

Pre-Leaving Certificate Examination
   
**Mathematics (Project Maths)**

Paper 2

Higher Level (**with solutions**)

February 2010    2½ hours

300 marks

Examination number
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Centre stamp
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Running total	
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For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade
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## Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions, as follows:

In Section A, answer all six questions

In Section B, answer:

Question 7

Question 8

**either** Question 9A **or** Question 9B.

Answers should be written in the spaces provided in this booklet. Extra paper may be used if needed.

The booklet *Formulae and Tables* may be used.

Marks will be lost if all necessary work is not clearly shown.

Where relevant, answers should include the appropriate units of measurement, and numerical answers should be simplified and given in standard form.

Answer **all six** questions from this section.

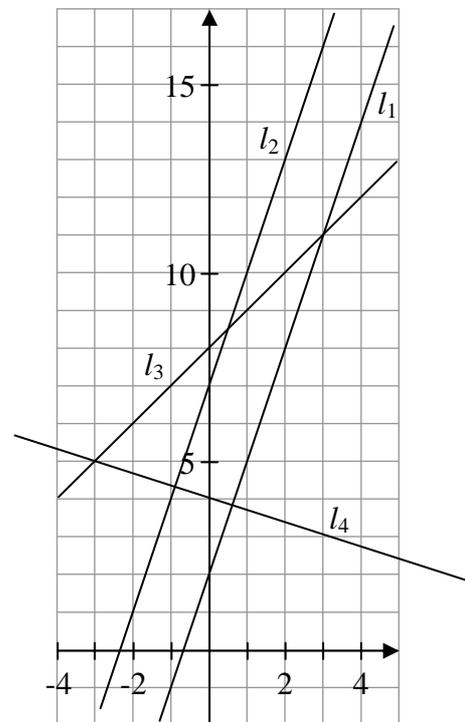
**Question 1****(25 marks)**

The line  $l_1$  in the diagram has slope 3 and y-intercept 2.

- (a) Write down the equation of this line, in the form  $y = mx + c$ . **5 B**

$$y = 3x + 2$$

- (b) On the diagram, draw and label the lines  $l_2$  and  $l_3$ , where: **10 C**  
 $l_2$  has slope 3 and y-intercept 7  
 $l_3$  has slope 1 and y-intercept 8.

**5 B**

- (c) On the diagram, draw and label the line  $l_4$ , which is perpendicular to  $l_1$  and passes through the point  $(0, 4)$ .

- (d) Determine whether  $l_4$  passes through the point  $(27, -4)$ . **5 C**

$$l_4: y = -\frac{1}{3}x + 4 \quad \text{Check } (27, -4)$$

$$\text{Is } -4 = -\frac{1}{3}(27) + 4? \quad -4 \neq -5 \quad (27, -4) \text{ is not on the } l_4.$$

**OR**

Get slope from  $(0, 4)$  to  $(27, -4)$  and show this  $\neq -\frac{1}{3}$ .

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**Question 2****(25 marks)** $P$  is the point  $(0, 7)$  and  $Q$  is the point  $(8, 11)$ .

- (a)**
- Find the equation of the circle with diameter
- $PQ$
- .
- 5 B**

Centre is  $C(4, 9)$ 

$$(x - 4)^2 + (y - 9)^2 = 20$$

Radius  $\sqrt{4^2 + 2^2} = \sqrt{20}$ 

$$[x^2 - 8x + y^2 - 18y + 16 + 81 - 20 = 0$$

$$x^2 - 8x + y^2 - 18y + 77 = 0]$$

- (b)**
- Find the equation of the tangent at
- $Q$
- .
- 5 C**

$$\text{Slope of the radius at } Q = \frac{11-9}{8-4} = \frac{2}{4} = \frac{1}{2}$$

Slope of the tangent to  $Q = -2$ Equation of the tangent:  $(y-11) = -2(x-8)$ 

$$(y-11) = -2x + 16$$

$$2x + y - 27 = 0$$

- (c)**
- This tangent crosses the
- $x$
- axis at the point
- $R$
- . Find the co-ordinates of
- $R$
- .
- 5 B**

$$R \Rightarrow y = 0 \Rightarrow 2x = 27 \Rightarrow x = 13\frac{1}{2} \quad (13\frac{1}{2}, 0)$$

- (d)**
- Find
- $\overrightarrow{PQ}$
- and
- $\overrightarrow{QR}$
- in terms of
- $\vec{i}$
- and
- $\vec{j}$
- , and verify that
- $\overrightarrow{PQ} \cdot \overrightarrow{QR} = 0$
- .
- 10 B**

$$\overrightarrow{PQ} = \vec{Q} - \vec{P} = (8\vec{i} + 11\vec{j}) - (0\vec{i} + 7\vec{j}) = 8\vec{i} + 4\vec{j}$$

$$\overrightarrow{QR} = \vec{R} - \vec{Q} = (13\frac{1}{2}\vec{i} + 0\vec{j}) - (8\vec{i} + 11\vec{j}) = 5\frac{1}{2}\vec{i} - 11\vec{j}$$

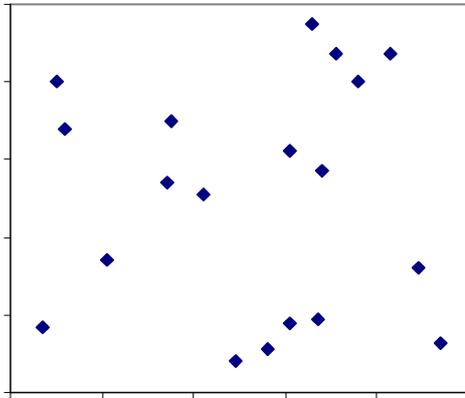
$$(8\vec{i} + 4\vec{j}) \cdot (5\frac{1}{2}\vec{i} - 11\vec{j}) = 44 - 44 = 0$$

**Question 3**

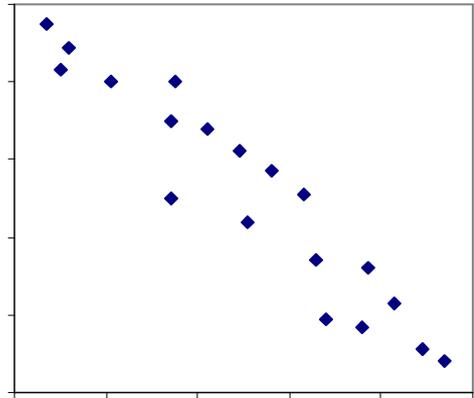
**(25 marks)**

(a) For each of the four scatter plots below, estimate the correlation coefficient.

