

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2019

Marking Scheme

Mathematics

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

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Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 2, 5	0, 2, 3, 5		
10 mark scales	0, 10	0, 5, 10	0, 4, 7, 10	0, 4, 5, 8, 10	
15 mark scales	0, 15	0, 7, 15	0, 5, 10, 15	0, 4, 7, 11, 15	
20 mark scales	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scales	0, 25	0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, in *scale 10C*, 9 marks may be awarded.

Throughout the scheme indicate by use of # where an arithmetic error occurs.

Summary of mark allocations and scales to be applied

Section A

Question 1
 (a) 10D
 (b) 15D

Question 2
 (a)(i) 5C
 (a)(ii) 5C
 (b) 15D

Question 3
 (a) 5B
 (b) 10D
 (c) 10D

Question 4
 (a) 5C
 (b)(i) 10D
 (b)(ii) 10D

Question 5
 (a) 10C
 (b)(i) 5C
 (b)(ii) 10C

Question 6
 (a)(i) 10C
 (a)(ii) 5C
 (b) 10D











Section B

Question 7 (45 Marks)
 (a)(i) 10C
 (a)(ii) 5C
 (a)(iii) 5C
 (b)(i) 10C
 (b)(ii) 5B
 (b)(iii) 10C

Question 8 (50 Marks)
 (a) 10C
 (b) 10D
 (c) 5C
 (d) 10C
 (e) 15D


Question 9 (55 Marks)
 (a)(i) 5C
 (a)(ii) 5C
 (b)(i)+(ii) 15D
 (b)(iii) 5C
 (c)(i) 5C
 (c)(ii) 5B
 (c)(iii) 15D

Palette of annotations available to examiners



Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
	Tick	Work of relevance	The work presented in the body of the script merits full credit
	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
	Hash	Rounding error Unit error Arithmetic error Misreading	
	Horizontal wavy	Error	
	Tick L		The work presented in the body of the script merits low partial credit
	Tick M		The work presented in the body of the script merits mid partial credit
	Tick H		The work presented in the body of the script merits high partial credit
	Left Bracket		Another version of this solution is presented elsewhere and is worth equal or higher credit
	Vertical wavy	No work on this page (portion of the page)	
	Oversimplify	The candidate has oversimplified the work	

Note: It may be necessary to use a combination of 2 symbols in the right margin to clearly show your judgement of the work in the body of the script:

 must be used to signify that Full Credit – 1 is merited by the work presented

 Signifies that the work in the body of the script is worth mid partial credit but another effort at the work has been awarded this or higher credit

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work

e.g. In a **C scale** where # and  and  appear in the body of the work then ✓¹ should be placed in the right margin.

In the case of a **D scale** with the same level of annotation then ✓^m should be placed in the right margin.

A ✓ in the body of the work may sometimes be used indicate where a portion of the work presented has value and has merited one of the levels of credit described in the marking scheme.

The level of credit is then indicated in the right margin.

Model Solutions & Detailed Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 25 Marks	Marking Notes
(a)	$(2x + 1)(x^2 + px + 4)$ $2x^3 + 2px^2 + 8x + x^2 + px + 4$ $8 + p = 2(2p + 1)$ $8 + p = 4p + 2$ $3p = 6$ $p = 2$ <p style="text-align: center;">Or</p> <p>Coefficient of x is $8 + p$ Coefficient of x^2 is $2p + 1$</p> $8 + p = 2(2p + 1)$ $8 + p = 4p + 2$ $3p = 6$ $p = 2$	<p>Scale 10D (0, 4, 5, 8, 10)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - Any relevant multiplication <p><i>Mid Partial credit:</i></p> <ul style="list-style-type: none"> - Multiplication completed without error(s) - Multiplication completed with errors and correctly identifies (in terms of p) the coefficient of either x^2 or x - Correctly identifies the coefficient of either x or x^2 <p><i>High Partial credit:</i></p> <ul style="list-style-type: none"> - Multiplication completed with error(s) but finishes correctly without further errors - Relevant coefficients equated (equation in p) - Multiplication completed and coefficients of x^2 and x identified but solves incorrect equation in p

