NCCA
An Chomhairle Náisiúnta Curaclaim agus Measúnachta
National Council for Curriculum and Assessment

Project Maths
Tionscadal Mata

Pre-Leaving Certificate Examination

Mathematics (Project Maths)

Paper 2

Ordinary Level (with solutions)

February 2010 2½ hours

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **three** sections in this examination paper:

Section 0	Area & Volume (old syllabus)	50 marks	1 question
Section A	Concepts and Skills	125 marks	5 questions
Section B	Contexts and Applications	125 marks	3 questions

Answer **all nine** questions, as follows:

In Section 0, answer question 1

In Section A, answer questions 2, 3, 4, 5 and 6

In Section B, answer:

question 7

question 8

either question 9A **or** question 9B.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. Extra paper may be used if needed. Label any extra work clearly with the question number and part.

The booklet *Formulae and Tables* may be used.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

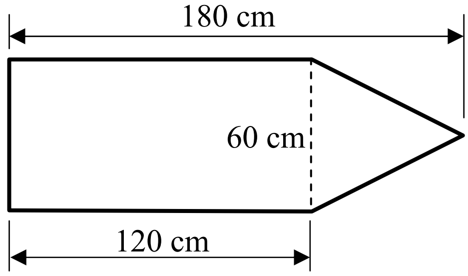
Answers should be given in simplest form, where relevant.

Section 0	Area and Volume (old syllabus)	50 marks

Answer **Question 1** from this section.

Question 1 **(50 marks)**

(a) Find the area of the figure on the right.
Express your answer in m^2.

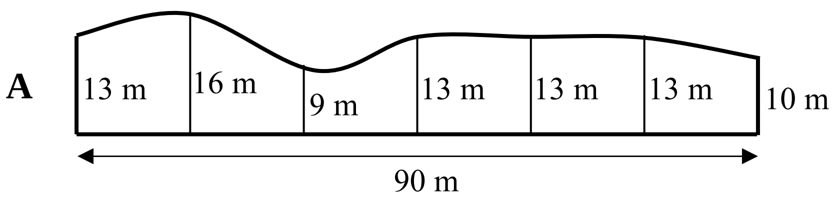


$120 \times 60 = 7200$	$[1.2 \times 0.6 = 0.72]$
$\frac{1}{2} \times 60 \times 60 = 1800$	$[\frac{1}{2} \times 0.6 \times 0.6 = 0.18]$
$7200 + 1800 = 9000$	$[0.72 + 0.18 = 0.9]$
$9000\text{ cm}^2 = 0.9\text{ m}^2$	Ans 0.9 m^2

10 marks (att 3) old syllabus

(b) Use Simpson's rule to determine which of the shapes A or B below has the greater area, and by how much.

10 marks (att 3) old syllabus

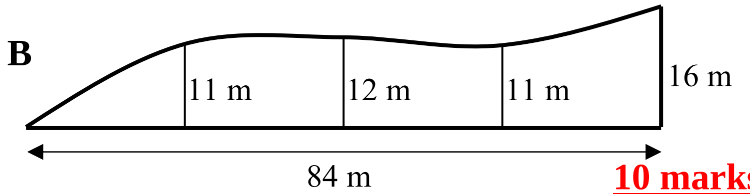


$90 \div 6 = 15$

$$\text{Area A} = \frac{15}{3}\{(13 + 10) + 2(9 + 13) + 4(16 + 13 + 13)\} = .$$

$$5(23 + 42 + 166) = 1175\text{m}^2$$

page	running



10 marks (att 3) old syllabus

$84 \div 4 = 21$

Area B $= \frac{24}{3}\{(0 + 16) + 2(12) + 4(11 + 11)\} = 1024 \text{ m}^2$.

Thus shape A has the greater area. It is greater by $1175 - 1024 = 151 \text{ m}^2$.

(c) **(i)** A cone has radius r cm and vertical height $\frac{K}{\pi}$ cm. **10 marks (att 3) old syllabus**

If the volume of this cone is $r^2 \text{ cm}^3$, find K.

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 \frac{K}{\pi} = \frac{K}{3} r^2$

If $V = r^2$ then $\frac{K}{3} = 1$, which means K = 3 Ans K = 3

10 marks (att 3) old syllabus

(ii) The vertical height of a cylinder is h cm and the radius of its base is $h\sqrt{2}$ cm.

If the total volume of four such cylinders is 125π, find the value of h.

$V = \pi r^2 h = \pi (h\sqrt{2})^2 h = 2\pi h^3$

$4V = 8\pi h^3 = 125\pi \rightarrow h^3 = \frac{125}{8} \rightarrow h = \frac{5}{2} = 2.5$ cm Ans.

Section A	Concepts and Skills	125 marks

Answer **all five** questions from this section.

