Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2012

Marking Scheme

Mathematics
(Project Maths – Phase 3)

Ordinary Level

Contents

Introduction

The Ordinary Level Mathematics examination for candidates in the 24 initial schools for *Project Maths* shared some content with the examination for all other candidates. The marking scheme used for the shared content was identical for the two groups.

This document contains the complete marking scheme for both papers for the candidates in the 24 schools.

Readers should note that, as with all marking schemes used in the state examinations, the detail required in any answer is determined by the context and the manner in which the question is asked, and by the number of marks assigned to the question or part. Requirements and mark allocations may vary from year to year.

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination, 2012

Mathematics
(Project Maths – Phase 3)

Paper 1

Ordinary Level

Friday 8 June   Afternoon 2:00 – 4:30

300 marks

Model Solutions – Paper 1

Note: the model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.

Instructions

There are **two** sections in this examination paper:

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You will lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **all six** questions from this section.

Question 1 (25 marks)

Alan pays income tax, a universal social charge (USC) and pay-related social insurance (PRSI) on his gross wages. His gross weekly wages are €510.

(a) Alan pays income tax at the rate of 20%. He has weekly tax credits of €63. How much income tax does he pay?

> Total tax: €510 × 0·2 = €102
>
> Tax paid: €102 − €63 = €39

(b) Alan pays the USC at the rate of 2% on the first €193, 4% on the next €115 and 7% on the balance. Calculate the amount of USC Alan pays.

> €193 × 0·02 = €3·86
> €115 × 0·04 = €4·60
>
> €510 − (€193 + €115) = €202
>
> €202 × 0·07 = €14·14
>
> USC: €3·86 + €4·60 + €14·14 = €22·60

(c) Alan also pays PRSI. His total weekly deductions amount to €76·92. How much PRSI does Alan pay?

> PRSI = €76·92 − (€39 + €22·60) = €15·32

Question 2 **(25 marks)**

Let $a = \sqrt{2}$.

(a) For each of the numbers in the table below, tick (✓) the correct box to say whether it is *rational* or *irrational*.

Number	rational	irrational
a		✓
$a - 1$		✓
$(-a)^2$	✓	
$(a-2)^2$		✓
$1 + a^2$	✓	

(b) Show the following numbers on the number line below.

$$a, \qquad -a, \qquad \sqrt{a}, \qquad a^{-2}$$

$\sqrt{a} \approx 1\cdot 189$ $a^{-2} = \frac{1}{2}$

(c) Verify that $3 - \sqrt{2}$ is a root (solution) of the equation $x^2 - 6x + 7 = 0$.

$$x^2 - 6x + 7 = 0$$
$$\left(3 - \sqrt{2}\right)^2 - 6\left(3 - \sqrt{2}\right) + 7 = 9 - 6\sqrt{2} + 2 - 18 + 6\sqrt{2} + 7$$
$$= 18 - 18 - 6\sqrt{2} + 6\sqrt{2} = 0$$

OR

$$x^2 - 6x + 7 = 0$$
$$\Rightarrow x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(7)}}{2} = \frac{6 \pm \sqrt{8}}{2} = \frac{6 \pm 2\sqrt{2}}{2} = 3 + \sqrt{2} \text{ or } 3 - \sqrt{2}$$

[8]

Question 3 **(25 marks)**

The complex number $z = 1 - 4i$, where $i^2 = -1$.

(a) Plot z and $-2z$ on the Argand diagram.

$-2z = -2(1 - 4i) = -2 + 8i$

(b) Show that $2|z| = |-2z|$.

$2|z| = 2|1 - 4i| = 2\sqrt{1^2 + (-4)^2} = 2\sqrt{17}$

$|-2z| = |-2 + 8i| = \sqrt{(-2)^2 + 8^2} = \sqrt{68} = 2\sqrt{17}$

$\therefore 2|z| = |-2z|$

(c) What does part **(b)** tell you about the points you plotted in part **(a)**?

$-2z$ is twice as far from the origin as z is.

(d) Let k be a real number such that $|z + k| = 5$. Find the two possible values of k.

$|z + k| = 5 \implies |1 - 4i + k| = 5$

$\implies |(1 + k) - 4i| = 5 \implies \sqrt{(1 + k)^2 + (-4)^2} = 5$

$\implies (1 + k)^2 + 16 = 25$

$\implies (1 + k)^2 = 9 \implies 1 + k = \pm 3 \implies k = 2$ or $k = -4$

OR

$(1 + k)^2 + 16 = 25 \implies 1 + 2k + k^2 + 16 - 25 = 0 \implies k^2 + 2k - 8 = 0$

$\implies (k - 2)(k + 4) = 0 \implies k = 2$ or $k = -4$

[9]

Question 4 **(25 marks)**

(a) Solve the equation $\frac{1}{2}(7x-2)+5 = 2x+7$.

$$\frac{1}{2}(7x-2)+5 = 2x+7$$
$$\Rightarrow 7x-2+10 = 4x+14$$
$$\Rightarrow 7x+8 = 4x+14$$
$$\Rightarrow 3x = 6$$
$$\Rightarrow x = 2$$

(b) Solve the equation $\dfrac{2}{3x-4} - \dfrac{1}{2x+1} = \dfrac{1}{2}$ and give your answers correct to one decimal place.

$$\frac{2}{3x-4} - \frac{1}{2x+1} = \frac{1}{2}$$
$$\Rightarrow \frac{2(2x+1)(2) - 1(3x-4)(2) = 1(3x-4)(2x+1)}{(3x-4)(2x+1)(2)}$$
$$\Rightarrow 8x+4-6x+8 = 6x^2+3x-8x-4$$
$$\Rightarrow 6x^2-7x-16 = 0$$
$$\Rightarrow x = \frac{7\pm\sqrt{(-7)^2-4(6)(-16)}}{2(6)} = \frac{7\pm\sqrt{433}}{12}$$
$$\Rightarrow x = 2{\cdot}3 \ \text{ or } \ x = -1{\cdot}2$$

[10]

Question 5 **(25 marks)**

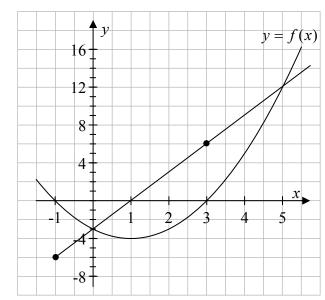The diagram shows the graph of a function f.

(a) The graph of another function g is a straight line.

$g(-1) = -6$ and $g(3) = 6$.

Draw the graph of g on the diagram.

(b) Use the graphs to find the two values of x for which $g(x) = f(x)$.

$$x = 0 \quad \text{or} \quad x = 5$$

(c) The functions g and f are defined for $x \in \mathbb{R}$ by:

$$g : x \mapsto ax + b$$
$$f : x \mapsto x^2 + px + q$$

where a, b, p, and q are constants.
The graph of f crosses the x-axis at -1 and 3, as shown.
By finding the values of a, b, p, and q, use algebra to solve $g(x) = f(x)$.

$g(x) = ax + b$

$g(-1) : -a + b = -6$

$g(3) : \quad 3a + b = 6$

$\quad \Rightarrow -4a = -12 \Rightarrow a = 3 \qquad \therefore -3 + b = -6$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad b = -3$

$\Rightarrow g(x) = 3x - 3$

$f : \quad x = -1, x = 3 \Rightarrow f(x) = (x+1)(x-3) = x^2 - 2x - 3$

OR

$f(x) = x^2 + px + q$

$f(-1) : \quad 1 - p + q = 0$

$f(3) : \quad 9 + 3p + q = 0$ $\quad \Rightarrow -4p = 8 \Rightarrow p = -2 \therefore q = -3$

$\qquad\qquad\qquad\qquad\qquad \Rightarrow f(x) = x^2 - 2x - 3$

$g(x) = f(x) \Rightarrow 3x - 3 = x^2 - 2x - 3 \Rightarrow x^2 - 5x = 0$

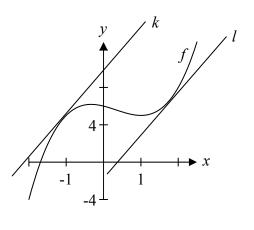$\Rightarrow x(x - 5) = 0 \Rightarrow x = 0 \text{ or } x = 5$

[11]

Question 6 **(25 marks)**

The diagram shows the graph of the cubic function f,
defined for $x \in \mathbb{R}$ as
$$f : x \mapsto x^3 - x^2 - x + 6.$$

(a) Find the co-ordinates of the point at which f cuts the y-axis.