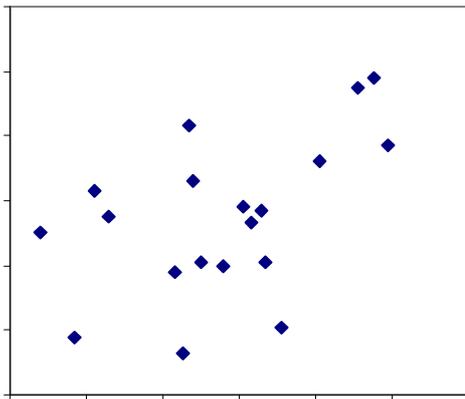
**15 C**



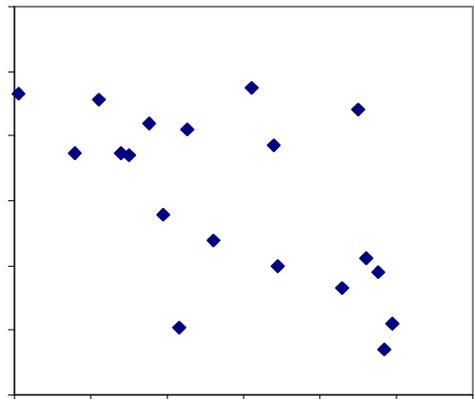
Correlation  $\approx 0$



Correlation  $\approx -0.9$



Correlation  $\approx +0.5$



Correlation  $\approx -0.6$

(b) Using your calculator, or otherwise, find the correlation coefficient for the data given in the table.

Give your answer correct to two decimal places. **10 B\***

x	y
0.0	0.5
5.0	1.3
5.2	3.3
6.1	6.7
9.3	4.5
9.5	4.6
9.9	6.5

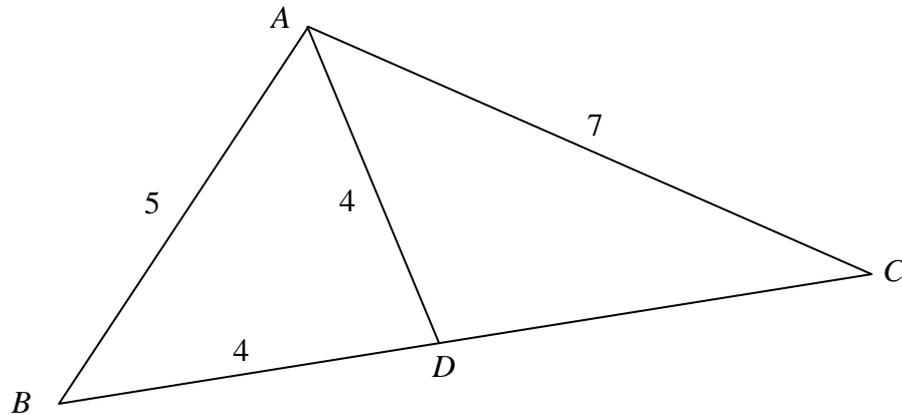
Answer:

$0.7582 \Rightarrow 0.76$

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Question 4 (25 marks)

- (a)  $ABC$  is a triangle, and  $D$  is a point on  $[BC]$ .  
The lengths  $|AB|$ ,  $|AD|$ ,  $|AC|$  and  $|BD|$  are as shown in the diagram.



Find  $|DC|$ , correct to one decimal place.

**5 B**

In triangle ABD

$$s^2 = a^2 + b^2 - 2ab \cos D$$

$$5^2 = 16 + 16 - 2(4)(4)\cos D$$

$$\frac{-7}{-32} = \cos D = \frac{7}{32}$$

In triangle ADC

$$7^2 = l^2 + 4^2 - 2(4)(l)\left(-\frac{7}{32}\right)$$

$$0 = l^2 + \frac{7}{4}l - 33$$

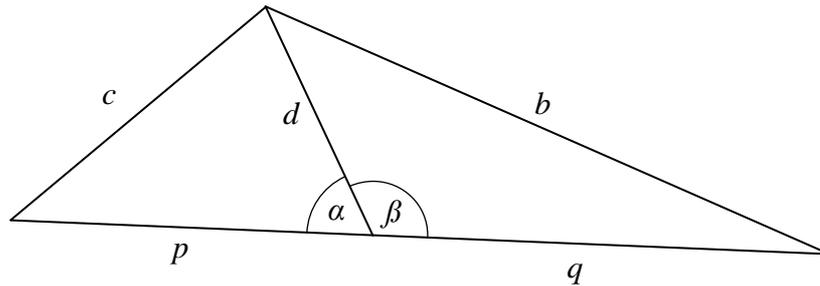
$$\frac{-1.75 \pm \sqrt{1.75^2 + 132}}{2}$$

$$l = 4.94 \text{ or } l = -6.69$$

$$|DC| = 4.9 \text{ to one decimal place}$$

**5 B\***

(b) Consider the diagram below.



(i) Express  $\cos \alpha$  and  $\cos \beta$  in terms of the labelled lengths.