Question 2 **(25 marks)**

Sarah is on a TV game show called *Take the Money and Run*.

She has won €10 000 so far. She now has four options:

Option 1: Leave the show with €10,000 – that is, *Take the Money and Run*.

Option 2: Play on and take a 50% chance of winning €50 000

Option 3: Play on and take a 30% chance of winning €75 000

Option 4: Play on and take a 20% chance of winning €100 000

If she plays on and does not win the higher amount, she loses the €10 000.

(a) Calculate the *expected value* of Sarah's winnings for each of the four options.

Option 1:	10,000 × 1 => €10,000	**5 A***
Option 2:	50,000 × 0.5 => €25,000	**5 B***
Option 3:	75,000 × 0.3 = €22,500	**5 B***
Option 4:	100,000 × 0.2 => €20,000	**5 B***

(b) What would you advise Sarah to do, and why? **5 B**

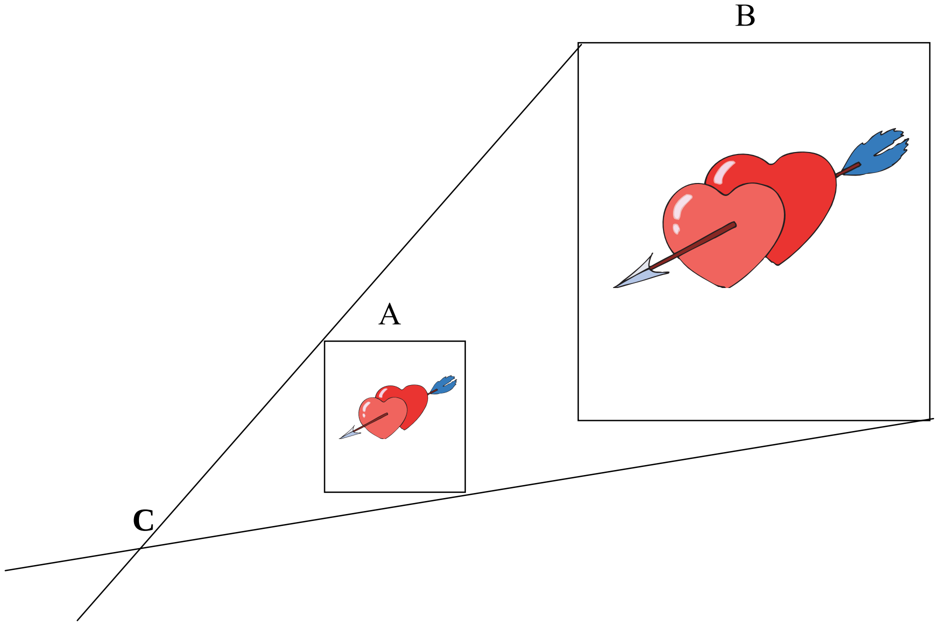
Possible answer: Take the €10,000 because it is certain.

OR take Option 2 because it offers the best expected value (although it is not certain)

page	running

Question 3 **(25 marks)**

The ray method was used to enlarge a design for a Valentine card. The original is labelled A and the image is labelled B.



(a) Find the centre of enlargement. Answer: Construction lines (as shown, or suitable alternatives) to get point C on the diagram. **10 B**

(b) Find the scale factor of the enlargement. Show your work. **5 B**

Measurements taken from diagram to give 2.5 (range 2.3-3.0) as the scale factor.

(c) Calculate the ratio $\dfrac{\text{Area of drawing B}}{\text{Area of drawing A}}$. Give your answer correct to one decimal place.

10 B*

Either measurements taken from each card design to find areas and ratio calculated as 6.3 : 1 (range 5.3 : 1 to 9 : 1, but consistent with answer given to (b) above)

OR

Square of scale factor used to get 6.25 and hence Answer = 6.3 : 1 (range consistent with answer to part (b) above)

Question 4 **(25 marks)**

Two spinners, each with four equal segments numbered 1 to 4, are spun.

(a) Using a list, table, tree diagram, or otherwise, show all the possible outcomes. **10 C**

Answer in the from of a tree diagram with main branches labelled 1, 2, 3 and 4 each with four smaller branches similarly labelled, to give the sixteen outcomes:

(1, 1) (1, 2) (1, 3) (1, 4) (2, 1) (2, 2) (2, 3) (2, 4)

(3, 1) (3, 2) (3, 3) (3, 4) (4, 1) (4, 2) (4, 3) (4, 4)

OR

A grid/matrix/list containing the sixteen outcomes above.

(b) If the spinners are fair, what is the probability of getting two fours? **5 B**

Answer $\frac{1}{16}$ [or appropriate fraction/decimal/percentage based on the result given for part (a) above]

(c) Jason thinks that one of the spinners is not fair. **10 C**
Describe an experiment that he could do to find out whether the spinner is fair.