> $f(x) = x^3 - x^2 - x + 6$
> $f(0) = 0 - 0 - 0 + 6 = 6 \quad \Rightarrow \quad (0, 6)$

(b) f has a minimum turning point at $(1, 5)$.
Find the co-ordinates of the maximum turning point.

> $f(x) = x^3 - x^2 - x + 6$
>
> $f'(x) = 3x^2 - 2x - 1 = 0$
> $\Rightarrow \quad (3x + 1)(x - 1) = 0$
> $\Rightarrow \quad x = -\frac{1}{3} \text{ or } x = 1$
>
> $f\left(-\frac{1}{3}\right) = \left(-\frac{1}{3}\right)^3 - \left(-\frac{1}{3}\right)^2 - \left(-\frac{1}{3}\right) + 6$
> $\qquad = -\frac{1}{27} - \frac{1}{9} + \frac{1}{3} + 6 = 6\frac{5}{27}$
>
> Maximum turning point $\left(-\frac{1}{3}, 6\frac{5}{27}\right)$.

[12]

(c) The lines k and l are tangents to the curve $y = f(x)$ and l is parallel to k. The equation of k is $4x - y + 9 = 0$. Find the x co-ordinate of the point at which l is a tangent to the curve.

$k : 4x - y + 9 = 0 \implies y = 4x + 9$

Therefore, the slope of $l = 4$.

$f'(x) = 3x^2 - 2x - 1 = 4$

$\implies 3x^2 - 2x - 5 = 0$

$\implies (3x - 5)(x + 1) = 0$

$\implies x = \frac{5}{3}$ or $x = -1$

$x = \frac{5}{3}$

[13]

Answer **all three** questions from this section.

Question 7   **(50 marks)**

Doctors sometimes need to work out how much medicine to give
a child, based on the correct dose for an adult. There are
different ways of doing this, based on the child's age, weight,
height, or some other measure.

(a)   One rule for working out the child's dose from the adult
dose is called *Clark's rule*. It is:

$$C = \left(\frac{W}{68}\right) \times A$$

where C is the child's dose, A is the adult's dose, and W is
the child's weight in kilograms.

The adult dose of a certain medicine is 125 mg per day. Calculate the correct dose for a child
weighing 30 kg, using Clark's rule. Give the answer correct to the nearest 5 mg.

$$C = \left(\frac{W}{68}\right) \times A$$

$$= \left(\frac{30}{68}\right) \times 125$$

$$= 55 \cdot 147 \approx 55$$

Answer: 55 mg

(b)   Another rule for working out the child's dose is called *Young's rule*. Below are three
different descriptions of Young's rule, taken from the internet. In each case, write down a
formula that *exactly* matches the description in words. State clearly the meaning of any letters
you use in your formulae.

(i)   **Young's rule:** a mathematical expression used to determine a drug dosage for children.
The correct dosage is calculated by dividing the child's age by an amount equal to the
child's age plus 12 and then multiplying by the usual adult dose.

Mosby's Dental Dictionary, 2nd edition.

Formula: $C = \left(\dfrac{Y}{Y+12}\right) \times A$

where C is child's dose, Y is the child's age in years, A is the adult's dose.

[14]

(ii) **Young's rule:** A rule for calculating the dose of medicine correct for a child by adding 12 to the child's age, dividing the sum by the child's age, then dividing the adult dose by the figure obtained.

The American Heritage Medical Dictionary

Formula: $C = \dfrac{A}{\left(\dfrac{Y+12}{Y}\right)}$ C, A and Y same meaning as (i)

(iii) **Young's rule:** the dose of a drug for a child is obtained by multiplying the adult dose by the child's age in years and dividing the result by the sum of the child's age plus 12.

Miller-Keane Encyclopedia and Dictionary of Medicine,
Nursing, and Allied Health, Seventh Edition.

Formula: $C = \dfrac{A \times Y}{Y+12}$ C, A and Y same meaning as (i)

(c) Explain why the three formulae in **(b)** above all give the same result.

By algebraic manipulation each formula can be written in the same form $C = \dfrac{YA}{Y+12}$

(d) The adult dose of a certain medicine is 150 mg per day. According to Young's rule, what is the correct dose for a six-year old child?

$C = \dfrac{YA}{Y+12} = \dfrac{6 \times 150}{6+12} = 50$ mg per day

(e) Young's rule results in a certain child being given one fifth of the adult dose of a medicine. How old is this child?

$C = \dfrac{YA}{Y+12} \Rightarrow \dfrac{A}{5} = \dfrac{YA}{Y+12} \Rightarrow Y+12 = 5Y \Rightarrow 4Y = 12 \Rightarrow Y = 3$

OR

$30 = \dfrac{150Y}{Y+12} \Rightarrow 30Y + 360 = 150Y \Rightarrow Y = 3$

The child is 3 years old.

[15]

(f) Another rule for working out a child's dose is based on "body surface area" (BSA). The rule is:

$$\text{child's dose} = \frac{\text{child's BSA in m}^2}{1 \cdot 73} \times \text{adult dose}$$

BSA is difficult to measure directly, but an estimate can be calculated from a person's height and weight. The chart below allows you to read off the BSA for a given height and weight, by drawing a straight line from the height on the left scale to the weight on the right. For example, the dotted line shows that a person of height 100 cm and weight 16 kg has a BSA of $0 \cdot 67$ m^2.

The correct adult dose of a certain medicine is 200 mg per day. Use the BSA rule to calculate the correct dose for a child of height 125 cm and weight 26 kg.

BSA = 0·95 (reading from chart)

$$C = \frac{BSA}{1 \cdot 73} \times A$$

$$= \frac{0 \cdot 95}{1 \cdot 73} \times 200$$

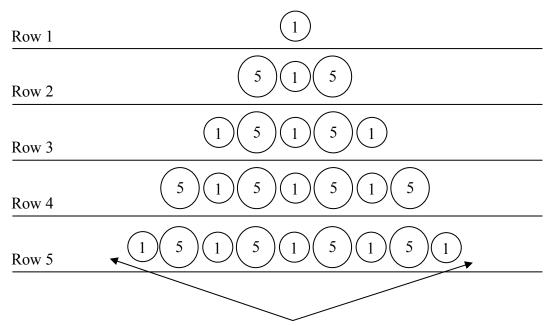$$\approx 109 \cdot 83 \text{ mg} \approx 110 \text{ mg}.$$

[16]

(g) The following apply in the case of a certain medicine and a certain child:
- the child is nine years old
- Clark's rule and Young's rule both give a dose of 90 mg per day
- the BSA rule gives a dose of 130 mg per day.

Find the weight and height of this child.

Young's rule: $C = \dfrac{YA}{Y+12} \Rightarrow \dfrac{9A}{9+12} = 90 \Rightarrow A = \dfrac{21}{9} \times 90 = 210\,\text{mg}$

Clark's rule: $C = \dfrac{W}{68} \times A \Rightarrow \dfrac{W}{68} \times 210 = 90 \Rightarrow W = \dfrac{90 \times 68}{210} = 29\frac{1}{7}\,\text{kg}$

Weight of child is $29\frac{1}{7}\,\text{kg}$

$C = \dfrac{BSA}{1\cdot73} \times A \Rightarrow \dfrac{BSA}{1\cdot73} \times 210 = 130 \Rightarrow BSA = \dfrac{130 \times 1\cdot73}{210} \approx 1\cdot07\ \text{m}^2$

$\therefore$ Height: 142 cm (using chart)

[17]

Question 8 (50 marks)

Lucy is arranging 1 cent and 5 cent coins in rows. The pattern of coins in each row is as shown below.
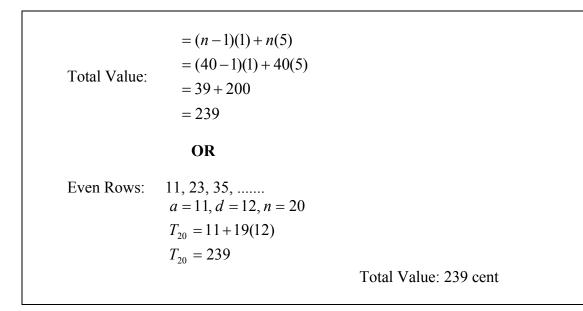
Row 1

Row 2

Row 3

Row 4

Row 5

(a) Draw the next row of coins above, continuing the same pattern.

(b) The table below gives the number of coins and the total value of the coins in each row. Complete the table for rows 4 to 7.