**5 B**

$$c^2 = p^2 + d^2 - 2pd \cos \alpha$$

$$b^2 = d^2 + q^2 - 2dq \cos \beta$$

$$\frac{c^2 - p^2 - d^2}{-2pd} = \cos \alpha$$

$$\frac{b^2 - d^2 - q^2}{-2dq} = \cos \beta$$

(ii) Show that  $pb^2 + qc^2 = (p+q)(pq + d^2)$

**10 C**

$$\cos \alpha = -\cos \beta$$

$$\frac{c^2 - p^2 - d^2}{-2pd} = -\left(\frac{b^2 - d^2 - q^2}{-2dq}\right)$$

$$-2dq c^2 + 2dqp^2 + 2d^2q = 2pdb^2 - 2pd^2 - 2pdq^2$$

$$qp^2 + qd^2 + pd^2 + pq^2 = pb^2 + qc^2$$

$$p(pq + d^2) + q(pq + d^2) = pb^2 + qc^2$$

$$(p+q)(pq + d^2) = pb^2 + qc^2$$

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**Question 5****(25 marks)****(a)** The following formula relates to the *binomial distribution*.

$$P(X = r) = \binom{n}{r} p^r q^{n-r}$$

**(i)** State what each of the letters  $p$ ,  $q$ ,  $n$ , and  $r$  represents in the formula above. **5 C** $p$  is The probability of success. $q$  is The probability of failure OR  $q = 1 - p$  $n$  is The number of trials. $r$  is Number of successes.**(ii)** Describe the type of experiment that results in a random variable that has a binomial distribution.**5 C**

Repeated trials that are independent of each other in which there are only two possible outcomes, success and failure, and where the probability of success is the same for each trial.

**(b)** In a certain type of archery competition, Laura hits the target with an average of two out of every three shots. The shots are independent of each other. During one such competition, she has ten shots at the target.**(i)** Find the probability that Laura hits the target exactly nine times.

Give your answer correct to three decimal places.

**(ii)** Find the probability that Laura hits the target fewer than nine times.

Give your answer correct to three decimal places.

**(i)**  $p = \frac{2}{3}$     $q = \frac{1}{3}$     $n = 10$     $r = 9$    **5 B\***

$$P(9 \text{ hits}) = \binom{10}{9} \left(\frac{2}{3}\right)^9 \left(\frac{1}{3}\right)^1 = \frac{10 \times 2^9}{3^{10}} = 0.0867$$

$$P(9 \text{ hits}) = 0.087$$

**(ii)**  $P(10 \text{ hits}) = \left(\frac{2}{3}\right)^{10} = 0.0173$ **10 C\***

$$P(9 \text{ hits}) = 0.0867$$

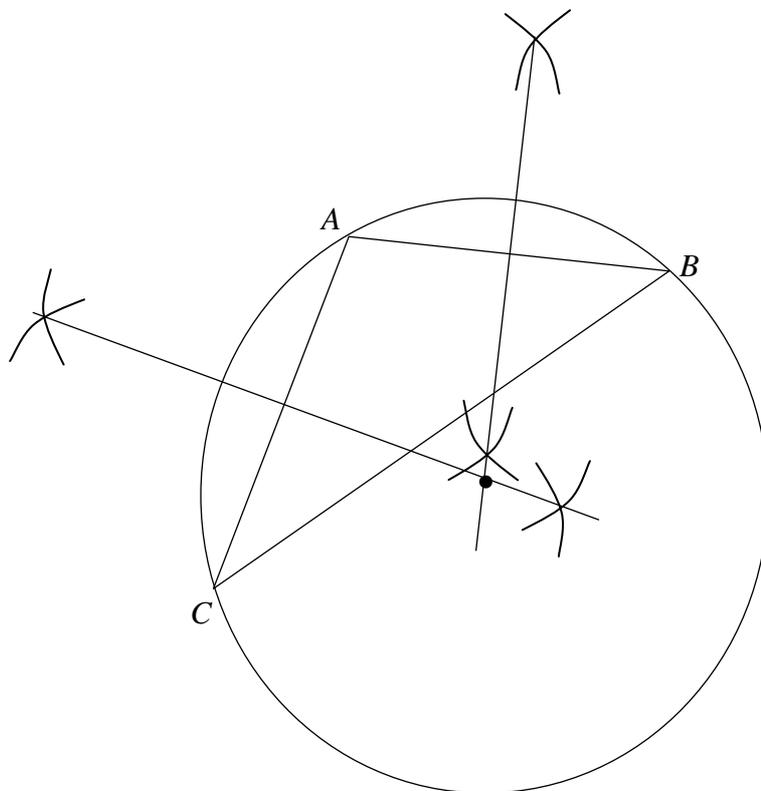
$$P(<9) = 1 - (0.0867 + 0.0173) = 0.8960$$

$$\text{Thus fewer than 9 times } \Rightarrow \text{probability} = 0.896$$

**Question 6**

**(25 marks)**

- (a) Construct the circumcircle of the triangle  $ABC$  below using only a compass and straight edge. Show all construction lines clearly. **10 C**



- (b) State the condition(s) under which the circumcentre of a triangle will lie **inside** the triangle, and justify your answer.

Condition(s):

**5 A**

The triangle must have all its angles acute OR equivalent.

Justification:

**10 C**

[If an angle  $(A) = 90^\circ$  the centre lies on the side opposite angle A, i.e. on BC.]

If an angle  $(A) > 90^\circ$  then the angle at the centre on the same arc is  $> 180^\circ$   
 $\Rightarrow$  the centre lies on the opposite side of BC from A.

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Answer Question 7, Question 8, and **either** Question 9A **or** Question 9B.

