2013 Leaving Cert Ordinary Level
Official Sample Paper 1

Section A	Concepts and Skills	150 marks

Question 1 **(25 marks)**

(a) The same number can be represented in different ways. Write two more representations of each of the numbers in the table below, by choosing only from the following list. Note: some of the representations may be approximations.

-36	9%	$\frac{1}{9}$	$\frac{1}{62}$	$\frac{1}{81}$
16^5	$(\sqrt{3})^4$	$2^4 \times 2^5$	$(0.25)^{-10}$	2.8%
36%	900%	0.027	1.05×10^6	1.05×10^{-5}

6^{-2}		
$81^{\frac{1}{2}}$		
2^{20}		

Firstly, we note the following

$$6^{-2} = \frac{1}{6^2} = \frac{1}{36} = 0.02777\cdots$$
$$81^{\frac{1}{2}} = \sqrt{81} = 9 = 3^2 = (\sqrt{3})^4$$
$$2^{20} = 1{,}048{,}576 = 4^{10} = 16^5$$

Now we can fill in our table from the choices above (some are approximations).

6^{-2}	0.027	2.8%
$81^{\frac{1}{2}}$	900%	$(\sqrt{3})^4$
2^{20}	1.05×10^6	16^5

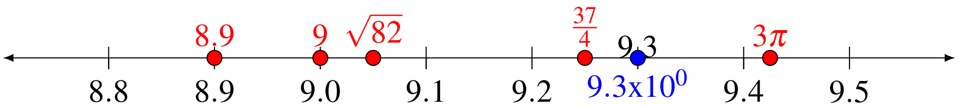
(b) Giving the number line below an appropriate scale, mark the following numbers on it:

$$\sqrt{82}, \quad 3\pi, \quad 8.9, \quad 9, \quad \frac{37}{4}, \quad 9.3\times 10^0$$

Firstly, we will write all of the above numbers in decimal format, correct to two decimal places.

$$\begin{aligned}\sqrt{82} &= 9.06\\ 3\pi &= 9.43\\ 8.9 &= 8.90\\ 9 &= 9.00\\ \tfrac{37}{4} &= 9.25\\ 9.3\times 10^0 &= 9.30\end{aligned}$$

Our number line becomes:



Question 2 **(25 marks)**

(a) A sum of €5,000 is invested in an eight-year government bond with an annual equivalent rate (AER) of 6%. Find the value of the investment when it matures in eight years' time.

We know that the future value F of a present investment P invested for n years at a rate of $i\%$ is given by

$$F = P\left(1+\frac{i}{100}\right)^n$$

In our case, $P =$€5000, $n = 8$ and $i = 6$. This gives $F = 5000(1.06)^8$ which is €7,989.24 correct to the nearest cent.

(b) A different investment bond gives 20% interest after 8 years. Calculate the AER for this bond.

We know that the future value of our investment is the original amount plus the 20% interest, so if our investment is P then the value after 8 years is $1.2P$
We wish to find a value for i such that

$$1.2P = P\left(1+\frac{i}{100}\right)^8$$

We divide across by P to get $1.2 = \left(1+\frac{i}{100}\right)^8$. We can then solve for i:

$$\begin{aligned} 1.2 = \left(1+\frac{i}{100}\right)^8 \quad &\Leftrightarrow \quad \sqrt[8]{1.2} = 1+\frac{i}{100} \\ &\Leftrightarrow \quad \sqrt[8]{1.2}-1 = \frac{i}{100} \\ &\Leftrightarrow \quad 100\left(\sqrt[8]{1.2}-1\right) = i \end{aligned}$$

From a calculator, this gives that the equivalent AER for this bond is $i = 2.305\%$ correct to three decimal places.

Question 3 **(25 marks)**

(a) Write each of the following complex numbers in the form $a+bi$, where $i^2 = -1$.

(i) $z_1 = (3+2i)(2-5i)$

$$z_1 = (3+2i)(2-5i) = 6-15i+4i+10 = 16-11i$$

(ii) $z_2 = (5+4i)(17-13i)-(5+3i)(17-13i)$

$$\begin{aligned} z_2 &= (5+4i)(17-13i)-(5+3i)(17-13i) \\ &= [(5+4i)-(5+3i)](17-13i) \\ &= i(17-13i) \\ &= 13+17i \end{aligned}$$

(iii) $z_3 = \left(\frac{5}{2}+\frac{7}{2}i\right)^2 - \left(\frac{5}{2}+\frac{1}{2}i\right)^2$

$$\begin{aligned} z_3 &= \left(\frac{5}{2}+\frac{7}{2}i\right)^2 - \left(\frac{5}{2}+\frac{1}{2}i\right)^2 \\ &= \left(\frac{25}{4}+\frac{35}{2}i-\frac{49}{4}\right) - \left(\frac{25}{4}+\frac{5}{2}i-\frac{1}{4}\right) \\ &= 0+15i-12 \\ &= -12+15i \end{aligned}$$

(iv) $z_4 = 1+i+i^2+i^3$

$$z_4 = 1+i+i^2+i^3 = 1+i+(-1)+(-i) = 0+0i$$

(b) Which of z_1 and z_2 above is farther from 0 on an Argand diagram? Justify your answer.