Row number n	Number of 1 cent coins	Number of 5 cent coins	Total number of coins in the row	Total value of the coins in the row
1	1	0	1	1
2	1	2	3	11
3	3	2	5	13
4	**3**	**4**	**7**	**23**
5	**5**	**4**	**9**	**25**
6	**5**	**6**	**11**	**35**
7	**7**	**6**	**13**	**37**

[18]

(c) Complete the following sentences to state, in terms of n, the number of 1 cent and 5 cent coins in row n.

(i) If n is odd, row n has __n__ 1 cent coins and __$(n-1)$__ 5 cent coins.

(ii) If n is even, row n has __$(n-1)$__ 1 cent coins and__n__ 5 cent coins.

(d) Find the total number of coins in the 40th row.

$n = 40$ which is an even number

$\therefore$ Number of coins: $n - 1 + n = 2n - 1 = 80 - 1 = 79$ coins

OR

$T_n = a + (n-1)d$
$T_{40} = 1 + (40-1)2$
$T_{40} = 79$

(e) Find the total value of the coins in the 40th row.

Total Value:
$$= (n-1)(1) + n(5)$$
$$= (40-1)(1) + 40(5)$$
$$= 39 + 200$$
$$= 239$$

OR

Even Rows: 11, 23, 35, ...
$a = 11, d = 12, n = 20$
$T_{20} = 11 + 19(12)$
$T_{20} = 239$

Total Value: 239 cent

[19]

(f) Which row has coins with a total value of 337 cent?

If n odd: $(n)(1) + (n-1)(5) = 337$

$\Rightarrow \quad n + 5n - 5 = 337$

$\Rightarrow \quad 6n = 342$

$\Rightarrow \quad n = 57$

If n even: $(n-1)(1) + n(5) = 337$

$\Rightarrow \quad 6n - 1 = 337$

$\Rightarrow \quad 6n = 338$

$\Rightarrow \quad n = 56\dfrac{1}{3} \notin \mathbb{N}.$

$\therefore$ Row 57 has a total value of 337 cent.

(g) Find the total value of the coins in the first 40 rows.

$S = 1 + 11 + 13 + 23 + 25 + 35 + ... + 239$

$= (1 + 13 + 25 + ...) \; + \; (11 + 23 + 35 + ...)$

$= \frac{20}{2}\left(2 + (20-1)12\right) \; + \; \frac{20}{2}\left(22 + (20-1)12\right)$

$= 10(230) + 10(250) = 4800$ cent.

OR

Number of 1 cent coins:

$S = 1 + 1 + 3 + 3 + 5 + 5 + ... + 39 + 39$

$= 2(1 + 3 + 5 + ...) = 2\left[\frac{20}{2}\left(2 + (20-1)2\right)\right] = 800$ coins

Number of 5 cent coins:

$S = 0 + 2 + 2 + 4 + 4 + ... + 38 + 40$

$= (0 + 2 + 4 + ...) + (2 + 4 + 6 + ...)$

$= \frac{20}{2}\left(0 + (20-1)2\right) + \frac{20}{2}\left(4 + (20-1)2\right) = 380 + 420 = 800$ coins

Value: $800(1) + 800(5) = 4800$ cent

[20]

Question 9 **(50 marks)**

(a) Investments can increase or decrease in value. The value of a particular investment of €100 was found to fit the following model:

$$V = 100 + 45t - 1 \cdot 5t^2$$

where V is the value of the investment in euro, and t is the time in months after the investment was made.

(i) Find the rate at which the value of the investment was changing after 6 months.

$$V = 100 + 45t - 1 \cdot 5t^2$$
$$\Rightarrow \frac{dV}{dt} = 45 - 3t$$
$$t = 6 \Rightarrow 45 - 3(6) = 27$$
$$\qquad \text{€27 per month}$$

(ii) State whether the value of the investment was increasing or decreasing after 18 months. Justify your answer.

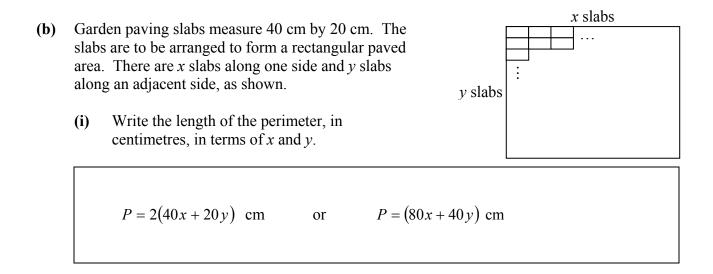$\frac{dV}{dt}$ at $t = 18$ \qquad $45 - 3(18) = -9$

Amount is negative meaning the value is decreasing.

(iii) The investment was cashed in at the end of 24 months. How much was it worth at that time?

$$V = 100 + 45t - 1 \cdot 5t^2$$
$$= 100 + 45(24) - 1 \cdot 5(24)^2$$
$$= 100 + 1080 - 864$$
$$= €316$$

[21]

(iv) How much was the investment worth when it had its maximum value?

$$\frac{dV}{dt} = 45 - 3t = 0 \implies t = 15$$

$$V = 100 + 45t - 1{\cdot}5t^2$$
$$= 100 + 45(15) - 1{\cdot}5(15)^2$$
$$= €437{\cdot}50$$

(b) Garden paving slabs measure 40 cm by 20 cm. The slabs are to be arranged to form a rectangular paved area. There are x slabs along one side and y slabs along an adjacent side, as shown.

x slabs

y slabs

(i) Write the length of the perimeter, in centimetres, in terms of x and y.

$$P = 2(40x + 20y) \text{ cm} \qquad \text{or} \qquad P = (80x + 40y) \text{ cm}$$

(ii) The material being used for edging means that the perimeter is to be 64 metres. Find y in terms of x.

$$P = 64\,\text{m} = 6400\,\text{cm}$$
$$\implies 80x + 40y = 6400$$
$$\implies 40y = 6400 - 80x$$
$$\implies y = 160 - 2x$$

[22]

(iii) Find the value of x for which the paved area is as large as possible.

$$A = (40x)(20y)$$
$$= 800x(160 - 2x)$$
$$= 128000x - 1600x^2$$

$$\frac{dA}{dx} = 128000 - 3200x$$

Let $\frac{dA}{dx} = 0$

$\Rightarrow 128000 - 3200x = 0$

$\Rightarrow \quad x = 40$

(iv) Find the number of slabs needed to pave this maximum area.

$$y = 160 - 2x = 80$$

Number of slabs $= xy = 40(80)$
3200 slabs are needed.

[23]

Marking Scheme – Paper 1

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5 mark scale		0, 3, 5	0, 3, 4, 5	
10 mark scale		0, 6, 10	0, 6, 8, 10	
15 mark scale			0, 8, 13, 15	
20 mark scale				
25 mark scale			0, 8, 20, 25	0, 6, 13, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)
- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, *scale 10C** indicates that 9 marks may be awarded.

Summary of mark allocations and scales to be applied

Section A

Question 1
 (a) 10C
 (b) 10C
 (c) 5B

Question 2
 (a) 10C
 (b) 10C
 (c) 5C

Question 3
 (a)   15C
 (b), (c), (d) 10C

Question 4
 (a) and (b) 25D

Question 5
 (a) 15C
 (b) 5C
 (c) 5C

Question 6
 (a) 10C
 (b) 10C
 (c) 5C

Section B

Question 7
 (a)   10B*
 (b) (i) 5B
 (b) (ii) 5B
 (b) (iii) 5B
 (c)   5B
 (d)   5B*
 (e)   5C
 (f)   5C*
 (g)   5C*

Question 8
 (a) 10B
 (b) 10C
 (c)   5C
 (d) 15C
 (e)   5C
 (f), (g) 5C

Question 9
 (a) (i) 5C*
 (a) (ii) 5C
 (a) (iii) 25C
 (a) (iv) 5C
 (b) 10C

Detailed marking notes

Section A

Question 1

(a) Scale 10C
   Low partial credit:
   - Error in calculating gross tax **and** misuse of tax credit
   - Calculates 20% of some figure

   High partial credit:
   - Error in calculating gross tax **or** misuse of tax credit

(b) Scale 10C
   Low partial credit:
   - One percentage calculated correctly
   - Adds 193 and 115 and stops
   - Any correct relevant step

   High partial credit:
   - Calculates two of the required percentages correctly but fails to finish