Spin it a very large number of times to see if each number comes up about the same number of times (about one quarter of the time). [Reference to relative frequency of each number being ¼ for a very large number of spins.]

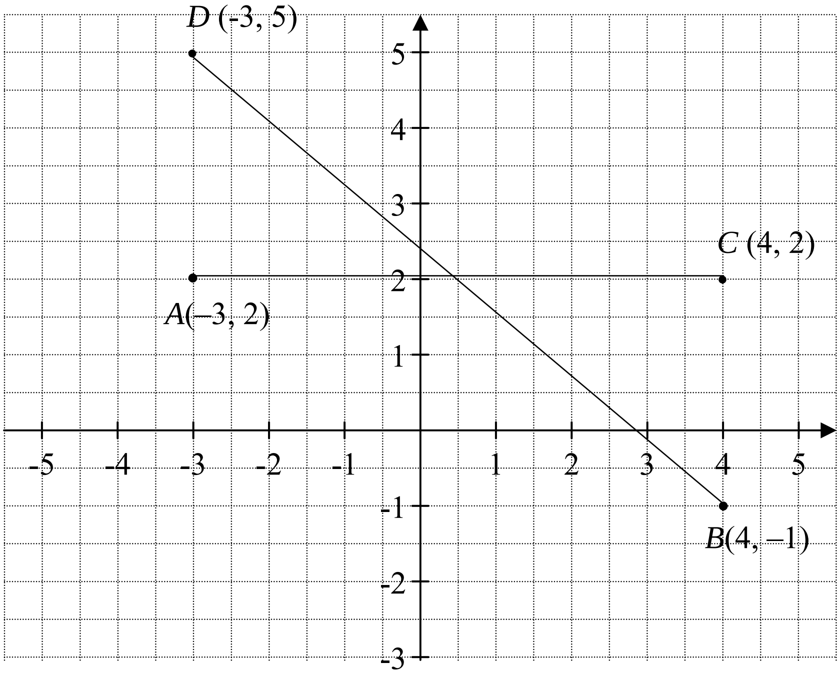
This approach could also be used to provide data on all outcomes in a given 'sample' and then carry out a hypothesis test to see whether there is evidence that the spinner is not fair.

page	running

Question 5 **(25 marks)**

(a) Two points $A(-3, 2)$ and $B(4, -1)$ are shown on the diagram below. Plot two suitable points C and D so that $ABCD$ is a parallelogram. Label the points and write down their co-ordinates.

5 B



(b) By performing suitable calculations, show that the figure you have drawn is indeed a parallelogram. **10 C**

Slope of side AB = $\frac{-1-2}{4+3} = -\frac{3}{7}$ Slope of side DC = $\frac{2-5}{4+3} = -\frac{3}{7}$

Thus AB and DC are parallel.

AD and BC are both parallel to the Y axis (or perpendicular to the X axis), so they are parallel++ to each other.

Hence ABCD is a parallelogram.

Other valid methods accepted. Example, AD and BC parallel as above, and their lengths are the same (read from grid). Therefore, ABCD is a parallelogram).

(c) Verify that the diagonals of the parallelogram bisect each other. **10 B**

Diagonals intersect at (½, 2) [from diagram]

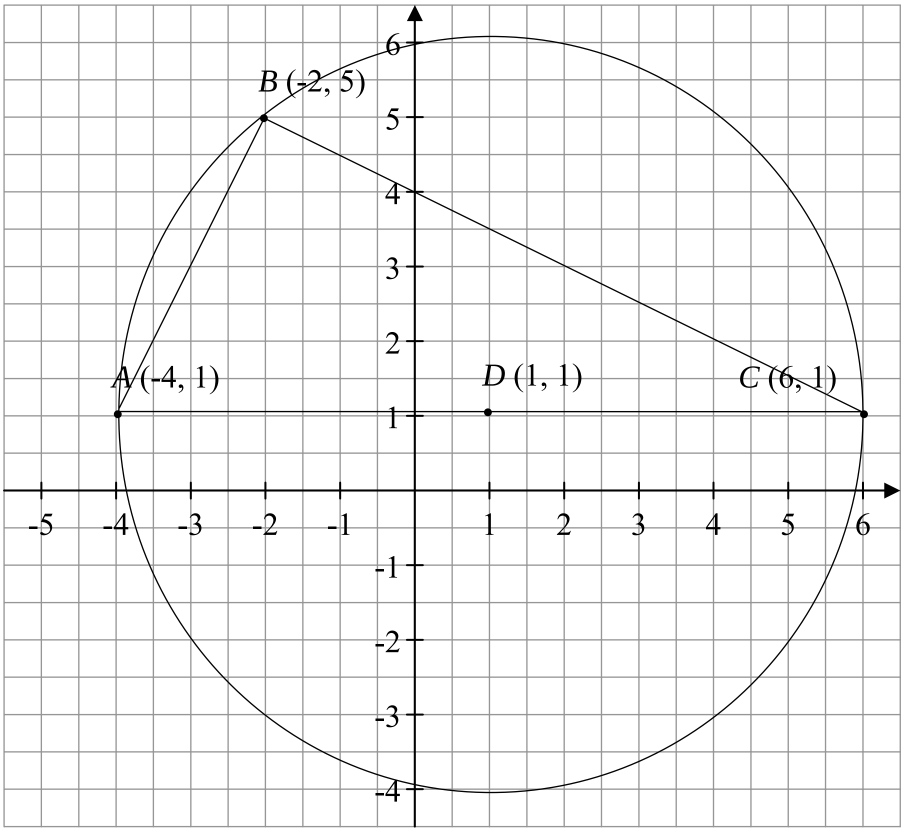
Midpoint of AC: $\left(\frac{-3+4}{2}, \frac{2+2}{2}\right)$ = (½, 2) and mid-point of DB: $\left(\frac{-3+4}{2}, \frac{5-1}{2}\right)$ = (½, 2)