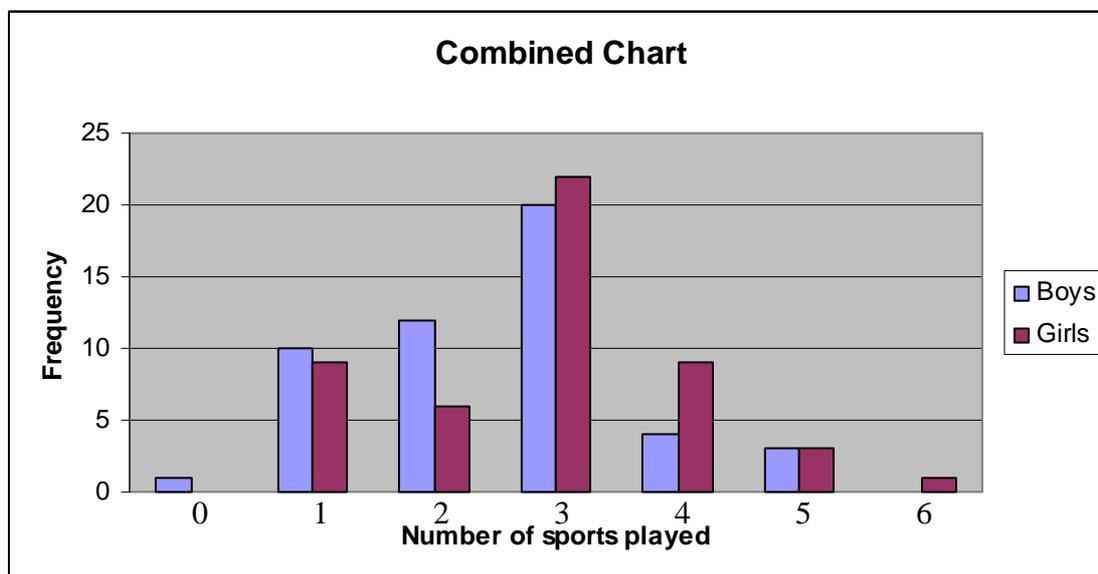
**Question 7****Probability and Statistics****(50 marks)**

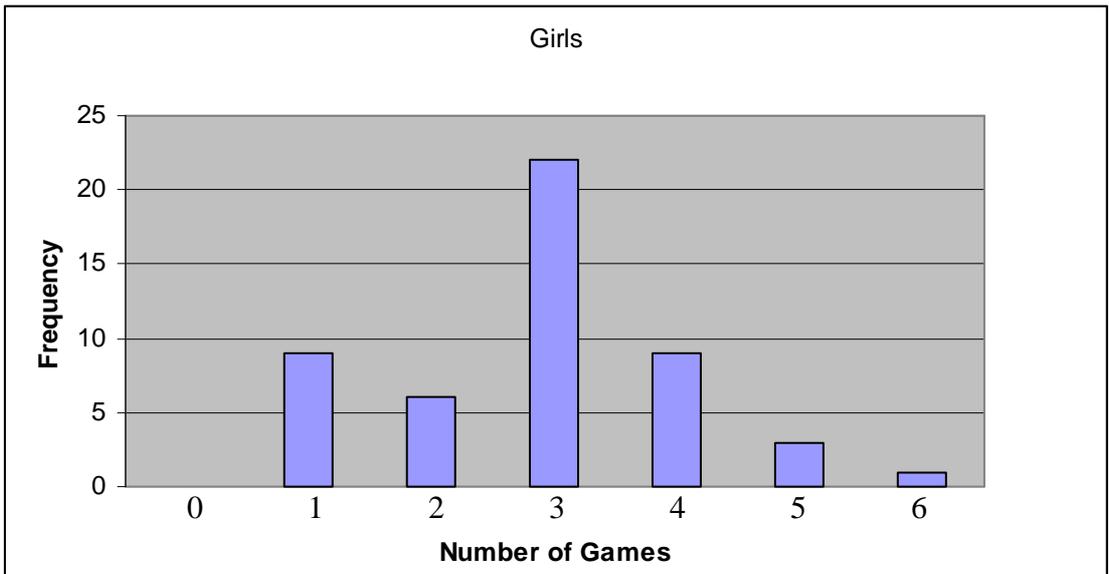
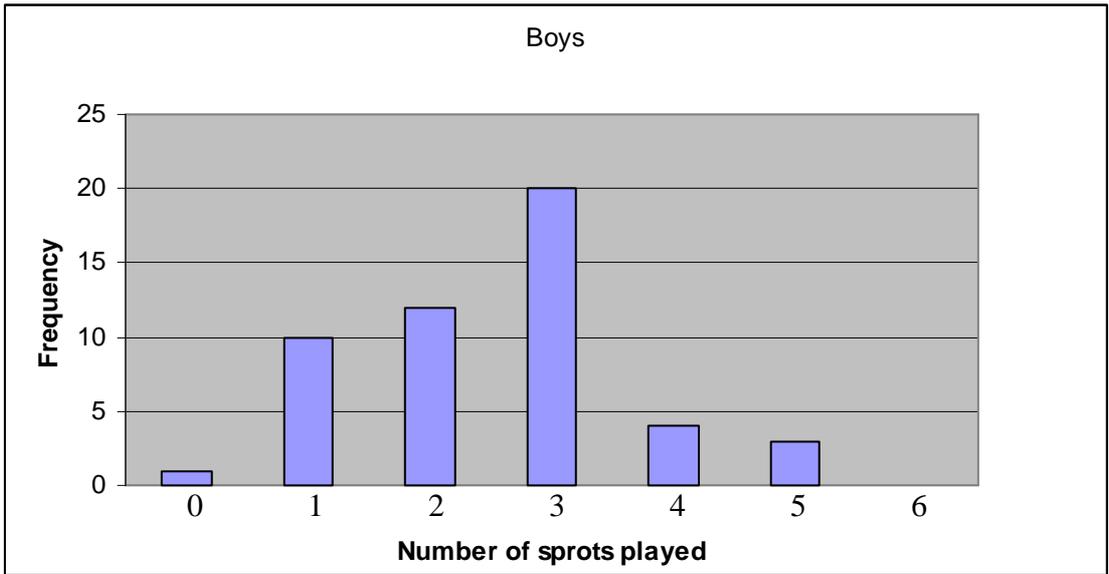
Some research was carried out into the participation of girls and boys in sport. The researchers selected a simple random sample of fifty male and fifty female teenagers enrolled in GAA clubs in the greater Cork area. They asked the teenagers the question: *How many sports do you play?*

The data collected were as follows:

Boys	Girls
0, 4, 5, 1, 4, 1, 3, 3, 3, 1,	3, 3, 3, 1, 1, 3, 3, 1, 3, 3,
1, 2, 2, 2, 5, 3, 3, 4, 1, 2,	2, 2, 4, 4, 4, 5, 5, 2, 2, 3,
2, 2, 2, 3, 3, 3, 4, 5, 1, 1,	3, 3, 4, 1, 6, 2, 3, 3, 3, 4,
1, 1, 1, 2, 2, 2, 2, 2, 3, 3,	4, 5, 3, 4, 3, 3, 3, 4, 4, 3,
3, 3, 3, 3, 3, 3, 3, 3, 3, 3	1, 1, 3, 2, 1, 3, 1, 3, 1, 3

(a) Display the data in a way that gives a picture of each distribution.