(c) Scale 5B
   Partial credit:
   - Adds the tax and the USC but fails to finish
   - Subtracts answer to part (a) or part (b) from total deductions
   - Any correct relevant step

Question 2

(a) Scale 10C

Low partial credit:
- Correctly identifies 1 or 2 numbers

High partial credit:
- Correctly identifies 3 or 4 numbers

(b) Scale 10C

Low partial credit:
- Correctly shows 1 or 2 numbers (see note)
- No numbers correctly shown but a and $-a$ equidistant from zero

High partial credit:
- Correctly shows 3 numbers

Note: Treat as one error if a and $-a$ are incorrectly shown but are equidistant from the origin

(c) Scale 5C

Low partial credit:
- Some correct substitution for x in the given equation
- Some correct substitution into quadratic formula

High partial credit:
- Fails to finish from $x = \dfrac{6 \pm \sqrt{8}}{2}$
- Correct substitution for x in the given equation but fails to finish
- Use of decimals after correct substitution

[27]

Question 3
(a) Scale 15C
Low partial credit:
- Plots z only
- Correct real or imaginary part plotted for either point
- Correct substitution for z

High partial credit:
- Plots -2z correctly only
- Distribution error in finding -2z but plots z and incorrect -2z correctly

(b), (c), **(d)** Scale 10C
Low partial credit:
- Substitutes for z at least once
- Some correct substitution into the modulus formula
- Modulus formula correctly stated
- Joins z and/or -2z to the origin
- Any correct relevant statement for (c)

High partial credit:
- At least one of (b), (c) or (d) fully correct
- Work of merit in at least 2 parts

Question 4
Scale 25D
Low partial credit:
Any correct relevant step

Middle partial credit:
Correct structure to the method but with at least two major errors in solving

High partial credit:
Clear proficiency in solving equations but with one major error

Question 5
(a)   Scale 15C
Low partial credit:
- Plots correct x or correct y coordinate of either point
- Plots one point only
- Plots (-6,-1) or (6,3)

High partial credit:
- Plots both points correctly but fails to join
- Joins a correctly plotted point to an incorrectly plotted point

(b)   Scale 5C   (Accept candidate's graph from (a))
Low partial credit:
- Only one x value given
- Marks one or both points of intersection on graph
- States answer is -3 or 12

High partial credit:
- Writes answer as (0,-3) and (5,12)
- Gives answers as -3 and 12

(c)   Scale 5C
Low partial credit:
- Any correct relevant step
- Solves for a, b, p or q

High partial credit:
- Solves correctly for a, b, p and q but fails to finish
- Correct structure to the method but with some errors

Question 6
(a) Scale 10C
Low partial credit:
- Reads incorrect coordinates from graph
- States $x = 0$ and stops
- Shows point of intersection on graph

High partial credit:
- Writes answer as 6 or (6,0)

(b) Scale 10C
Low partial credit:
- Writes down answer by reading from graph
- Any correct differentiation and stops
- Indicates the local maximum on the graph

High partial credit:
- Correct structure to answer but with some errors
- Puts first derivative (without errors) = 0 and stops

(c) Scale 5C
Low partial credit:
- Attempts to find slope of k
- Writes down first derivative of f
- States slope of $k =$ slope of l (or equivalent statement)

High partial credit:
- Fails to solve $3x^2 - 2x - 1 = 4$ (or candidate's equivalent answer)

Section B

Question 7

(a) Scale 10B*
Partial credit:
- Some correct substitution

Note: Penalise once only for omission of units in parts (a), (d) and (f)
Failure to round: penalty is one mark (provided otherwise full marks)

(b)(i) Scale 5B
Partial credit:
- Partially correct formula
- Fail to state meaning of letters used in formula
[Needs to be stated once only in part (b)]

(b)(ii) Scale 5B
Partial credit:
- Partially correct formula

(b)(iii) Scale 5B
Partial credit:
- Partially correct formula

(c) Scale 5B
Partial credit:
- Any reasonable explanations with error(s)

(d) Scale 5B*
Partial credit:
- Some correct substitution [Accept candidate's rule from (b)]

(e) Scale 5C
Low partial credit:
- Finds one fifth of a dose of medicine
- Attempt at setting up a relevant equation

High partial credit:
- Correct equation but error(s) in solving
- Correct answer without work

(f) Scale 5C*
Low partial credit:
- States BSA from chart and stops
- Substitutes for *A*
- Correct height or weight indicated on chart

High partial credit:
- Substitutes BSA into rule and calculates dose with errors

[31]

(g)   Scale 5C*

Low partial credit:
- Some correct substitution into a relevant rule

High partial credit:
- Correct weight only
- Incorrect weight found but then finds height using correct method

Question 8
(a) Scale 10B
Partial credit:
  - Error in pattern

(b) Scale 10C
Low partial credit:
  - At least one correct entry
  - Error(s) in more than one column (but at least one entry correct)

High partial credit:
  - Error(s) in only one column

(c) Scale 5C
Low partial credit:
  - 1 space only correctly filled
  - Correct sentence regarding a specific odd/even case
  - Correct sentence(s) but not in terms of n

High partial credit:
  - 2 or 3 spaces correctly filled

(d) Scale 15C
Low partial credit:
  - Attempt at continuing pattern beyond row 7
  - Writes $a + (n-1)d$ with partial or no substitution
  - Correct number of 1 cent coins or correct number of 5 cent coins

High partial credit:
  - Correct substitution into $a + (n-1)d$ but fails to finish correctly
  - Correct number of 1 cent coins and correct number of 5 cent coins but no/incorrect addition
  - Correct answer without work

(e) Scale 5C
Low partial credit:
  - Correct value of 5 cent coins or correct value of 1 cent coins
  - Relevant formula with some substitution
  - Recognition of sequence of even rows

High partial credit
  - Correct value of 5 cent coins and correct value of 1 cent coins but no/incorrect addition
  - Correct answer without work
  - $40(1) + 39(5) = 235$

(f) and **(g)** Scale 5C

Low partial credit:

- Correct relevant statement involving 337
- Attempt at listing
- Any correct relevant step

- Writes $\dfrac{n}{2}\left[2a+(n-1)d\right]$ with some substitution for (g)

- Solves $(n-1)(1)+n(5)=337$ and stops

- Correct number of 1 cent coins and 5 cent coins but fail to find correct value
- Correct answer to one part without work

High partial credit

- Correct answer to both parts without work
- One part correct with work shown
- Work of merit for both parts

Question 9
(a)(i)   Scale 5C*
  Low partial credit:
  - Substitutes $t = 6$ into V formula
  - Effort at differentiation

  High partial credit:
  - Correct derivative with or without substitution for t

(a)(ii)   Scale 5C
  Low partial credit:
  - Substitutes $t = 18$ into V formula
  - Finds first derivative and stops
  - States investment was decreasing but incorrect justification

  High partial credit:
  - First derivative with substitution for t but subsequent error(s)
  - Correct justification but fails to state that investment was decreasing
  - Uses substitution of $t = 17$, 18 and 19 into V formula but no conclusion

  Full credit:
  - Correct answer with justification, by calculus or otherwise

(a)(iii)   Scale 25C
  Low partial credit:
  - Some correct substitution into formula for V
  - Substitutes $t = 24$ into expression for $\dfrac{dV}{dt}$

  High partial credit:
  - Correct substitution but errors in evaluating worth

(a)(iv)   Scale 5C
  Low partial credit:
  - Writes down first derivative
  - Solves for t and stops
  - Substitutes a value for t ($\neq 15$) into V formula

  High partial credit:
  - Substitutes correct value for t into V formula with error(s) in calculations
  - Uses substitution of $t = 14$, 15 and 16 into V formula with no conclusion
  - Correct answer without justification of using $t = 15$

(b)   Scale 10C
Low partial credit:
- States Perimeter = 2 length + 2 breadth (or equivalent)
- Perimeter = $2x + 2y$
- Perimeter = $40x + 20y$
- Changes 64 metres to centimetres
- Changes $80x + 40y$ to $0.8x + 0.4y$
- Correct expression for area in terms of x and y or in terms of x only
  [Accept candidate's answer from (ii)]
- Some correct differentiation
- Substitutes value for x from part (iii) into a relevant formula

High partial credit:
- Correct structure to answer with some errors
- At least 2 parts fully correct (allowing for candidate error in an earlier part)

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination, 2012

Mathematics
(Project Maths – Phase 3)

Paper 2

Ordinary Level

Monday 11 June   Morning 9:30 – 12:00

300 marks

Model Solutions – Paper 2

Note: the model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.