This is the same point, hence they bisect each other.

Other valid methods accepted. Example: calculation of diagonal lengths and half lengths.

Question 6 **(25 marks)**

(a) On the diagram below, show the triangle *ABC*, where *A* is (–4, 1), *B* is (–2, 5) and *C* is (6, 1). **5 B**



(b) Find *D*, the midpoint of [*AC*], and label this point on the diagram. **5 B**

Mid-point of AC is *D*(1, 1) [reading from grid] OR calculated as $\left(\frac{-4+6}{2}, \frac{1+1}{2}\right) = (1,1)$

(c) Hence, construct on the diagram the circle with diameter [*AC*]. See circle on diagram. **5 B**

(d) Show that angle $\angle ABC$ is a right angle. **10 C**

Since length of DB = $\sqrt{[(5-1)^2 + (-2-1)^2]} = \sqrt{25} = 5$ = radius $\Rightarrow$ circle passes through point B. AC is a diameter $\Rightarrow |\angle ABC|$ in semi-circle is 90°.

OR

Slope of side AB: $\frac{5-1}{-2+4} = \frac{4}{2} = 2$ Slope of side BC: $\frac{5-1}{-2-6} = \frac{4}{-8} = -\frac{1}{2}$

Product of slopes = 2 × -½ = -1 and so sides AB and BC are perpendicular. Hence the $|\angle ABC| = 90°$.

page	running

Section B	Contexts and Applications	125 marks

in this section, answer Question 7 and Question 8, and **either** Question 9A **or** Question 9B.

Question 7 **Probability and Statistics** **(40 marks)**

(a) A teacher asked the students in her class to estimate the height of the church opposite the school in metres.

The stem-and-leaf diagram shows all the results:

3	5	9				
4	0	2	6	8	8	
5	3	3	5	7	7	9
6	0	5	5	5		
7	4	8				
8	2	7				

Key: 3 | 5 represents 35 m

(i) How many students were in the class? **21** 5 A

(ii) Describe the **shape** of the distribution of the data. 5 B

The shape is reasonably normal, but slightly skewed to the right.

(iii) What was the median estimate? **57 m** 5 A

(iv) Explain the answer to part **(iii)** to someone who does not know what the word "median" means. 5 B

The median is the middle value when they are all arranged in increasing order. If there are two 'middle' numbers, you use the average of these as the median.

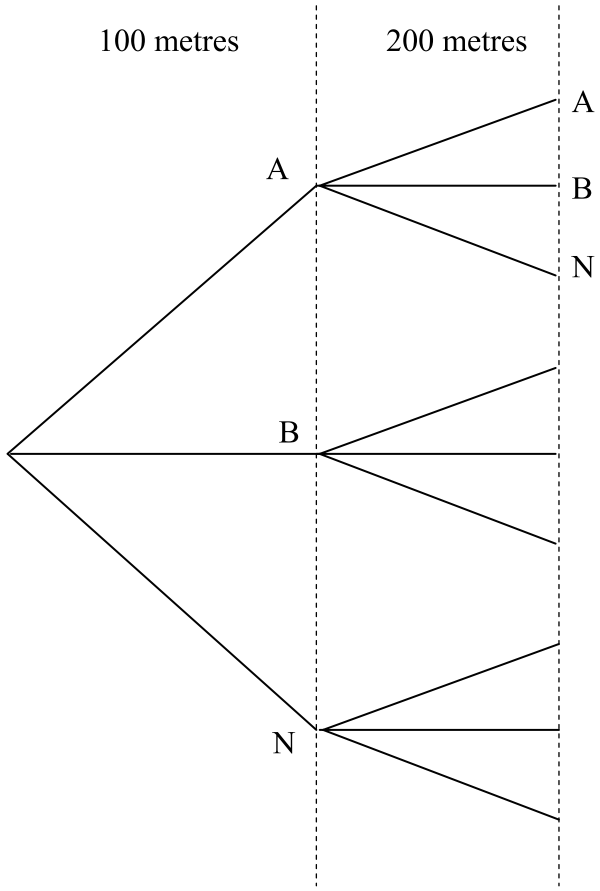
(b) Alex and Bobby are running in the final of a 100 metre race and a 200 metre race.
The probabilities of each of them winning each race are given in the table below.
The probability that neither of them wins the 100 metre race is also given.