(b)

$$\frac{3}{2x+1} + \frac{2}{5} = \frac{2}{3x-1}$$

$$\text{CD: } 5(2x+1)(3x-1)$$

$$15(3x-1) + (4x+2)(3x-1) \\ = 10(2x+1)$$

$$12x^2 + 27x - 27 = 0$$

$$4x^2 + 9x - 9 = 0$$

$$(x+3)(4x-3)=0$$

$$x = -3 \text{ or } x = \frac{3}{4}$$

Or

$$\frac{3}{2x+1} + \frac{2}{5} = \frac{2}{3x-1}$$

$$\frac{15 + 2(2x+1)}{5(2x+1)} = \frac{2}{3x-1}$$

$$\frac{4x+17}{10x+5} = \frac{2}{3x-1}$$

$$(4x+17)(3x-1) = 2(10x+5)$$

$$12x^2 + 47x - 17 = 20x + 10$$

$$12x^2 + 27x - 27 = 0$$

$$4x^2 + 9x - 9 = 0$$

$$(x+3)(4x-3)=0$$

$$x = -3 \text{ or } x = \frac{3}{4}$$

Scale 15D (0, 4, 7, 11,15)

Low Partial Credit:

- CD or partial CD identified
- Cross multiply on LHS
- Multiplies one term correctly by one of the denominators
- $x = -3$ or $x = \frac{3}{4}$ substituted and justified as a solution


Mid Partial Credit:

- Equation without fractions

High Partial Credit:

- Relevant quadratic in the form:
 $ax^2 + bx + c = 0$

Note: No quadratic \Rightarrow low partial credit at most, except in the case where the candidate has reached the mid partial stage

Q2	Model Solution – 25 Marks	Marking Notes
<p>(a) (i)</p>	<p>(0, 1) (2, 9)</p> 	<p>Scale 5C (0, 2, 3, 5) <i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - 1 point on line found <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - 2 points on line found - 1 point found and plotted (apart from (0, 1) and (2, 9)) <p><i>Full Credit -1:</i></p> <ul style="list-style-type: none"> - Freehand graph drawn
<p>(a) (ii)</p>	$g(1.9) = 4(1.9) + 1 = 8.6$ $f(1.9) = 3^{1.9} = 8.06$ $f(x) < g(x)$	<p>Scale 5C (0, 2, 3, 5) <i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - $g(1.9)$ written or found - $f(1.9)$ written or found <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - $g(1.9)$ and $f(1.9)$ found

Q3	Model Solution – 25 Marks	Marking Notes
(a)	$(3x + 4)(y - 3)$	<p>Scale 5B (0, 2, 5) <i>Mid Partial Credit:</i></p> <ul style="list-style-type: none"> - Any relevant factorisation
(b)	$3x \ln x - 9x + 4 \ln x - 12 =$ $3x(\ln x - 3) + 4(\ln x - 3) =$ $(3x + 4)(\ln x - 3)$ $3x + 4 = 0 \Rightarrow x = -\frac{4}{3} \text{ (not possible)}$ $\ln x - 3 = 0$ $\ln x = 3$ $x = e^3$	<p>Scale 10D (0, 4, 5, 8, 10) <i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - Any relevant factorisation of $g(x)$ - Trial and improvement with at least two values tested - Substitutes $20 \leq x \leq 20.1$ - $y = \ln x$ <p><i>Mid Partial Credit</i></p> <ul style="list-style-type: none"> - Expression fully factorised <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - $\ln x = 3$ <p><i>Full Credit –1:</i></p> <ul style="list-style-type: none"> - Both solutions presented <p><u>Note:</u> Accept $x = 20.1$ for $x = e^3$ in the last line of the solution</p> <p><u>Note:</u> If no reference is made to $3x + 4$ in the solution, then award high partial credit at most</p>

