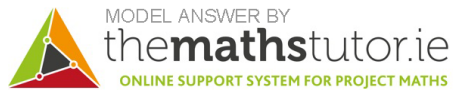
correct to one decimal place.

(b) Find the time it will take to cross by each route.

Time from G to A:

The distance from *G* to *A* is 48.4 metres and her speed is 0.9m/s.
Since $time = \dfrac{distance}{speed}$ we get

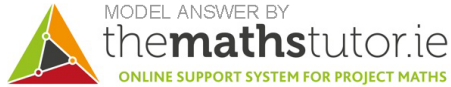$$time = \frac{48.4}{0.9} = 53.78 \text{ seconds}$$

Time from G to B:

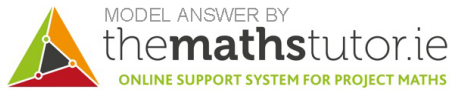The distance from *G* to *B* is 69.3 metres and her speed is 3.2m/s.

Since $time = \dfrac{distance}{speed}$ we get

$$time = \frac{69.3}{3.2} = 21.66 \text{ seconds}$$

(c) Gráinne wants to get home as fast as possible. Give one possible reason why she might **not** choose the faster of the two routes across the river.

There are many possible reasons. Perhaps point *B* is further away from her car or home.

(d) Suppose that the diagram at the start of this question is co-ordinated in such a way that the origin is at *B*, the point *A* lies on the positive *x*-axis, and the units are metres.

(i) Construct such a co-ordinate diagram, showing the positions of *B*, *A*, and *G*.
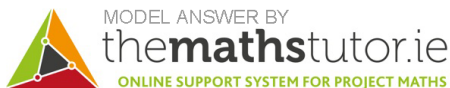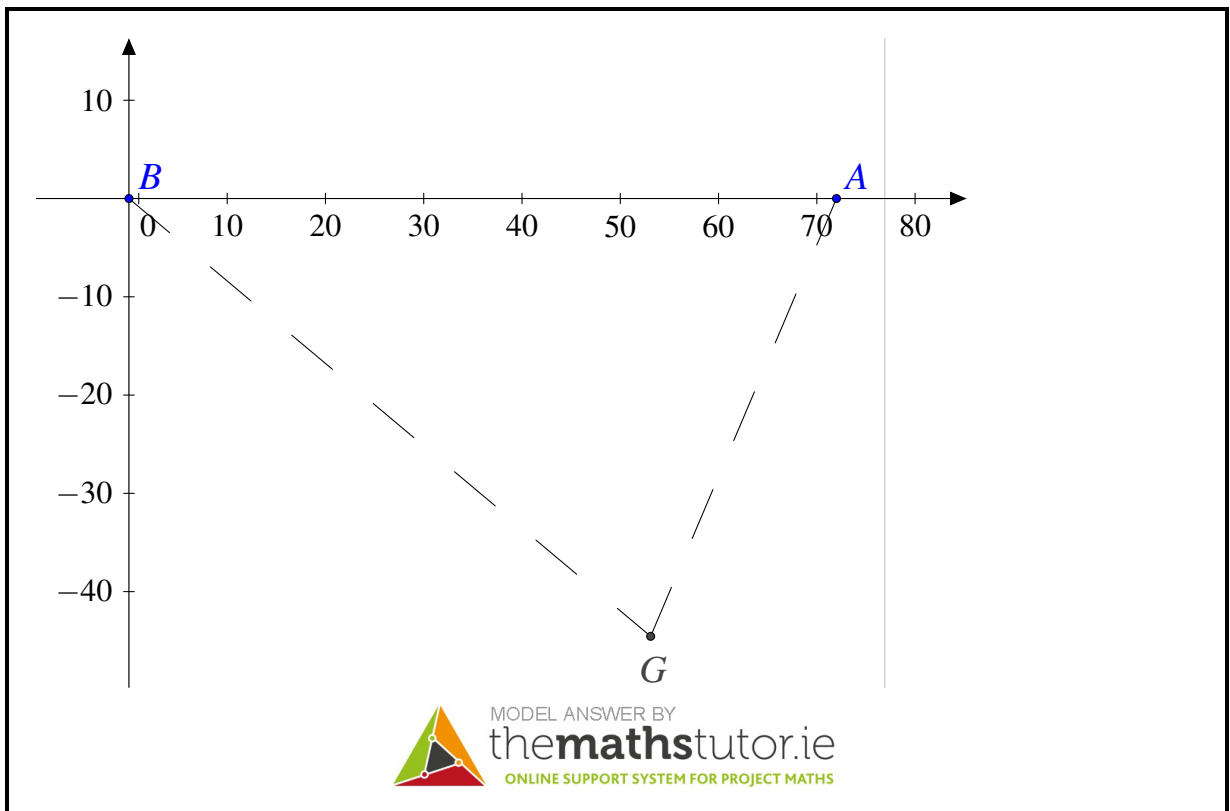
(ii) Calculate the co-ordinates of *G*.