	Alex	Bobby	Neither
100 metre race	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{7}{12}$
200 metre race	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{3}{8}$

(i) Complete the table above, by inserting the probability that someone other than Alex or Bobby wins the 200 metre race. **5 A**

(ii) Using the tree diagram, or otherwise, complete the list of outcomes below.
For example, the outcome that Alex wins the first race and the second race is recorded as (A, A), as shown. **10 C**

Write the probability of each outcome in the space beside it.



Outcome	Probability
(A, A)	$\frac{1}{24}$
(A, B)	$\frac{1}{16}$
(A, N)	$\frac{1}{16}$
(B, A)	$\frac{1}{16}$
(B, B)	$\frac{3}{32}$
(B, N)	$\frac{3}{32}$
(N, A)	$\frac{7}{48}$
(N, B)	$\frac{7}{32}$
(N, N)	$\frac{7}{32}$

(c) What is the probability that Alex and Bobby win a race each?

$$\frac{1}{16} + \frac{1}{16} = \frac{1}{8}$$

5 B

page	running

Question 8 **Geometry and Trigonometry** **(40 marks)**

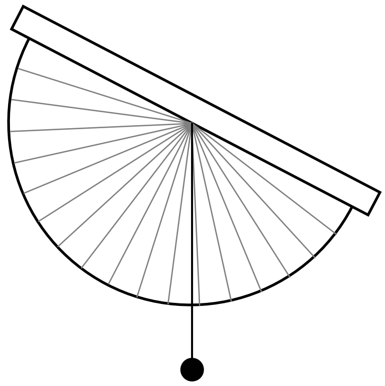
The students mentioned in Question **7(a)** above went to measure the height of the church.

(a) Peter explained his group's method:

"We made a clinometer from a protractor, a pen tube, some thread and a weight.

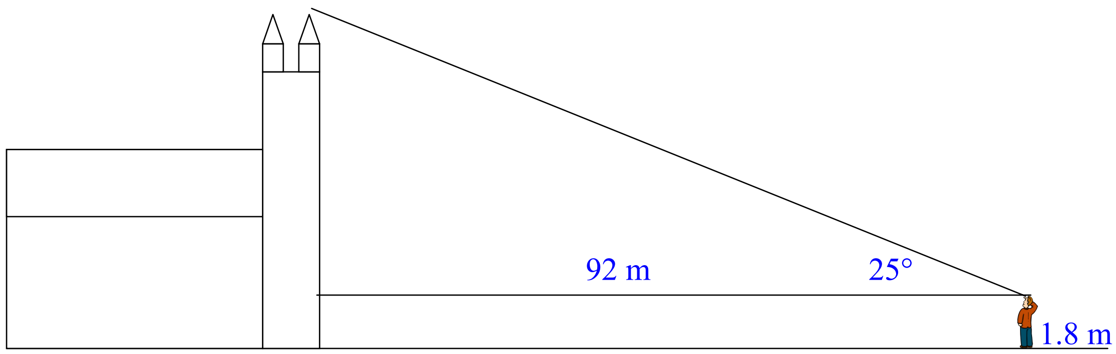
We measured the distance from here to the church and it was 92 metres.

We made sure the ground was flat. Then we used the clinometer to look up at the top of the spire of the church. The weight had moved from 90° to 65°, so we knew the angle up was 25°. We worked out the height from that. But we had to remember to add on my height of 1·8 metres at the end"



(i) On the diagram below, show the measurements that Peter's group made.

5 B



(ii) Show how Peter's group used these measurements to find the height of the church.

$\tan 25 = \dfrac{h}{92}$ so $h = 92 \tan 25 = 42.9$

Add Peter's height (1.8)

Height of church is 44.7 m

10 C*

(b) Hannah was in a different group from Peter. She explained her group's method for finding the height of the church: **15 C***

"It was really sunny and we used the shadows cast by the sun.