<p>(c)</p>	$g'(x) = 3x \left(\frac{1}{x}\right) + (3)\ln x - 9 + 4 \left(\frac{1}{x}\right)$ $g'(e) = 3(e) \left(\frac{1}{e}\right) + (3)\ln(e) - 9 + 4 \left(\frac{1}{e}\right)$ $g'(e) = 3 + 3 - 9 + \frac{4}{e} = -1.53$	<p>Scale 10D (0, 4, 5, 8, 10)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - Any relevant differentiation - $g(e)$ evaluated correctly to at least 2 decimal places <p><i>Mid Partial Credit</i></p> <ul style="list-style-type: none"> - Expression fully differentiated - Product rule not applied but finishes correctly <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - Derivative fully substituted
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Q4	Model Solution – 25 Marks	Marking Notes
(a)	$\frac{4x^4}{4} - \frac{6x^2}{2} + 10x + C$ $x^4 - 3x^2 + 10x + C$	<p>Scale 5C (0, 2, 3, 5)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - Any relevant integration <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - 3 correct terms
(b) (i)	$\int (6x^2 - 54x + 109) dx$ $= 2x^3 - 27x^2 + 109x + C = f(x)$ <p>$(2, 0) \in f(x)$</p> $2(2)^3 - 27(2)^2 + 109(2) + C = 0$ $2(8) - 27(4) + 218 + C = 0$ $16 - 108 + 218 + C = 0$ $16 + 110 + C = 0$ $126 + C = 0$ $C = -126$ $\therefore f(x) = 2x^3 - 27x^2 + 109x - 126$	<p>Scale 10D (0, 4, 5, 8, 10)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - Any relevant integration <p><i>Mid Partial Credit</i></p> <ul style="list-style-type: none"> - 3 correct terms <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - Relevant equation in C <p><u>Note:</u> Must integrate or indicate integration to gain any credit</p>

<p>(b) (ii)</p>	<p>2 is a root $\Rightarrow (x - 2)$ is a factor $2x^3 - 27x^2 + 109x - 126 = 0$ $2x^2(x - 2) - 23x(x - 2) + 63(x - 2)$ $2x^2 - 23x + 63 = 0$ $(2x - 9)(x - 7) = 0$ $x = 4.5$ or $x = 7$ $\therefore B(4.5, 0)$ and $C(7, 0)$</p>	<p>Scale 10D (0, 4, 5, 8, 10) <i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - 2 identified as root - 0 given as the y co-ordinate - Sets up equation - Any integer fully substituted in $f(x)$ fully worked - $(x - 2)$ is a factor - Sets up the correct equation <p><i>Mid Partial Credit</i></p> <ul style="list-style-type: none"> - Division completed with no remainder - 7 identified as a root - One coordinate pair found <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - x values found from factors
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Q5	Model Solution – 25 Marks	Marking Notes
(a)	$3 - 2i = \text{other root}$ $-p = (3 + 2i) + (3 - 2i) = 6$ $p = -6$ $q = (3 + 2i)(3 - 2i) = 13$ <p style="text-align: center;">Or</p> $(3 + 2i)^2 + p(3 + 2i) + q = 0$ $5 + 12i + 3p + 2pi + q = 0$ $2p = -12 \Rightarrow p = -6$ $5 + 3p + q = 0 \Rightarrow q = 13$ <p style="text-align: center;">Or</p> $\frac{-p \pm \sqrt{p^2 - 4q}}{2} = 3 \pm 2i$ $-p \pm \sqrt{p^2 - 4q} = 6 \pm 4i$ $-p = 6$ $\therefore p = -6$ $\sqrt{4q - p^2} = 4$ $4q - p^2 = 16$ $4q - (-6)^2 = 16$ $4q = 52$ $\therefore q = 13$	<p>Scale 10C (0, 4, 7, 10)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - Second root identified <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - Sum and product of roots formulated into equations for p and q - p or q found correctly <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - Root substituted into equation - Any correct substitution <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - Real and imaginary terms formulated into equations for p and for q <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - Some substitution into quadratic formula <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - Finds p - Full substitution into quadratic formula and equated to either root.