Find a fourth point *X* by adding a line vertically upwards from *G* until it hits the *x*-axis.
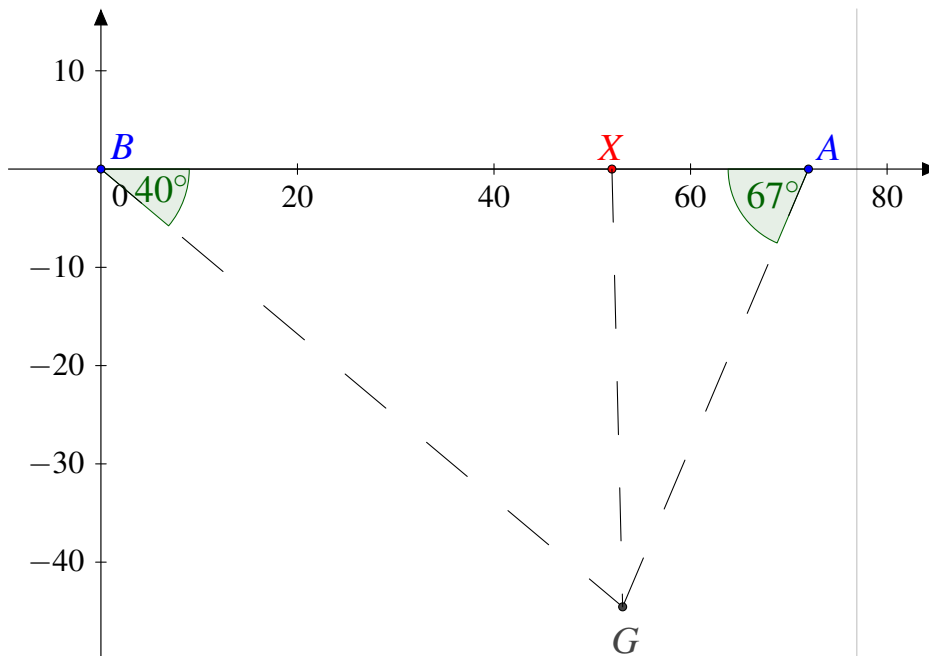
Now we can use the triangle $\triangle BXG$ to find the length of *BX* which is the *x*-coordinate of *G*.

$$\cos 40° = \frac{adjacent}{hypotenuse} = \frac{|BX|}{|BG|} = \frac{|BX|}{69.3}$$

which means $|BX| = (69.3)(\cos 40) = (69.3)(0.766) = 53.09$.

We can use the same triangle to find the length of *GX* which will tell us the *y*-coordinate of *G* (remember to make this negative since the *y*-coordinate is below the *x* axis)

$$\sin 40° = \frac{opposite}{hypotenuse} = \frac{|GX|}{|BG|} = \frac{|GX|}{69.3}$$

which means $|BX| = (69.3)(\sin 40) = (69.3)(0.64278761) = 44.55$.

Thus the co-ordinates of *G* are $(53.09, -44.55)$

Question 8 **(50 marks)**

Old Syllabus