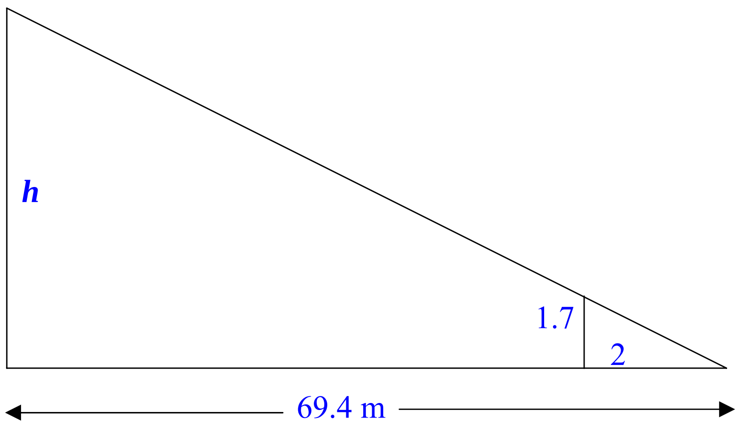
Amy stood with her back to the sun and we used a tape measure to measure Amy's shadow along the ground from the tips of her toes to top of her shadow's head. We also measured Amy's height and recorded the results in the table.

Then we recorded the length of the shadow cast by the church. We measured along the ground from the base of the church out to the end of its shadow and recorded this measurement."

Amy's shadow	2 m
Amy's height	1·7 m
Church's shadow	69·4 m

Show how Hannah's group used their results to calculate the height of the church.

$$\frac{h}{69.4} = \frac{1.7}{2} \Rightarrow h = 58.99 \text{ m}$$



(c) The church is actually 50 metres high. Calculate the percentage error in each group's result.

Peter's group	Hannah's group
$\frac{50 - 44.7}{50} \times 100$	$\frac{58.99 - 50}{50} \times 100$
$= 10.6\%$ **5 B**	$= 17.98\%$ **5 B**

page	running

Question 9A **Probability and Statistics** **(45 marks)**

Oxygen levels in a polluted river were measured at randomly selected locations before and after a clean-up. These results are given in the table.

Before (mg/l)				After (mg/l)			
20	25	20	9	26	10	10	9
23	23	10	11	11	15	11	11
2	10	11	5	3	8	11	4
11				13			

(a) Construct a back-to-back stem-and-leaf plot of the above data. **10 C**

Before	Stem	After
9 5 2	0	3 4 8 9
1 1 1 0 0	1	0 0 1 1 1 1 3 5
5 3 3 0 0	2	6

(b) State **one difference** and **one similarity** between the distributions of the measurements before and after cleanup.

Difference: The measurements made afterwards are generally lower than the measurements taken before cleanup.

Similarity: The two sets of measurements have the same range:

Before: 25 – 2 = 23 After: 26 – 3 = 23 **5 B**

(c) Perform a *Tukey Quick Test* on the data to see if there is evidence that the clean-up worked.

No. in higher set (after) which are bigger than the largest value in lower set (before) = 1

No. in lower set (before) which are smaller than the lowest value in higher set (after) = 1

Total = 2. Since this is less than 7, we reject the hypothesis that they are different.

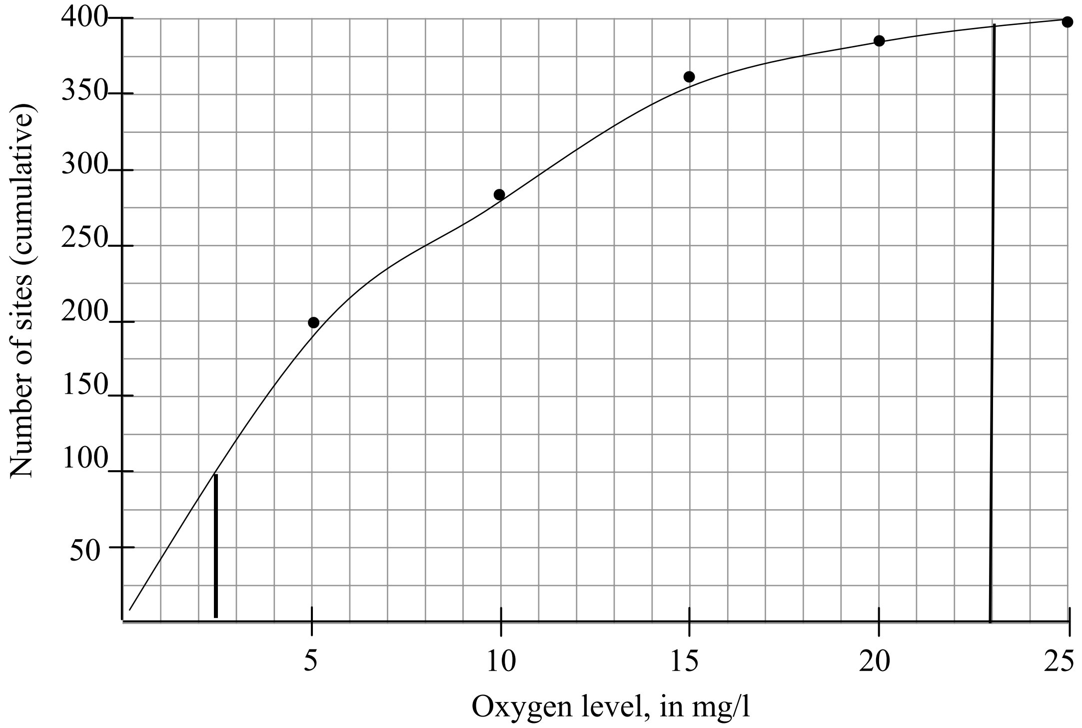
Thus, there is not sufficient evidence that the cleanup worked. **5 B**