<p>(b) (i)</p>	$ v = \sqrt{4 + 12} = 4$ $\theta = 300^\circ$ $v = 4(\cos 300^\circ + i \sin 300^\circ)$ <p style="text-align: center;">Or</p> $ v = \sqrt{4 + 12} = 4$ $\theta = \frac{5\pi}{3}$ $v = 4\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right)$	<p>Scale 5C (0, 2, 3, 5)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - Correct plot on the Argand diagram - Some use of Pythagoras to find modulus - Some use of trigonometry to find argument <p><i>High Partial Credit:</i></p> <p>Modulus or argument found</p> <p><u>Note:</u> Accept $4\left(\cos -\frac{\pi}{3} + i \sin -\frac{\pi}{3}\right)$ and $4(\cos -60^\circ + i \sin -60^\circ)$</p>
<p>(b) (ii)</p>	$w = \pm v^{\frac{1}{2}}$ $w = \pm 2(\cos 300 + i \sin 300)^{\frac{1}{2}}$ $w = \pm 2(\cos 150 + i \sin 150)$ $w = \pm(-\sqrt{3} + i)$ $w = -\sqrt{3} + i \text{ or } \sqrt{3} - i$ <p style="text-align: center;">Or</p> $w = [4(\cos(300 + 360n) + i \sin(300 + 360n))]^{\frac{1}{2}}$ $w = 4^{\frac{1}{2}}[\cos(150 + 180n) + i \sin(150 + 180n)]$ <p><u>$n = 0$</u></p> $w = 2\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = -\sqrt{3} + i$ <p><u>$n = 1$</u></p> $w = 2\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \sqrt{3} - i$	<p>Scale 10C (0, 4, 7, 10)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - w written in polar form with index - Some use of De Moivre's Theorem - $w = v^{\frac{1}{2}}$ <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - De Moivre's theorem applied to w - One solution found - Solutions in polar form <p><u>Note:</u> Accept candidates answer from (b)(i)</p>

Q6	Model Solution – 25 Marks	Marking Notes
(a) (i)	$x + 5x = \sqrt{128} + \sqrt{32}$ $6x = 8\sqrt{2} + 4\sqrt{2}$ $6x = 12\sqrt{2}$ $x = 2\sqrt{2}$ <p style="text-align: center;">Or</p> $x - \sqrt{32} = \sqrt{128} - 5x$ $(x - \sqrt{32})^2 = (\sqrt{128} - 5x)^2$ $(x - 4\sqrt{2})^2 = (8\sqrt{2} - 5x)^2$ $x^2 - 8\sqrt{2}x + 32 = 128 - 80\sqrt{2}x + 25x^2$ $x^2 - 3\sqrt{2}x + 4 = 0$ $(x - \sqrt{2})(x - 2\sqrt{2}) = 0$ $x = \sqrt{2} \text{ or } x = 2\sqrt{2}$ <p>Check solutions:</p> $x = \sqrt{2}$ $\sqrt{2} - \sqrt{32} = \sqrt{128} - 5\sqrt{2}$ $-3\sqrt{2} = 3\sqrt{2} \text{ (False)}$ <p>Solution: $x = 2\sqrt{2}$</p>	<p>Scale 10C (0, 4, 7, 10)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - Any relevant transposing - $\sqrt{32}$ or $\sqrt{128}$ in the form $a\sqrt{2}$ <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> - x term isolated in equation <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - $\sqrt{32}$ or $\sqrt{128}$ in the form $a\sqrt{2}$ - Any relevant multiplication <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - LHS and RHS squared correctly - Solution not in the form $a\sqrt{2}$ <p><i>Full Credit –1:</i></p> <ul style="list-style-type: none"> - Both solutions presented <p><u>Note:</u> If $\sqrt{128}$ and $\sqrt{32}$ are converted to decimals, then award low partial credit at most</p>
(a) (ii)	$\sqrt{32k^2}, \sqrt{128k^2}, \sqrt{98k^2}, \sqrt{50k^2}$ $4\sqrt{2}k, \quad 8\sqrt{2}k, \quad 7\sqrt{2}k, \quad 5\sqrt{2}k$ $4\sqrt{2}k, \quad 5\sqrt{2}k, \quad 7\sqrt{2}k, \quad 8\sqrt{2}k$ $\text{Mean} = \frac{24\sqrt{2}k}{4} = 6\sqrt{2}k$ $\text{Median} = 6\sqrt{2}k$	<p>Scale 5C (0, 2, 3, 5)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - List in ascending or descending order - Any term written in the form $a\sqrt{2}k$ or in the form $a\sqrt{2k^2}$ <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - Mean or median found - Verified for a particular value of k <p><u>Note:</u> If decimals are used then award low partial credit at most</p>