Instructions

There are **two** sections in this examination paper:

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	2 questions

Answer **all eight** questions, as follows:

In Section A, answer:

Questions 1 to 5 and

either Question 6A **or** Question 6B.

In Section B, answer Questions 7 and 8.

Write your answers in the spaces provided in this booklet. You will lose marks if you do not do so. There is space for extra work on the back cover of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **all six** questions from this section.

Question 1   **(25 marks)**

Peter and Niamh go to a large school. One morning, they arrive early. While they are waiting, they decide to guess whether each of the next three students to come in the door will be a boy or a girl.

(a) Write out the sample space showing all the possible outcomes. For example, BGG is one outcome, representing Boy, Girl, Girl.

> BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG

(b) Peter says these outcomes are equally likely. Niamh says they are not. What do you need to know about the students in the school to decide which of them is correct?

> The number of boys and the number of girls in the school.

(c) If all the outcomes are equally likely, what is the probability that the three students will be two girls followed by a boy?

> P(GGB) = $\frac{1}{8}$ or 0·125 or 12·5%

(d) Niamh guesses that there will be at least one girl among the next three students. Peter guesses that the next three students will be either three boys or two boys and a girl.
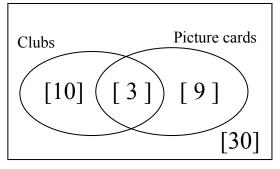Who is more likely to be correct, assuming all outcomes are equally likely?
Justify your answer.

> P(at least one girl) $= \frac{7}{8}$ or 0·875 or 87·5%
>
> P(three boys or two boys and a girl $= \frac{4}{8}$ or $\frac{1}{2}$ or 0·5 or 50%
>
> Niamh is more likely to be correct because of the greater probability.

Question 2 **(25 marks)**

(a) In the Venn diagram below, the universal set is a normal deck of 52 playing cards. The two sets shown represent *clubs* and *picture cards* (kings, queens and jacks).

Show on the diagram the number of elements in each region.

Clubs   Picture cards

[10] ([3]) [9]

[30]

(b) (i) A card is drawn from a pack of 52 cards.
Find the probability that the card drawn is the king of clubs.

P(king of clubs) $= \frac{1}{52}$

(ii) A card is drawn from a pack of 52 cards.
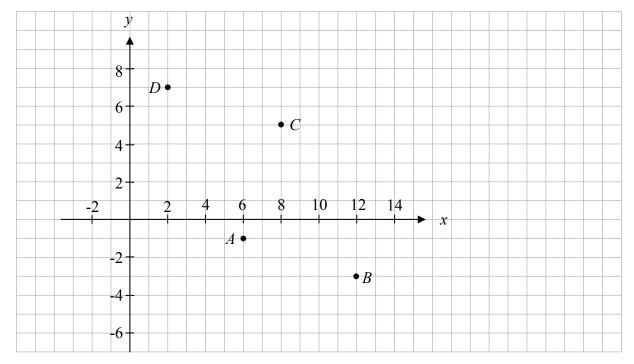Find the probability that the card drawn is a club or a picture card.

P(club or picture card) $= \frac{22}{52} = \frac{11}{26}$

(iii) Two cards are drawn from a pack of 52 cards. Find the probability that neither of them is a club or a picture card. Give your answer correct to two decimal places.

P(not club or picture card) $= \frac{30}{52} \times \frac{29}{51} \approx 0.33$

Question 3 **(25 marks)**

$A(6, -1)$, $B(12, -3)$, $C(8, 5)$ and $D(2, 7)$ are four points.

(a) Plot the four points on the diagram below.

(b) Describe two different ways of showing, using co-ordinate geometry techniques, that the points form a parallelogram $ABCD$.

> Any TWO of:
>
> - Show that opposite sides are parallel by showing the slopes of opposite lines are equal.
>
> - Show that the diagonals bisect each other by showing the midpoint of $[AC]$ equals the midpoint of $[DB]$.
>
> - Show that the opposite sides are equal in length using the length formula.
>
> - Show that $\overrightarrow{AB}$ maps D onto C or similar.

[41]

(c) Use one of the ways you have described to show that $ABCD$ is a parallelogram.

Slope $AB = \frac{-3+1}{12-6} = -\frac{2}{6}$, Slope $DC = \frac{7-5}{2-8} = -\frac{2}{6}$ $\Rightarrow AB \parallel DC$

Slope $BC = \frac{5+3}{8-12} = -2$, Slope $AD = \frac{7+1}{2-6} = -2$ $\Rightarrow BC \parallel AD$

Hence, $ABCD$ a parallelogram

or

Midpoint $[AC] = (\frac{6+8}{2}, \frac{-1+5}{2}) = (7,2)$, Midpoint $[BD] = (\frac{12+2}{2}, \frac{-3+7}{2}) = (7,2)$,

$\Rightarrow$ Diagonals bisect. Hence, $ABCD$ a parallelogram

or

Length $[AB]$ $= \sqrt{(12-6)^2 + (-3+1)^2} = \sqrt{40}$

Length $[DC]$ $= \sqrt{(2-8)^2 + (7-5)^2} = \sqrt{40}$

Length $[AD]$ $= \sqrt{(6-2)^2 + (-1-7)^2} = \sqrt{80}$

Length $[BC]$ $= \sqrt{(12-8)^2 + (-3-5)^2} = \sqrt{80}$

$\Rightarrow$ Opposite sides are equal. Hence, $ABCD$ a parallelogram

or

$A(6,-1) \rightarrow B(12,-3)$ maps $D(2,7) \rightarrow (2+6, 7-2) = C(8,5)$ or similar

$\Rightarrow \overrightarrow{AB} = \overrightarrow{DC}$. Hence, $ABCD$ a parallelogram
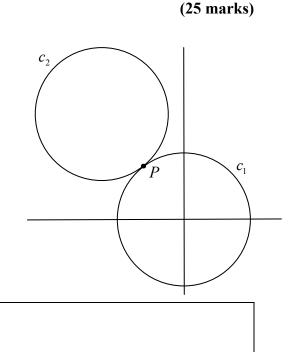

Question 4 (25 marks)

The diagram shows two circles c_1 and c_2 of equal radius.

c_1 has centre $(0, 0)$ and it cuts the x-axis at $(5, 0)$.

(a) Find the equation of c_1.

$x^2 + y^2 = 5^2 = 25$

(b) Show that the point $P(-3, 4)$ is on c_1.

$x^2 + y^2 = (-3)^2 + 4^2 = 9 + 16 = 25 = r^2$

[42]

(c) The two circles touch at $P(-3, 4)$.
P is on the line joining the two centres.
Find the equation of c_2.

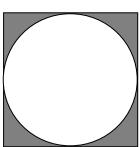$(0, 0) \rightarrow (-3, 4)$ maps $(-3, 4) \rightarrow (-6, 8)$

$c_2 : \quad (x+6)^2 + (y-8)^2 = 25$

(d) Find the equation of the common tangent at P.

Slope of line of centres: $\dfrac{8-0}{-6-0} = -\dfrac{4}{3}$

Perpendicular slope, the slope of the tangent: $\dfrac{3}{4}$

Equation of tangent: $y - 4 = \frac{3}{4}(x + 3) \implies 3x - 4y + 25 = 0$

[43]

Question 5 (25 marks)

(a) The diagram shows a circle inscribed in a square.
The area of the square is 16 cm^2.

(i) Find the radius length of the circle.

$$l^2 = 16 \quad \Rightarrow \quad l = 4 \quad \Rightarrow \quad \text{radius} = 2 \text{ cm}$$

(ii) Find the area of the shaded region, in cm^2, correct to one decimal place.

Shaded area: $16 - \pi(2)^2 = 16 - 12 \cdot 566 = 3 \cdot 433 = 3 \cdot 4 \text{ cm}^2$

(b) A solid wax candle is in the shape of a cylinder with a cone on top, as shown in the diagram.

The diameter of the base of the cylinder is 3 cm and the height of the cylinder is 8 cm.

The volume of the wax in the candle is 21π cm^3.

(i) Find the height of the candle.

8 cm

3 cm

Volume of cylinder $= \pi(1 \cdot 5)^2 8 = 18\pi$

Volume of cone $= 21\pi - 18\pi = 3\pi$

$\frac{1}{3}\pi r^2 h = 3\pi \Rightarrow \frac{1}{3}\pi(1 \cdot 5)^2 h = 3\pi \Rightarrow h = \dfrac{9}{2 \cdot 25} = 4$ cm

Height of candle: $8 + 4 = 12$ cm

(ii) Nine of these candles fit into a rectangular box. The base of the box is a square. Find the volume of the smallest rectangular box that the candles will fit into.