Answer: z_2 is farther from 0.

Justification: The distance of a complex number from 0 is known as the modulus. Hence, the complex number with the greatest modulus will be the one which is farthest from 0. Calculating these we find that z_2 is farther than z_1 from 0:

$$|z_1| = |16-11i| = \sqrt{(16)^2+(-11)^2} = \sqrt{377}$$
$$|z_2| = |13+17i| = \sqrt{(13)^2+(17)^2} = \sqrt{458}$$

Question 4 **(25 marks)**

(a) Solve the equation $x^2-6x-23=0$. Give your answer in the form $a \pm b\sqrt{2}$ where $a, b \in \mathbb{Z}$.

We will use the quadratic formula to solve this equation.

$$
\begin{aligned}
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad &\Leftrightarrow \quad x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-23)}}{2(1)} \\
&\Leftrightarrow \quad x = \frac{6 \pm \sqrt{36+92}}{2} \\
&\Leftrightarrow \quad x = \frac{6 \pm \sqrt{128}}{2} \\
&\Leftrightarrow \quad x = \frac{6 \pm \sqrt{64}\sqrt{2}}{2} \\
&\Leftrightarrow \quad x = \frac{6 \pm 8\sqrt{2}}{2} \\
&\Leftrightarrow \quad x = 3 \pm 4\sqrt{2}
\end{aligned}
$$

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(b) Solve the simultaneous equations:

$$
\begin{aligned}
2r - s &= 10 \\
rs - s^2 &= 12
\end{aligned}
$$

We will need to write one variable in terms of the other from one of the two equations and substitute that value into the second equation. We will write r in terms of s from the first equation, and substitute that value of r into the second equation and then solve for s. The first equation tells us that $2r = 10 + s$ or $r = 5 + \frac{1}{2}s$.

The second equation now reads $\left(5 + \frac{1}{2}s\right)s - s^2 = 12$ which expands to $5s + \frac{1}{2}s^2 - s^2 = 12$, or $5s - \frac{1}{2}s^2 = 12$. We can multiply both sides by 2 and rearrange the equation to get $s^2 - 10s + 24 = 0$. This factorises into $(s-4)(s-6) = 0$, i.e. $s = 4$ or $s = 6$.

We finally use these values to solve for r. If $s = 4$, then $r = 5 + \frac{1}{2}(4) = 7$. If $s = 6$, then $r = 5 + \frac{1}{2}(6) = 8$. Thus our solutions are

$$r = 7, s = 4 \qquad \text{or} \qquad r = 8, s = 6$$

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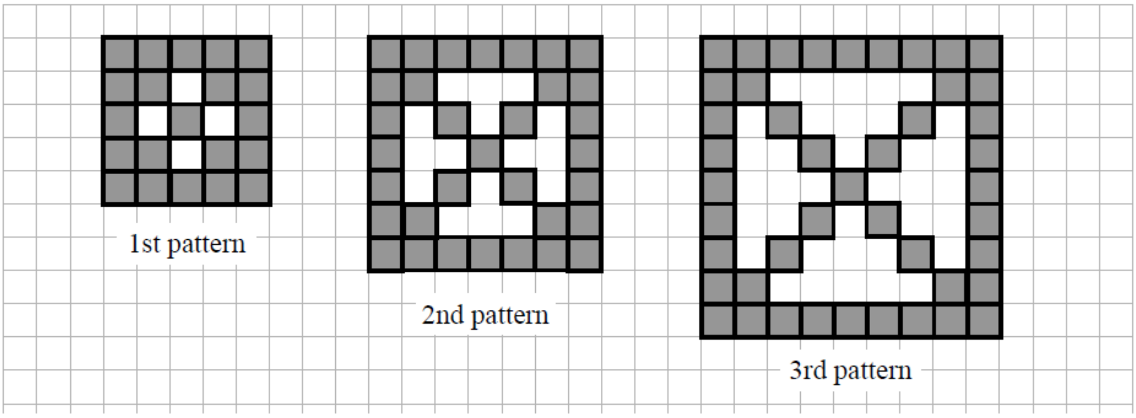
Section B	Contexts and Applications	150 marks

Answer **all three** questions from this section

Question 5 **(50 marks)**

Síle is investigating the number of grey square tiles needed to make patterns in a sequence. The first three patterns are shown below, and the sequence continues in the same way. In each pattern, the tiles form a square and its two diagonal. There are no tiles in the white areas in the pattern - there are only the grey tiles.