(d) Oxygen levels were measured at 400 different sites on different rivers.

The measurements are summarised in the table below. **10 C**

Oxygen level (mg/l)	<5	<10	<15	<20	<25
Number of sites	200	285	365	385	400

Draw a cumulative frequency diagram to represent this data, using the scale indicated.



(e) Use your cumulative frequency curve to estimate: **5 A**

(i) the number of sites with oxygen levels below 23 mg/l — 395± 10

(ii) the interquartile range 11 - 2.4 — 8.6

5 B

(f) An oxygen level of between 2 and 8 mg/l indicates that the site is *moderately polluted*. If a site is chosen at random from the 400 sites in part **(d)**, what is the probability that it is moderately polluted? **5 B**

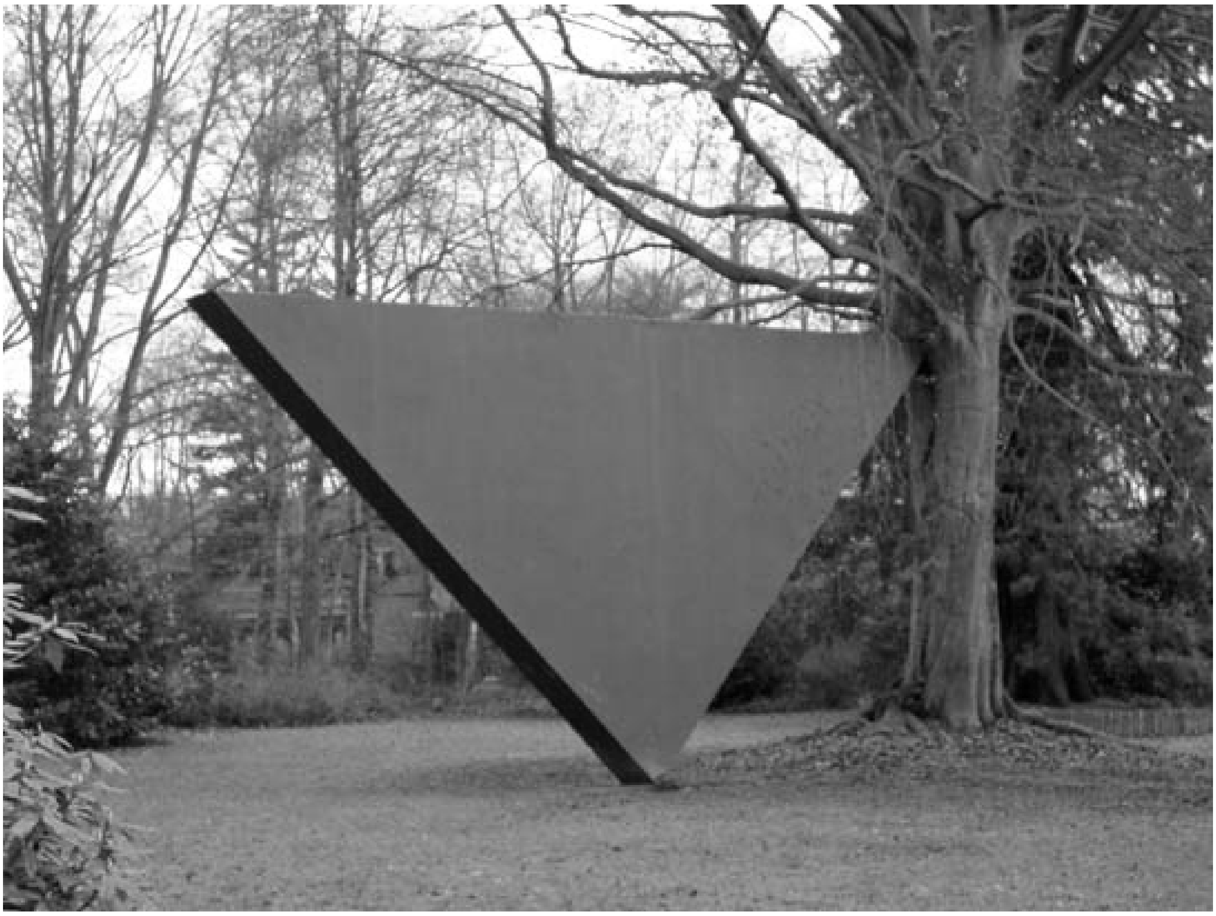
No. sites moderately polluted = 250 – 80 = 170

Probability of selecting one of these from the 400 sites = 170 ÷ 400 = 0.425

page | running

Question 9B **Geometry and Trigonometry** **(45 marks)**

Noel and Anne were taking part in a mathematics competition with other students from the *Project Maths* schools. They were finding the area of the face of the triangular sculpture shown below.