Square base $\Rightarrow$ 3 candles wide $\times 3$ candles deep.

Dimensions of base $= 3(3) \times 3(3)$.

Area of base of box: $9 \times 9 = 81$ cm^2
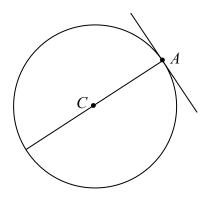
Height of box: 12 cm

Volume: $81 \times 12 = 972$ cm^3

---

[45]

Question 6 **(25 marks)**

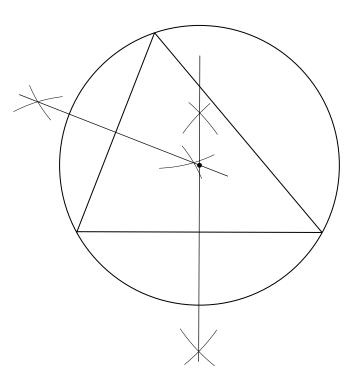Answer **either** 6A **or** 6B.

Question 6A

(a) **(i)** Write down a geometrical result that can be used to construct a tangent to a circle at a point.

> The tangent is perpendicular to the tangent at the point of contact

(ii) On the diagram shown, construct the tangent to the circle at A.

(b) Construct the circumcentre and circumcircle of the triangle below, using only a straight edge and compass. Show all construction marks clearly.
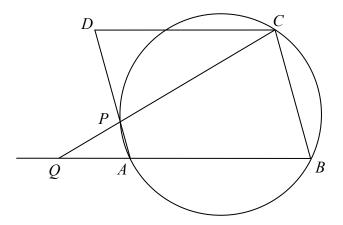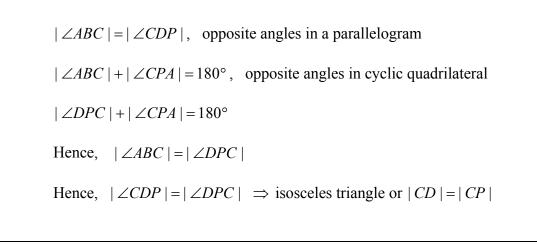
OR

Question 6B

ABCD is a parallelogram.

The points *A*, *B* and *C* lie on the circle
which cuts [*AD*] at *P*.
The line *CP* meets the line *BA* at *Q*.

Prove that |*CD*| = |*CP*|.

$|\angle ABC| = |\angle CDP|$, opposite angles in a parallelogram

$|\angle ABC| + |\angle CPA| = 180°$, opposite angles in cyclic quadrilateral

$|\angle DPC| + |\angle CPA| = 180°$

Hence, $|\angle ABC| = |\angle DPC|$

Hence, $|\angle CDP| = |\angle DPC| \Rightarrow$ isosceles triangle or $|CD| = |CP|$

Answer Question 7 and Question 8.
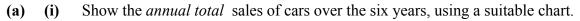
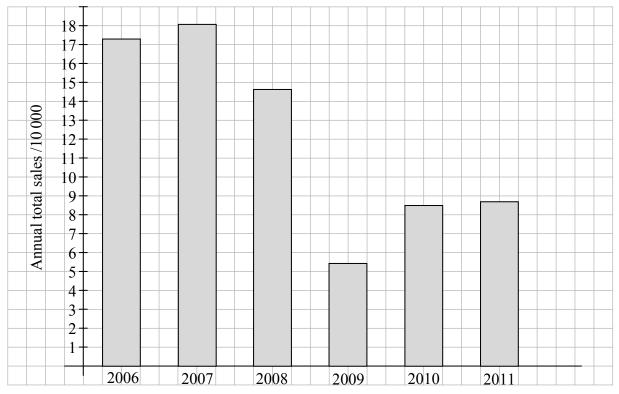**Question 7**   **(75 marks)**

The following table gives data on new private cars sold in Ireland in each quarter of each year from 2006 to 2011.

New private cars sales							
Number of cars sold					Engine type of cars sold		
Year	January to March	April to June	July to Sept.	October to Dec.	Annual Total	Petrol	Diesel	Other
2006	75 769	54 572	32 873	10 059	173 273	128 634	44 010	629
2007	81 750	57 124	32 418	9 462	180 754	128 346	50 560	1 848
2008	77 441	37 128	27 361	4 540	146 470	92 298	50 283	3 889
2009	27 140	15 225	9 049	3 018	54 432	22 802	30 645	985
2010	34 555	26 806	17 011	6 535	84 907	27 124	53 998	3 785
2011	39 484	29 770	13 467	4 211	86 932	23 246	61 730	1 956

(Source: Central Statistics Office, http://www.cso.ie)

(a)   **(i)**   Show the *annual total* sales of cars over the six years, using a suitable chart.

(ii)   Find the mean number of cars sold per year over the six years.

$$\text{Mean} \quad \tfrac{1}{6}\left(173273 + 180754 + 146470 + 54432 + 84907 + 86932\right) = \tfrac{1}{6}\left(726768\right) = 121128$$

[48]

(iii) Calculate the percentage increase in annual car sales between 2009 and 2011.

Increase = 86 932 − 54 432 = 32 500 ⇒ % increase = $\dfrac{32500}{54432} \times 100 = 59\cdot71\%$

(iv) Aoife says that this increase shows car sales are currently going well. Paul says that car sales are currently going badly. He says that sales have fallen by 52% since 2007 and that they are well below average.
Complete the sentences below to give a criticism of each argument.

Aoife's argument does not recognise that this increase comes from a very low base, and that sales had been much better before 2009.

Paul's argument does not recognise that, although sales are lower now than in 2007, they have recovered a lot since their lowest point in 2009.

(v) Give a more balanced description of the pattern of car sales over the six years.

Sales fell dramatically from 2007 to 2009; they recovered a lot since then, but are still much lower than they were at the start.

(b) (i) Describe how the sales of the cars are distributed over the four quarters of each year.

Highest quarterly sales are in the first quarter and decrease significantly in each subsequent quarter,
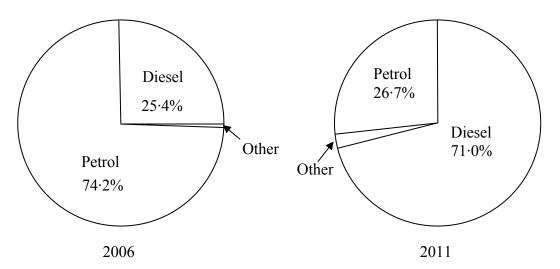
(ii) Suggest a reason for this pattern of sales.

People like to buy new cars early in the year so that they have a new year number plate.

(iii) The sales for the first quarter of 2012 are 36 081.
Find, with justification, an estimate for the total annual sales for 2012.

First quarter sales in 2012 are about one-third of the range between those quarters in 2010 and 2011. Assuming annual sales will retain that proportion would give an estimate of 85 500 for annual sales.

(c) **(i)** Two pie charts are being used to show the change from 2006 to 2011 in the popularity of petrol and diesel cars. Complete the second pie chart.

2006

2011

$$\text{Diesel} = \frac{61730}{86932} \times 360 = 256°$$

$$\text{Petrol} = \frac{23246}{86932} \times 360 = 96°$$

$$\text{Other} = \frac{1956}{86932} \times 360 = 8°$$

(ii) Which of the following statements best describes the change over time in the popularity of diesel cars as a percentage of the total?

A. Diesel cars have suddenly become very popular in the last year or two.

B. Diesel cars have increased very steadily in popularity over the last six years.

C. Diesel cars have become very popular since car sales started to improve.

D. Diesel cars got more popular each year, with an especially big increase in 2009.

E. Diesel cars became popular as car sales fell but have been getting less popular as they rise again.

Write the letter corresponding to the correct answer in the box.   D

[50]

(d) A survey of some of the most popular models of private cars sold in 2011 examined the CO_2 emissions in g/km from diesel engines and petrol engines. The data are as follows:

Diesel engines	Petrol engines
117, 125, 120, 125, 134, 110, 118, 114, 119, 119, 116, 107.	139, 133, 150, 157, 138, 159, 129, 138, 134, 129, 129, 136.