<p>(b)</p> <p>Assume $\sqrt{2}$ is rational</p> <p>i.e. $\sqrt{2} = \frac{p}{q}$ where p and q have no common factors (simplest form)</p> <p>$\Rightarrow 2 = \frac{p^2}{q^2}$</p> <p>$\Rightarrow 2q^2 = p^2$</p> <p>$\Rightarrow p^2$ is even</p> <p>$\Rightarrow p$ is even</p> <p>$\Rightarrow p = 2k$ for some $k \in \mathbb{Z}$</p> <p>$2q^2 = p^2$ becomes $2q^2 = 4k^2$</p> <p>$\Rightarrow q^2 = 2k^2$</p> <p>$\Rightarrow q^2$ is even</p> <p>$\Rightarrow q$ is even</p> <p>$\Rightarrow q = 2m$ for some $m \in \mathbb{Z}$</p> <p>$\therefore \sqrt{2} = \frac{p}{q} = \frac{2k}{2m}$</p> <p>$\Rightarrow$ common factor of 2 (contradiction)</p> <p>$\therefore \sqrt{2}$ cannot be rational.</p>	<p>Scale 10D (0, 4, 5, 8, 10)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - $\sqrt{2} = \frac{p}{q}$ or similar <p><i>Mid Partial Credit</i></p> <ul style="list-style-type: none"> - deduces that p is even or equivalent - $p = 2k$ or equivalent deduced - $p^2 = 2q^2$ <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - $q = 2m$ or equivalent deduced
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<p>(b) (i)</p>	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="background-color: #cccccc;">Label</th> <th style="background-color: #cccccc;">A</th> <th style="background-color: #cccccc;">B</th> <th style="background-color: #cccccc;">C</th> <th style="background-color: #cccccc;">D</th> <th style="background-color: #cccccc;">E</th> <th style="background-color: #cccccc;">F</th> </tr> </thead> <tbody> <tr> <td style="background-color: #cccccc;">End-point</td> <td>$\frac{2}{3}$</td> <td>$\frac{2}{9}$</td> <td>$\frac{7}{9}$</td> <td>$\frac{8}{9}$</td> <td>$\frac{7}{27}$</td> <td>$\frac{25}{27}$</td> </tr> </tbody> </table>	Label	A	B	C	D	E	F	End-point	$\frac{2}{3}$	$\frac{2}{9}$	$\frac{7}{9}$	$\frac{8}{9}$	$\frac{7}{27}$	$\frac{25}{27}$	<p>Scale 10C (0, 4, 7, 10) <i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - 1 correct fraction given in table - All denominators correct <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - 4 correct fractions given in table
Label	A	B	C	D	E	F										
End-point	$\frac{2}{3}$	$\frac{2}{9}$	$\frac{7}{9}$	$\frac{8}{9}$	$\frac{7}{27}$	$\frac{25}{27}$										
<p>(b) (ii)</p>	<p>It is the end point (start point) of a segment</p> <p style="text-align: center;">Or</p> <div style="border: 1px solid black; padding: 10px; width: fit-content; margin: 10px auto;"> $\frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81} = \frac{20}{81}$ $\frac{6}{27} \quad \circ \quad \circ \quad \mathbf{E} \frac{7}{27}$ $\frac{18}{81} \quad \frac{19}{81} \quad \frac{20}{81} \quad \frac{21}{81}$ </div> <p style="text-align: center;">Or</p> <p>$\frac{7}{27} - \frac{1}{81} = \frac{20}{81}$ is a point in the Cantor Set</p>	<p>Scale 5B (0, 2, 5) <i>Mid Partial Credit:</i></p> <ul style="list-style-type: none"> - Relevant but incomplete reason given - Sum of fractions = $\frac{20}{81}$ 														
<p>(b) (iii)</p>	$S_{\infty} = \frac{a}{1-r} = \frac{\frac{1}{3}}{1 - \left(-\frac{1}{3}\right)} = \frac{1}{4}$ <p style="text-align: center;">Or</p> $\frac{1}{3} + \frac{1}{27} + \frac{1}{243} + \dots = \frac{\frac{1}{3}}{1 - \frac{1}{9}}$ $= \frac{3}{8}$ $-\left(\frac{1}{9} + \frac{1}{81} + \frac{1}{729} + \dots\right) = -\left(\frac{\frac{1}{9}}{1 - \frac{1}{9}}\right)$ $= -\frac{1}{8}$ $S_{\infty} = \frac{3}{8} - \frac{1}{8}$ $= \frac{1}{4}$	<p>Scale 10C (0, 4, 7, 10) <i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - S_{∞} indicated - S_{∞} formula with some substitution - Correct a or correct r <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - S_{∞} formula fully substituted 														