Noel said: "We should measure the height and the base of the triangle. Then use the formula that says the area is half the base by the height."

Sarah said: "Ok, but how do we know which side is the base?"

Noel said: "It doesn't matter, because of the theorem we did."

(a) State the theorem that Noel is talking about. **5 B**

In a triangle, base times height does not depend on the choice of base. [Theorem 16]

Could be stated as: In any triangle, the base multiplied by the altitude does not depend on the choice of base [is the same for all bases; is constant].

(b) Noel and Sarah trace the triangle from the photograph onto a page to find its area. Their drawing is shown here. By making suitable measurements on the drawing, verify the theorem you stated in part **(a)**.

Measurements for the lengths of the three sides and the corresponding heights.

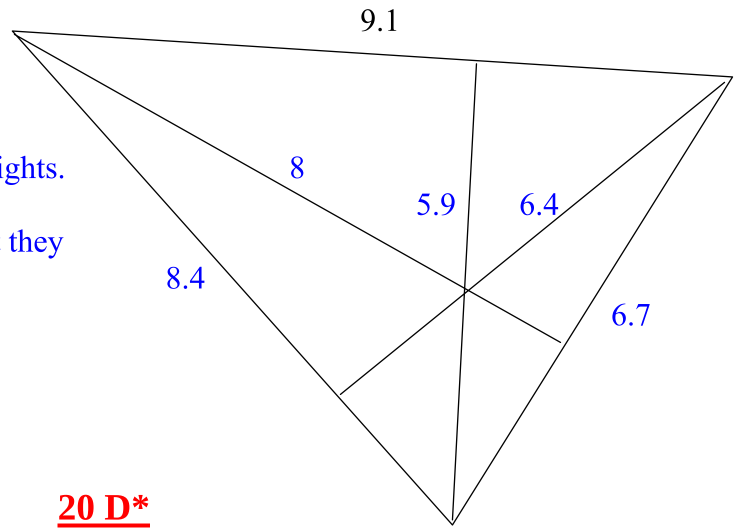
Calculate the products and show that they are (approximately) equal.

$6.7 \times 8 = 53.6$

$9.1 \times 5.9 = 53.69$

$8.4 \times 6.4 = 53.76$

20 D*



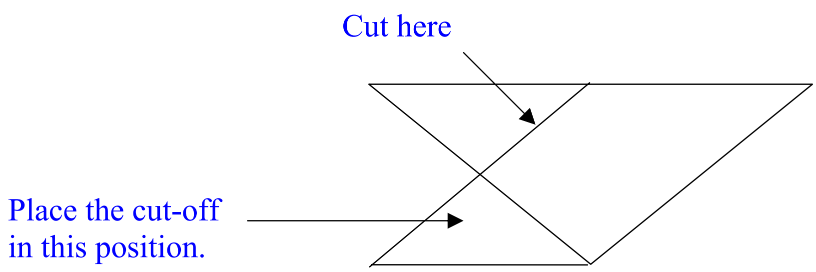
(c) Suppose that the drawing were a true representation of the face of the sculpture.
If each centimetre in the drawing represents 70 centimetres in reality, find the area of the face of the sculpture.

$\frac{1}{2} \times (6.7 \times 70) \times (8 \times 70) = 131320 \text{ cm}^2 = 13.132 \text{ m}^2$ **10 B***

(d) The true shape of the face of the sculpture is shown below. The people who made it have changed their minds and now want a parallelogram instead! **10 B**

Show how the triangle could be turned into a parallelogram by making **one cut** and moving one of the two pieces. You should make it clear exactly where the cut is to be made, and show the new position of the piece moved.

Cut along the line joining the mid-points of two sides, as shown. Move the piece cut off to form the parallelogram, as shown.



page	running

You may use this page for extra work

You may use this page for extra work

page	running

Note to readers of this document:

This pre-Leaving Certificate paper is intended to help teachers and candidates in the 24 *Project Maths* initial schools prepare for the June 2010 examination. The content and structure of the paper do not necessarily reflect the 2011 or subsequent examinations in the initial schools or in all other schools.

Mathematics (Project Maths) – Paper 2

Pre-Leaving Certificate Paper – Ordinary Level
February 2010