(i) Construct a back-to-back stem-and-leaf plot of the above data.

```
                    7 | 10 |
      9, 9, 8, 7, 6, 4, 0 | 11 |
               5, 5, 0 | 12 | 9, 9, 9
                    4 | 13 | 3, 4, 6, 8, 8, 9
                      | 14 |
                      | 15 | 0, 7, 9
```

Key: 12| 9 means 129

(ii) Does the information suggest that diesel engines produce lower CO_2 emissions than petrol engines? In your answer you should refer to the stem-and-leaf plot and to an appropriate measure of central tendency.

Yes. The diesel engines grouped at the top of the plot have a smaller median [/mean] value.

(iii) Does the information suggest that there is a greater variation in the CO_2 emissions of diesel engines than petrol engines? In your answer you should refer to the stem-and-leaf plot and an appropriate measure of variability.

No. The emissions for the petrol engines are *more* spread out than for the diesel ones. The range [/interquartile range /standard deviation] for the petrol engines is greater than that for the diesel engines.
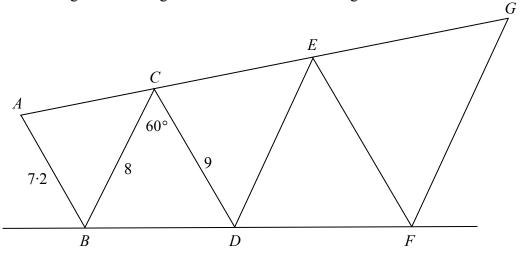
Question 8 (75 marks)

(a) The planned supports for the roof of a building form scalene triangles of different sizes.

   (i) Explain what is meant by a **scalene triangle**.

> A triangle in which the three sides have different lengths

The triangle *EFG* is the image of the triangle *CDE* under an enlargement and the triangle *CDE* is the image of the triangle *ABC* under the same enlargement.

The proposed dimensions for the structure are $|AB| = 7 \cdot 2$ m, $|BC| = 8$ m, $|CD| = 9$ m and $|\angle DCB| = 60°$.

   (ii) Find the length of $[FG]$.

> Scale factor $= \frac{9}{7.2} = 1 \cdot 25$
>
> $|DE| = 1 \cdot 25\,|BC| = 1 \cdot 25(8) = 10 \implies |FG| = 1 \cdot 25|DE| = 1 \cdot 25(10) = 12 \cdot 5$ m

   (iii) Find the length of $[BD]$, correct to three decimal places.

> $|BD|^2 = 8^2 + 9^2 - (2)(8)(9)\cos 60° = 64 + 81 - 72 = 73$
>
> $\implies |BD| = \sqrt{73} = 8 \cdot 544\,\text{m}$

(iv) The centre of the enlargement is O. Find the distance from O to the point B.

$$\frac{|OD|}{|OB|} = \frac{x+8{\cdot}544}{x} = 1{\cdot}25$$

$$\Rightarrow \quad x+8{\cdot}544 = 1{\cdot}25x$$

$$\Rightarrow \quad 0{\cdot}25x = 8{\cdot}544$$

$$\Rightarrow \quad x = 34{\cdot}176 \ \text{m}$$

[53]

(v) A condition of the planning is that the height of the point G above the horizontal line BF cannot exceed $11 \cdot 6$ m.

Does the plan meet this condition? Justify your answer by calculation.

$| \angle GFH | = \alpha = | \angle CBD |$

In triangle CBD:

$$\frac{\sin \alpha}{9} = \frac{\sin 60°}{8 \cdot 544} \implies \sin \alpha = \frac{9 \sin 60°}{8 \cdot 544}$$

In triangle GFH:

$$\sin \alpha = \frac{h}{12 \cdot 5} = \frac{9 \sin 60°}{8 \cdot 544}$$

$$\implies h = \frac{12 \cdot 5 \times 9 \sin 60°}{8 \cdot 544} = 11 \cdot 4 < 11 \cdot 6$$

Yes, the plan meets the condition.

[54]

(b) In order to estimate the area of the irregular shape shown below, a horizontal line was drawn across the widest part of the shape and five offsets (perpendicular lines) were drawn at equal intervals along this line.

35 mm   45 mm   60 mm   40 mm   40 mm

120 mm

(i) Find the lengths of the horizontal line and the offsets, taking each grid unit as 5 mm, and record the lengths on the diagram.

(ii) Use the trapezoidal rule to estimate the area of the shape.

$$A = \tfrac{20}{2}\left(0 + 0 + 2(35 + 45 + 60 + 40 + 40)\right)$$
$$= \tfrac{20}{2}(440)$$
$$= 4400 \text{ mm}^2$$

[55]

Marking Scheme – Paper 2

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5 mark scale	0, 5	0, 2, 5	0, 2, 4, 5	
10 mark scale		0, 5, 10	0, 4, 7, 10	0, 2, 5, 8, 10
15 mark scale			0, 5, 10, 15	0, 4, 7, 11, 15
20 mark scale				
25 mark scale			0, 8, 17, 25	

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)
- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, *scale 10C** indicates that 9 marks may be awarded.

[56]

Summary of mark allocations and scales to be applied

Section A

Question 1
- (a)   10C
- (b)   5B
- (c)   5B
- (d)   5C

Question 2
- (a)   5C
- (b) (i)   10B
- (b) (ii)   5C
- (b) (iii)   5C*

Question 3
- (a)   15C
- (b)   5B
- (c)   5C

Question 4
- (a)   10C
- (b)   5C
- (c)   5C
- (d)   5C

Question 5
- (a) (i)   5B*
- (a) (ii)   5B*
- (b) (i)   10D*
- (b) (ii)   5B*

Question 6A
- (a) (i)   5B
- (a) (ii)   5B
- (b)   15D

Question 6B
- 25C

Section B

Question 7
- (a) (i)   10C
- (a) (ii)   10C
- (a) (iii)   5C
- (a) (iv)   5B
- (a) (v)   5B
- (b) (i)   5B
- (b) (ii)   5A
- (b) (iii)   5B
- (c) (i)   5C
- (c) (ii)   5B
- (d) (i)   5C
- (d) (ii)   5C
- (d) (iii)   5C

Question 8
- (a) (i)   5A
- (a) (ii)   10C
- (a) (iii)   10C
- (a) (iv)   10C
- (a) (v)   15C
- (b) (i)   10C
- (b) (ii)   15C

Detailed marking notes

Section A

Question 1

(a) Scale 10C (0, 4, 7,10)
Low partial credit:
- Any work of merit e.g. one correct outcome other than BGG.

High partial credit:
- An almost correct response such as one/two outcomes missing or extra outcomes.

(b) Scale 5B (0, 2, 5)
Partial credit:
- Any work of merit e.g. the number of boys or the number of girls.

(c) Scale 5B (0, 2, 5)
Partial credit:
- Any work of merit e.g. correct numerator or denominator in fraction format e.g. P(2 girls and a boy) = $\frac{3}{8}$

Full credit:
- Correct answer without work shown.

(d) Scale 5C (0, 2, 4, 5)
Low partial credit:
- Any work of merit e.g. outcomes for Niamh or Peter given or a correct answer with no work shown.

High partial credit:
- An almost correct response such as the outcomes for both Niamh and Peter given.
- Answer with one element missing.

Question 2

(a)   Scale 5C (0, 2, 4, 5)
Low partial credit:
- Any work of merit such as one correct element.

High partial credit:
- An almost correct response such as two or three correct elements.

(b)(i)   Scale 10B (0, 5, 10)
Partial credit:
- Any work of merit such as correct or consistent numerator or denominator in fraction format.

Full credit:
- Correct answer without work shown.

(b)(ii) Scale 5C (0, 2, 4, 5)
Low partial credit:
- Any work of merit such as a correct or consistent partial probability e.g. $\frac{10}{52}$.

High partial credit:
- Almost correct or consistent response e.g. $= \frac{13}{52} + \frac{9}{52}$ and stops.

Full credit:
- Correct answer without work shown.

(b)(iii) Scale 5C* (0, 2, 4, 5)
Low partial credit:
- Any work of merit such as a correct or consistent numerator or denominator in fraction format.

High partial credit:
- Almost correct or consistent response e.g. $= \frac{30}{52} \times \frac{29}{51}$ and stops.

Full credit:
- Correct answer without work shown.

Question 3

(a) Scale 15C (0, 5, 10, 15)
Low partial credit:
▪ Any work of merit such as showing knowledge of plotting a point.

High partial credit:
▪ An almost correct response such as two or three points plotted correctly or a consistent error in plotting all four points.

(b) Scale 5B (0, 2, 5)
Partial credit:
▪ Any work of merit e.g. incomplete statement(s) with some merit.