**15 C**





- (b) State **one difference** and **one similarity** between the distributions of the two samples.

*Difference:*

**5 A**

Any statement of difference about the overall distributions (not about individual data points).

For example:

12 out of 50 girls play more than 3 sports, whereas only 7 out of 50 boys play more than 3 sports.

*Similarity:*

**5 A**

Any statement of similarity about the overall distributions (not about individual data points).

For example:

The range/spread (or median, or mode) of the distribution is the same for both.

- (c) Do you think that there is evidence that there are differences between the two populations? Explain your answer.

*Note: you are not required to conduct a formal hypothesis test.*

**10 B**

Answer:

Yes

No

Justification:

The samples are sufficiently different that this could not be just by chance.

The samples are similar enough that the difference could be due to chance.

- (d) The researchers are planning to repeat this research on a larger scale. List **two** improvements they could make to the design of the research in order to reduce the possibility of *bias* in the samples. Explain why each improvement you suggest will reduce the likelihood of bias.

**15 C**

They could include other boys and girls not just those in GAA clubs. This would be more representative and help to reduce bias.

Widen the sample to include those from rural as well as urban areas This would be more representative and help to reduce bias.

Change the question to be more precise about what is meant by 'playing sports'. This would reduce bias by ensuring that answers will more accurately reflect participation in sports.

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**Question 8****Geometry and Trigonometry****(50 marks)**

The *Wonder Building* is an arched building that does not need any support inside, due partly to the fact that its shape is an arc of a circle.

The photograph shows a *Wonder Building* being used in Antarctica.



The arc for a *Wonder Building* can be a full semicircle or less than a semicircle. It cannot be more than a semi-circle. The “span” of the building is the total width from one side of the arch to the other.

- (a) A particular *Wonder Building* has a span of 30 metres and a height of 10 metres.  
Find the radius of the arc.

**15 C\***

$$r^2 = 15^2 + (r - 10)^2$$

$$r^2 = 225 + r^2 - 20r + 100$$

$$20r = 325$$

$$r = 16.25 \text{ m}$$

- (b) A customer wants a building with a span of 18 metres and a height of 10 metres.

- (i) What arc radius would be required to give such a building? **10 B\***

$$r^2 = 9^2 + (r - 10)^2$$

$$r^2 = 81 + r^2 - 20r + 100$$

$$20r = 181$$

$$r = 9.05 \text{ m}$$

(ii) Explain why the *Wonder Building* that the customer wants is not possible.

### 5 B

Statement that shows understanding of relationship of height and width for given conditions.

For example:

$R - 10$  cannot be negative. The height can not be greater than half the width.

The radius of 9.05 is less than the height required, so it needs more than a semi-circle, which is the limit of design.

The span is greater than 18 m at the diameter for a radius of 9.05

(c) An air force needs a *Wonder Building* to house a *Tornado* military jet.

The dimensions of the aircraft are as follows:

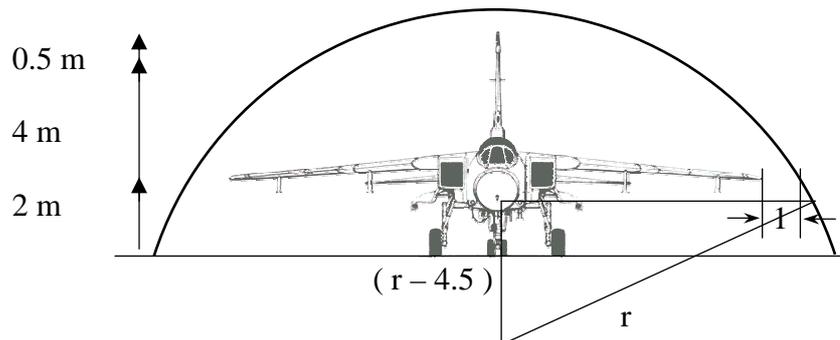
- Wingspan: 14 metres
- Height: 6 metres
- Height of wingtips above ground: 2 metres.



The shelter must be at least 0.5 metres above the top of the tail, and at least 1 metre clear horizontally of the wingtips.

For the shelter to have **minimum height**, find the smallest possible radius of the arc.

20 D\*



$$\begin{aligned}r^2 &= 8^2 + (r - 4.5)^2 \\ &= 64 + r^2 - 9r + 20.25 \\ 9r &= 84.25 \\ r &= 9.3611... \text{ metres}\end{aligned}$$

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**Question 9A****Probability and Statistics****(50 marks)**

A national newspaper reported in January 2010 that the average rent for a 3 bed-roomed house in Ireland was €824 per month. A Dublin estate agent conducted a survey of 40 such properties in the greater Dublin area to see if the same was true there. The estate agent's sample had a mean rent of €1090 and a standard deviation of €480.

- (a) Using the estate agent's sample, construct a 95% confidence interval for the mean monthly rent of a 3-bed-roomed house in Dublin. **15 C\***

95%  $\Rightarrow$  z value = 1.96 (thus  $\mu$  lies in the region  $\bar{x} \pm 1.96\sigma_{\bar{x}}$  )

$$1090 \pm 1.96 \left( \frac{480}{\sqrt{40}} \right) = 1090 \pm 148.75$$

$$1090 - 148.75 = 941.25$$

}  $\Rightarrow$  between €941.25 and €1238.75

$$1090 + 148.75 = 1238.75$$

- (b) Explain what your 95% confidence interval means. **5 A\***

You can be 95% certain (confident) that the mean rent for a 3-bedroomed house in Dublin lies between € 941.25 and €1238.75

- (c) The estate agent says that the survey shows that the average rent in Dublin is not the same as the average rent in the country as a whole.