Q8	Model Solution – 50 Marks	Marking Notes
(a)	$r(20) = 22500 \cos\left(\frac{\pi}{26}(20)\right) + 37500$ $= 22500 \cos\left(\frac{20\pi}{26}\right) + 37500$ $= \text{€}20658.51$ $\approx \text{€}20659$	<p>Scale 10C (0, 4, 7, 10)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - Any relevant substitution - $r(20)$ or $t = 20$ <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - Correct substitution <p><i>Full Credit –1:</i></p> <ul style="list-style-type: none"> - Uses degrees as unit of measurement, giving an answer of €59980
(b)	$22500 \cos\left(\frac{\pi}{26}t\right) + 37500 = 26250$ $22500 \cos\left(\frac{\pi}{26}t\right) = -11250$ $\cos\left(\frac{\pi}{26}t\right) = -\frac{1}{2}$ $\frac{\pi}{26}t = \frac{2\pi}{3} \text{ and } \frac{\pi}{26}t = \frac{4\pi}{3}$ $t = \frac{52}{3} \text{ and } t = \frac{104}{3}$	<p>Scale 10D (0, 4, 5, 8, 10)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - Equation formed - Trial and improvement with at least two values tested <p><i>Mid Partial Credit:</i></p> <ul style="list-style-type: none"> - Equation simplified to: <ul style="list-style-type: none"> $\cos\left(\frac{\pi}{26}t\right) = -\frac{1}{2}$ - Equation simplified to: <ul style="list-style-type: none"> $\cos\left(\frac{\pi}{26}t\right) = -\frac{11250}{22500}$ <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - 1 correct solution to equation found