(a) In the table below, write the number of tiles needed for each of the first five patterns.

In pattern 1 the number of diagonal tiles is 9. This comes from 5 on one diagonal and 4 on the other, since the one in middle has already been counted. In pattern 2 the number of diagonal tiles is 13: 7 on one diagonal and 6 on the other. In pattern 3 the number of diagonal tiles is 17: 9 on one diagonal and 8 on the other.

For pattern 4, we should have 11 on one diagonal and 10 on the other, so a total of 21 diagonal tiles. Similarly, for pattern 5, we will have 13 on one diagonal and 12 on the other, giving a total of 25 diagonal tiles.

When we have counted the two diagonals, we need to look at the four edges of the square. Remember though that the corner tiles of the square have already been counted when we considered the diagonals. In pattern 1, each edge has 3 remaining tiles, so a total of 12 tiles. In pattern 2, each edge has 5 remaining tiles, so a total of 20 tiles. In pattern 3, each edge has 7 remaining tiles, so a total of 28 tiles.

For pattern 4, each edge will have 9 remaining tiles, so a total of 36 tiles. For pattern 5, each edge will have 11 remaining tiles, so a total of 44 tiles.

Adding the tiles in the diagonals to the remaining tiles on the edges of the square gives us the total tiles needed.

Pattern	1	2	3	4	5
No. of Tiles	21	33	45	57	69

(b) Find, in terms of n, a formula that gives the number of tiles needed to make the nth pattern.

Consider the above sequence of the number of tiles needed: 21, 33, 45, 57, 69. There is a constant common difference of $d = 12$, so this series is an arithmetic one. The first term is $a = 21$, so the general expression for the nth term is given by

$$T_n = a + (n-1)d = 21 + (n-1)12 = 12n + 9$$

(c) Using your formula, or otherwise, find the number of tiles in the tenth pattern.

Setting $n = 10$ in the above formula gives us that $T_{10} = 12(10) + 9 = 129$. Thus, the tenth pattern will need 129 tiles.

(d) Síle has 399 tiles. What is the biggest pattern in the sequence that she can make?

We wish to find the largest positive whole number n such that $T_n \leqslant 399$. Substituting our formula, we get

$$12n + 9 \leqslant 399 \quad \Leftrightarrow \quad 12n \leqslant 390 \quad \Leftrightarrow \quad n \leqslant \frac{390}{12} = 32.5$$

The largest positive whole number which satisfies this inequality is $n = 32$. Thus, the biggest pattern that Síle can make is pattern 32.

We can verify this by noting that $T_{32} = 12(32) + 9 = 393$ and $T_{33} = 12(33) + 9 = 405$.

(e) Find in terms of n a formula for the total number of tiles in the first n patterns.

To find the total number of tiles need to make the first n pattens, we need to find the sum of the first n terms in the sequence T_n. The formula for this S_n is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where a and d are the first term and common difference of T_n respectively. From part **(b)**, we know that $a = 21$ and $d = 12$, so

$$S_n = \frac{n}{2}[2(21) + (n-1)12] = \frac{n}{2}[30 + 12n] = 6n^2 + 15$$

(f) Síle starts at the beginning of the sequence and makes as many of the patterns as she can. She does not break up the earlier patterns to make the new ones. For example, after making the first two patterns, she has used up 54 tiles $(21 + 33)$. How many patterns can she make in total with her 339 tiles?

As with part **(d)** above, we wish to find the greatest positive whole number such that $S_n \leqslant 399$. Substituting our formula for S_n, we get

$$6n^2 + 15n \leqslant 399 \quad \Leftrightarrow \quad 6n^2 + 15n - 399 \leqslant 0 \quad \Leftrightarrow \quad 2n^2 + 5n - 133 \leqslant 0$$

We will consider the function $2x^2 + 5x - 133$, and find the largest positive whole number for which the function is negative. We can factorise the function as $2x^2 + 5x - 133 = (2x + 19)(x - 7)$, and so the function will cross the x-axis when $x = -\frac{19}{2}, 7$. Thus, the function will be negative when $-\frac{19}{2} \leqslant x \leqslant 7$.

The largest positive whole number which satisfies this inequality is $n = 7$. Thus, the largest number of patterns that Síle can make is 7 patterns. We can verify this by noting that $S_7 = 6(7)^2 + 15(7) = 399$.

Question 6 **(50 marks)**

John is given two sunflower plants. One plant is 16 cm high and the other is 24 cm high. John measures the height of each plant at the same time every day for a week. He notes that the 16 cm plant grows 4 cm each day, and the 24 cm plant grows 3·5 cm each day.