Assuming the information reported by the newspaper is accurate, is there evidence to support the estate agent's statement? Conduct a hypothesis test at the 5% level of significance.

Clearly state the null hypothesis, the alternative hypothesis, and the conclusion. **10 C**

$H_0$  :the average rent in Dublin is € 824 per month

$H_1$  :the average rent in Dublin is not €824 per month.

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{1090 - 824}{\frac{480}{\sqrt{40}}} = 3.50 \text{ which is } \geq 1.96, \text{ so we reject } H_0.$$

The estate agent was justified in his statement.

Other variations are also acceptable, such as using interval found in part (a) above, getting p value for two-tailed test, etc. (cf. SEC Trialling Report p. 42)

(d) Describe **two** different ways that the estate agent might have collected the data in the sample. In each case, state clearly:

- what list is used to draw the sample from
- how the relevant information about each item is found, and
- what possible sources of bias there are. **10 C × 2**

Valid methods which include the three specified elements. For example:

He could have randomly selected the properties from his own list. However, this may be limited to particular types of properties or particular locations, or the asking rent price, which may cause bias.

He could have randomly selected areas of Dublin and then randomly selected properties from local estate agents' lists of rented properties in each area. This may be biased due to the types of properties the estate agents handle in particular areas (or the fact that the properties are not actually rented).

He could look up newspapers or surf websites and randomly select 40 properties, then check their rent prices. This may be biased since it does not include privately rented properties.

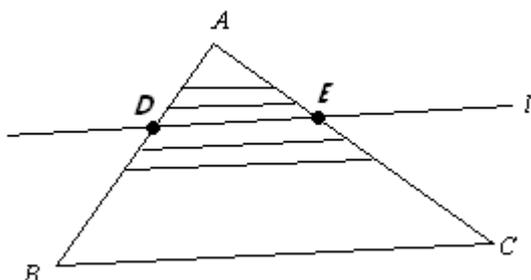
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**Question 9B**

**Geometry and Trigonometry**

**(50 marks)**

- (a) Let  $\triangle ABC$  be a triangle. The line  $l$  is parallel to  $BC$  and cuts  $|AB|$  in the ratio  $s : t$ , where  $s$  and  $t$  are natural numbers. Prove that  $l$  also cuts  $AC$  in the ratio  $s : t$ .



Let  $D$  be the point where line  $l$  cuts  $AB$  and  $E$  the point where it cuts  $AC$ .

Construction: Mark off points along  $AB$  to form  $s + t$  equal segments so that  $s$  of these segments lie in  $AD$  and  $t$  of them lie in  $DB$ .

Draw lines from these points parallel to line  $l$ , which will divide  $AC$  into  $s + t$  segments. **10 B**

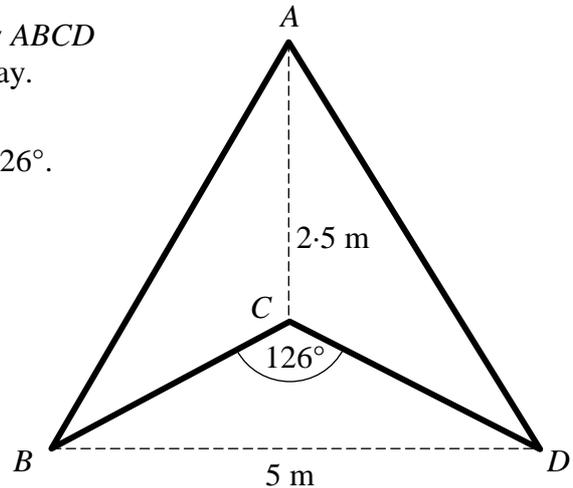
Proof: All of the  $s + t$  segments on  $AC$  will be of equal length (Theorem 11).

There will be  $s$  of them along  $AE$  and  $t$  of them along  $EC$  (axiom of parallels).

Thus  $l$  divides  $AC$  in the ratio  $s : t$  **15 C**

- (b) The diagram represents a large symmetrical arrow  $ABCD$  that is painted on the ground at one end of a runway.

The distance  $BD$  is 5 metres and distance  $CA$  is 2.5 metres, as shown. The angle  $ADB$  measures  $126^\circ$ .



- (i) Find the length of the perimeter of the arrow.  
Give your answer correct to one decimal place.

**15 C\***

$$\angle CBD = \angle CDB \Rightarrow \angle CBD = \frac{1}{2}(180 - 126) = 27^\circ$$

$$\tan 27^\circ = \frac{h}{2.5} \Rightarrow h = 2.5 (\tan 27) = 1.27381$$

$$|CB| = \sqrt{2.5^2 + h^2} = 2.80582 = |CD|$$

$$|AB| = \sqrt{2.5^2 + (h+2.5)^2} = 4.52677 = |AD|$$

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$$7.33259$$

$$\text{Perimeter} = 7.33259 \times 2 = 14.66518 = 14.7 \text{ m to one decimal place.}$$

(Note: alternative solutions exist, e.g. using sin rule in isosceles triangle  $BCD$ , or  $\cos \angle CBD$  in right triangle with  $BC$  as hypotenuse.)