(c) Scale 5C (0, 2, 4, 5)
Low partial credit:
▪ Any work of merit such as identification of a relevant formula.

High partial credit:
▪ Almost correct response such as one pair of lines shown as parallel or a correct response using a method not described in (b).

Question 4

(a) Scale 10C (0, 4, 7, 10)
Low partial credit:
- Any work of merit such as identification of a relevant formula.

High partial credit:
- An almost correct response e.g. mishandles r^2.

(b) Scale 5C (0, 2, 4, 5)
Low partial credit:
- Any work of merit such as a correct substitution.

High partial credit:
- Almost correct response such as required conclusion not given.

(c) Scale 5C (0, 2, 4, 5)
Low partial credit:
- Any work of merit e.g. use of translation indicated.

High partial credit:
- Almost correct response such as correct centre and radius indicated.
- Correct answer without work shown.

(d) Scale 5C (0, 2, 4, 5)
Low partial credit:
- Any work of merit such as identification of a relevant formula.

High partial credit:
- An almost correct response such as slope of tangent found correctly but fails to finish.
- Correct answer without work shown.

Question 5

(a)(i)   Scale 5B* (0, 2, 5)
Partial credit:
- Any work of merit e.g. leaves diameter $= 4$ or works with $l = \frac{16}{4}$.

    Full credit:
- Correct answer without work shown.

(a)(ii)   Scale 5B* (0, 2, 5)
Partial credit:
- Any work of merit such as identification of r in this part.

    Full credit:
- Correct answer without work shown.

(b)(i)   Scale 10D* (0, 2, 5, 8, 10)
Low partial credit:
- Any work of merit such as r identified as 1.5.

Middle partial credit:
- Further work of merit e.g. correct substitution for r and h or volume of cylinder or cone correct.

High partial credit:
- An almost correct response such as h isolated.

(b)(ii)   Scale 5B* (0, 2, 5)
Partial credit:
- Any work of merit e.g. finds base or height of the box.

Question 6A

(a)(i)   Scale 5B (0, 2, 5)
Partial credit:
- Any work of merit such as an incomplete statement with radius or diameter or point of contact mentioned.

(a)(ii)   Scale 5B (0, 2, 5)
Partial credit:
- Any work of merit e.g. any incorrect line through the point of contact.

(b)   Scale 15D (0, 4, 7, 11, 15)
Low partial credit:
- Any work of merit such as an arc drawn from any vertex of the triangle.
- Midpoint of a triangle side indicated or finds the incentre or centroid.

Middle partial credit:
- Further work of merit such as one perpendicular bisector of the triangle side constructed correctly.

High partial credit:
- An almost correct response where the circumcentre is established correctly but no circle drawn or both mediators and circle drawn correctly but no evidence of construction.

Question 6B

Scale 25C (0, 8, 17, 25).
Low partial credit:
- Any one correct step.

High partial credit:
- One or two steps missing or incorrect.

Section B

Question 7

(a)(i) Scale 10C (0, 4, 7, 10)
Low partial credit:
- Any work of merit such as a rectangle for one year represented correctly or a correct scaled axis shown.

High partial credit:
- An almost correct response such as one or two incorrect rectangles or scales omitted.

(a)(ii) Scale 10C (0, 4, 7, 10).
Low partial credit:
- Any work of merit e.g. at least two correct elements identified.

High partial credit:
- An almost correct response such as formula for mean with fully correct substitution.

Full credit:
- 121 128 without work.

(a)(iii) Scale 5C (0, 2, 4, 5)
Low partial credit:
- Any work of merit e.g. a correct relevant figure identified 86 932 or 54 432.

High partial credit:
- An almost correct response such as one step missing.

Full credit:
- Correct answer without work shown.

(a)(iv) Scale 5B (0, 2, 5)
Partial credit:
- Any work of merit which includes any mention of the year 2009, a decline in sales or an increase in sales.

(a)(v) Scale 5B (0, 2, 5)
Partial credit:
- Any work of merit which includes any mention of an increase of sales in year 2007, a decline in sales in years 2008/2009 or an increase in sales in years 2010/2011.

(b)(i) Scale 5B (0, 2, 5)
Partial credit:
- Any work of merit such as a correct relevant observation that is incomplete.

(b)(ii) Scale 5A (0, 5)
No credit:
- No reason given.

(b)(iii) Scale 5B (0, 2, 5)
Partial credit:
- Any work of merit such as an estimate within the range of values 72 000 to 90 000 without justification or any work with 36 081.

(c)(i) Scale 5C (0, 2, 4, 5)
Low partial credit:
- Any work of merit such as finding any or all of the correct percentages or fractions.

High partial credit:
- An almost correct response such as all angles indicated but drawn incorrectly or not drawn.
- Correct answer without work shown

(c)(ii) Scale 5B (0, 2, 5)
Partial credit:
- Selects B or C.

(d)(i) Scale 5C (0, 2, 4, 5)
Low partial credit:
- Any work of merit e.g. a correct element of stem-leaf plot indicated.

High partial credit:
- An almost complete response such as an incomplete plot with one/two elements missing or incorrect or no key indicated.

(d)(ii) Scale 5C (0, 2, 4, 5)
Low partial credit:
- Any work of merit such as an answer with no reference to stem-leaf or central tendency.

High partial credit:
- An almost complete response such as reference to median/mean but no conclusion.

(d)(iii) Scale 5C (0, 2, 4, 5).
Low partial credit:
- Any work of merit such as an answer with no reference to stem-leaf or measure of variability.

High partial credit:
- An almost complete response such as reference to range or measurement of variability but no conclusion.

Question 8

(a)(i)   Scale 5B (0, 2, 5).
Partial credit:
- Any work of merit such as any triangle drawn with no measurement of sides indicated or defines an isosceles triangle.

Full credit:
- A triangle drawn with three different lengths indicated.

(a)(ii)   Scale 15D* (0, 4, 7, 11, 15).
Low partial credit:
- Any work of merit e.g. an indication that $[FG]$ is an enlargement of $[BC]$ or $[ED]$ or work towards scale factor.

Middle partial credit:
- Further work of merit such as scale factor found.

High partial credit:
- An almost complete response such as scale factor applied to $[BC]$ to get $[ED]$ correctly or fails to finish e.g. leaves $|FG|$ as $1 \cdot 25^2 \times 8$ or $1 \cdot 25 \times 10$.

Full credit:
- Correct answer without work shown.

(a)(iii) Scale 15D* (0, 4, 7, 11, 15)
Low partial credit:
- Any work of merit such as identifying the cosine rule as a method of solution.

Middle partial credit:
- Further work of merit such as significant correct substitution into the cosine formula.

High partial credit:
- An almost complete response such as cosine rule worked to $\sqrt{73}$.
- Correct answer without work shown.

(a)(iv) Scale 5B* (0, 2, 5)
Partial credit:
- Any work of merit such as an effort at finding or explaining the centre of enlargement or an effort at the use of the scale factor to find the centre of enlargement.
- Correct answer without work shown.

[66]

(a)(v)   Scale 10D (0, 2, 5, 8, 10)
Low partial credit:
- Any work of merit e.g. vertical height indicated (from point *G* or in triangles *CBD, EDF*) or correct relevant formula identified or $|FG| = 12 \cdot 5$.
- Correct answer without work shown.

Middle partial credit:
- Further work of merit such as α or sine α found.

High partial credit:
- An almost complete response whereby *h* is found but no conclusion is given.

(b)(i)   Scale 10C* (0, 4, 7, 10)
Low partial credit:
- Any work of merit such as one or two correct measurements shown.
- If (b) (i) not done but at least one correct measurement is shown in (b) (ii).

High partial credit:
- An almost correct response such as one or two incorrect measurements.
- If (b) (i) not done but all correct measurements are shown in (b) (ii).

(b)(ii)   Scale 15D* (0, 4, 7, 11, 15)
Low partial credit:
- Any work of merit such as a correct substitution of *h*, F + L or OTHERS.
- Correct answer without work shown.

Middle partial credit.
- Further work of merit such as two correct substitutions of *h*, F + L or OTHERS.
- One correct substitution followed by some correct calculation.

High partial credit:
- An almost correct response such as fully correct substitution followed by an error in calculation.
- Two correct substitutions followed by fully correct calculation.

Marcanna Breise as ucht Freagairt trí Ghaeilge

(Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. 198 marc × 5% = 9·9 ⇒ bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle [300 – bunmharc] × 15%, agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0