(a) Draw up a table showing the heights of the two plants each day for the week, starting on the day that John got them.

We will refer to the plant which was initially 16 cm as plant A and the other as plant B. In the table below, day 0 will refer to the day that John bought the plants.

Day	0	1	2	3	4	5	6	7
A	16cm	20cm	24cm	28cm	32cm	36cm	40cm	44cm
B	24cm	27.5cm	31cm	34.5cm	38cm	40.5cm	44cm	48.5cm

(b) Write down two formulas – one for each plant – to represent the heights of the two plants on any given day. State clearly the meaning of any letters used in your formulas.

Let $f(x)$ represent the height in centimetres of plant A, x days after John bought it. Similarly, let $g(x)$ represent the height in centimetres of plant B, x days after John bought it. We know the initial values $f(0) = 16$ and $g(0) = 24$. As we increase x by one, f increases by 4 and g increases by 3.5, so our general formulae are

$$f(x) = 16 + 4x \qquad \text{and} \qquad g(x) = 24 + 3.5x$$

We could also determine these by noting that the graphs will be straight lines. So for f, for example, two points on the line will be $(0,16)$ and $(1,20)$. We could then use the formula $(y - y_1) = m(x - x_1)$, where $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and solve for $y = f(x)$. A similar approach would give g.

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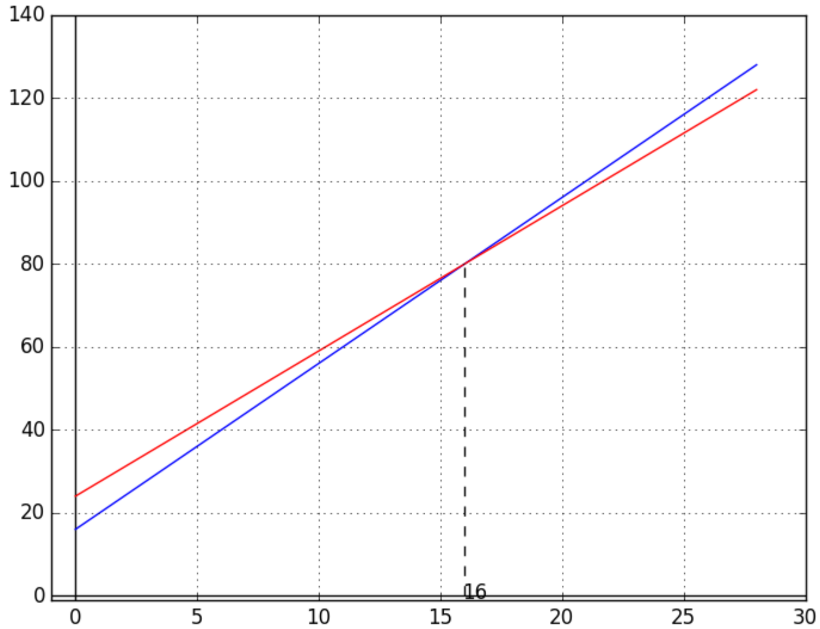
(c) John assumes that the plants will continue to grow at the same rates. Draw graphs to represent the heights of the two plants over the first *four weeks*.



Here, the blue line represents $f(x)$ (the height of plant A) and the red line represents $g(x)$ (the height of plant B).

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(d) **(i)** From your diagram, write down the point of intersection of the two graphs.

The two graphs intersect when $x = 16$. We can verify this by noting that $f(16) = 16+4(16) = 80$ and $g(16) = 24+3.5(16) = 80$.

(ii) Explain what the point of intersection means, with respect to the two plants. Your answer should refer to the meaning of *both* co-ordinates.

The point of intersection $(16, 80)$ tells us the day at which the heights of the two plants will be equal. Since the x coordinate represents the day and the y coordinate represents the height in centimetres of each plant, on day 16, both plants will be 80 cm tall.

(e) Check your answer to part **(d)(i)** using your formulas from part **(b)**.

We wish to determine the point x for which $f(x) = g(x)$, i.e.

$$16+4x = 24+3.5x \quad \Leftrightarrow \quad 0.5x = 8 \quad \Leftrightarrow \quad x = 16$$

as expected.

(f) The point of intersection can be found either by reading the graph or by using algebra. State one advantage of finding it using algebra.

By using algebra, we can find the point of intersection exactly with no errors. When reading from a graph, the point has to be estimated, which can introduce errors.

(g) John's model for the growth of the plants might not be correct. State one limitation of the model that might affect the point of intersection and its interpretation.

This model assumes that both plants grow at constant rates. This may not be true in practice, as plants may grow at varying speeds. Thus, the graphs for the functions may not be straight lines, which would affect where the point of intersection lies.