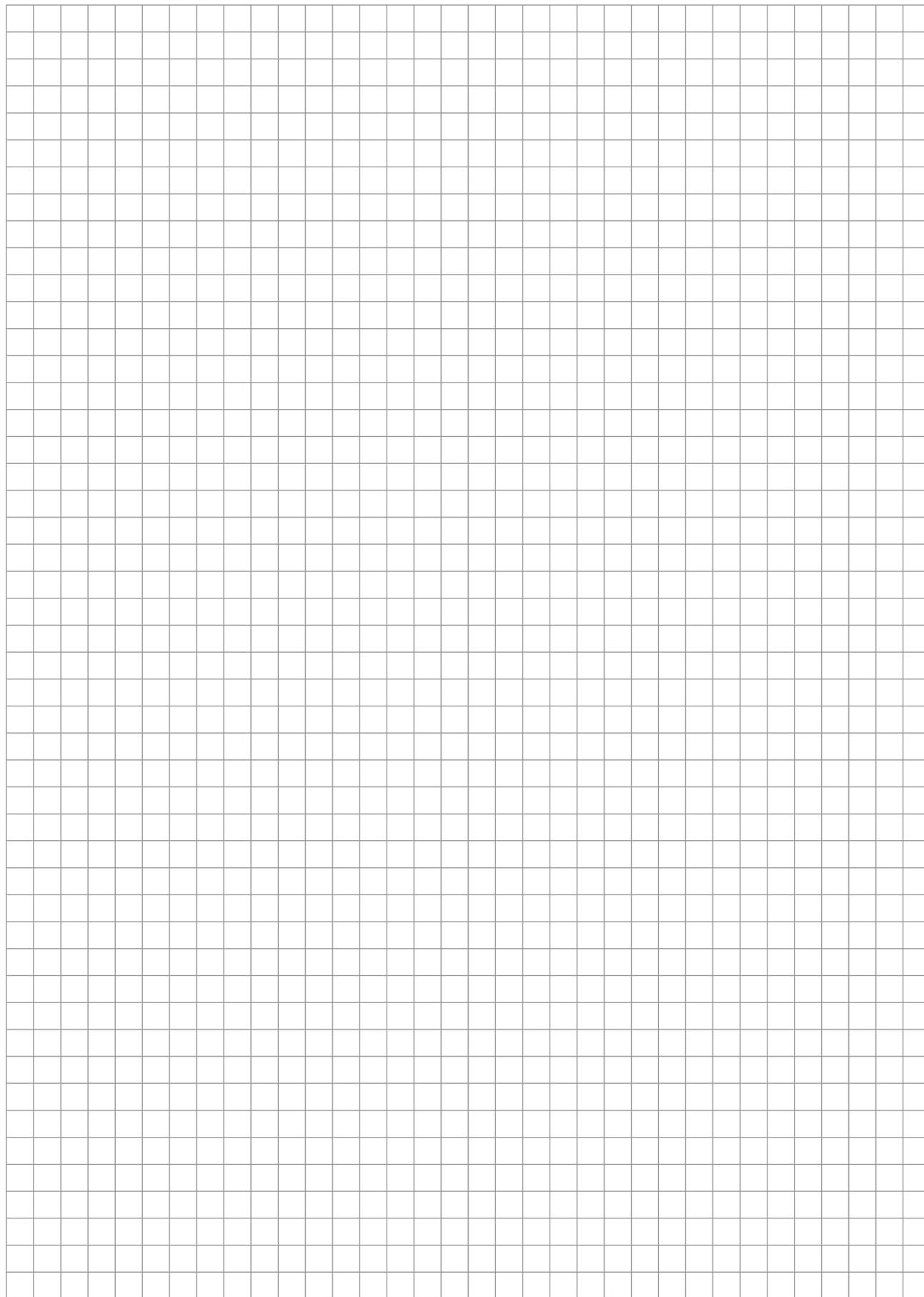
- (ii) *Cat's eyes* are reflective devices used in road markings. They are being laid along the perimeter of the arrow. One is placed at each vertex, and others are placed at intervals of no more than half a metre along the perimeter. How many are needed?

**10 C**

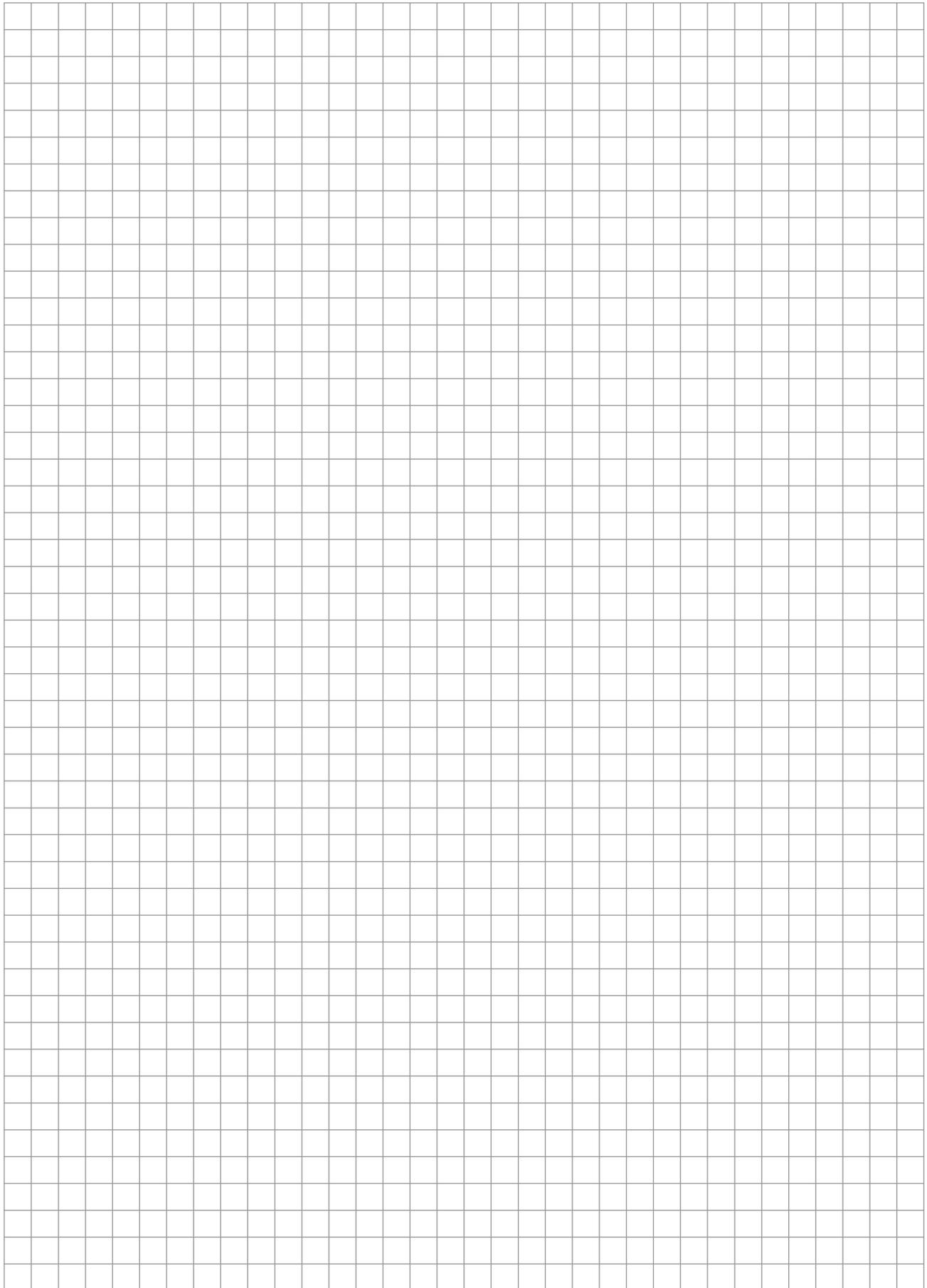
4 at the vertices	=	4
5 x 2 along $CB, CD$	=	10
9 x 2 along $AB, AD$	=	18
Total	=	32

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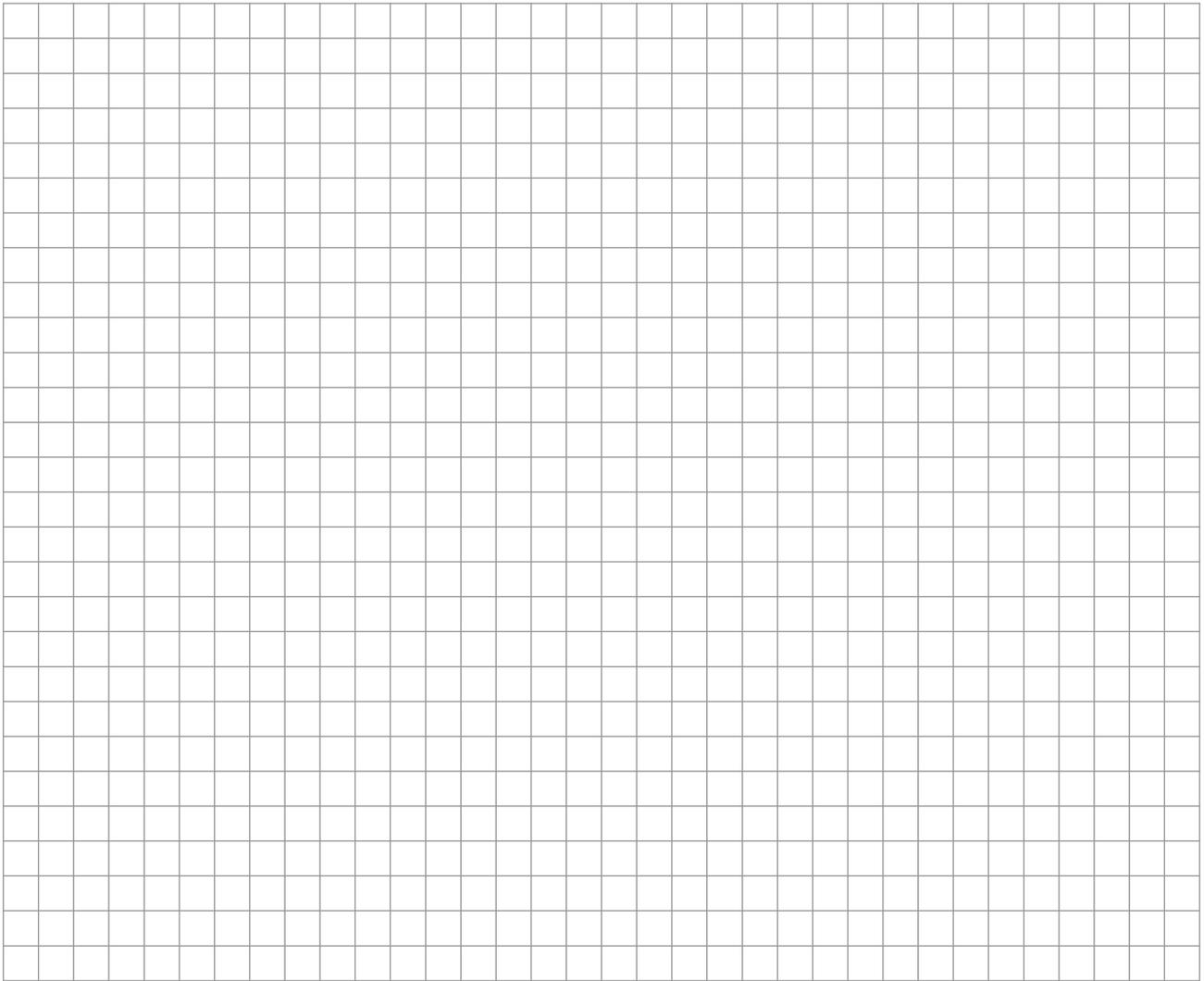
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*Note to readers of this document:*

This pre-Leaving Certificate paper is intended to help teachers and candidates in the 24 *Project Maths* initial schools prepare for the June 2010 examination. The content and structure of the paper do not necessarily reflect the 2011 or subsequent examinations in the initial schools or in all other schools.

## Mathematics (Project Maths) – Paper 2

Pre-Leaving Certificate Paper – Higher Level  
February 2010