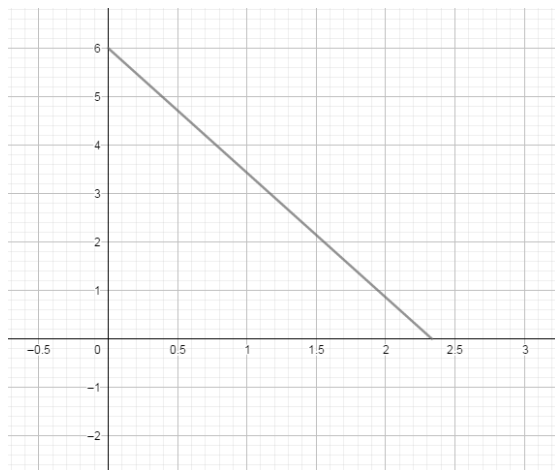
<p>(c)</p>	$r'(t) = 22500 \left[-\sin\left(\frac{\pi}{26}t\right) \right] \left(\frac{\pi}{26}\right)$ $= -\frac{11250}{13} \pi \left[\sin\left(\frac{\pi}{26}t\right) \right]$	<p>Scale 5C (0, 2, 3, 5)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - Some relevant differentiation <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - Chain rule applied
<p>(d)</p>	$r'(30) = -\frac{11250}{13} \pi \left[\sin\left(\frac{\pi}{26}(30)\right) \right]$ $= 402.164\pi$ $= 1263.44$ > 0 $\Rightarrow \text{Increasing}$	<p>Scale 10C (0, 4, 7, 10)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - Some relevant substitution into answer from (c) - $r'(t) > 0$ - $\frac{dy}{dx} > 0$ <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - $r'(30)$ found but no conclusion or incorrect conclusion <p><u>Note:</u> If calculus is not used then award no credit for the solution</p>

Q9	Model Solution – 55 Marks	Marking Notes
<p>(a) (i)</p>	$= 2(x) + 2(y) + \frac{1}{2} (2\pi)(x)$ $= 2x + 2y + \pi x$	<p>Scale 5C (0, 2, 3, 5) <i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - Some relevant substitution into perimeter formula - Circumference of circle of radius x found i.e. $2\pi x$ <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - Two of the three terms found
<p>(a) (ii)</p>	$2x + 2y + \pi x = 12$ $2y = 12 - 2x - \pi x$ $y = \frac{12 - 2x - \pi x}{2}$ $y = \frac{12 - (2 + \pi)x}{2}$	<p>Scale 5C (0, 2, 3, 5) <i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - Some relevant substitution into equation <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - y term isolated correctly in equation <p><u>Note:</u> Accept candidates answer from (a)(i) provided it doesn't oversimplify the work.</p> <p><u>Note:</u> Must draw a relevant conclusion from incorrect work</p>

(b)
(i)
+
(ii)

Table and Graph

x	0	$\frac{12}{2 + \pi}$
$y = \frac{12 - (2 + \pi)x}{2}$	6	0



Scale 15D (0, 4, 7, 11, 15)

Low Partial Credit:

- One correct table entry
- One correct plot of incorrect point

Mid Partial Credit:

- 2 table entries correct
- 2 incorrect points plotted and joined

High Partial Credit:

- 2 correct points plotted but not joined with correct table entries

Full Credit –1:

- Two correct points plotted and joined but the function is not graphed in the stated domain

Note: Accept $2.25 \leq x \leq 2.5$ for x -intercept

<p>(c) (ii)</p>	$a(x) = \frac{1}{2}(24x - (\pi + 4)x^2)$ $a'(x) = \frac{1}{2}(24 - 2(\pi + 4)x)$ $= 12 - (\pi + 4)x$	<p>Scale 5B (0, 2, 5) <i>Mid Partial Credit:</i></p> <ul style="list-style-type: none"> - Some correct differentiation
<p>(c) (iii)</p>	$a'(x) = 0$ $12 - (\pi + 4)x = 0$ $(\pi + 4)x = 12$ $x = \frac{12}{\pi + 4} \quad (1.68)$ $y = \frac{12 - (2 + \pi)x}{2} \quad (= \frac{12 - (5 \cdot 14) 1.68}{2} \approx 1.68)$ $= \frac{12 - (2 + \pi)\left(\frac{12}{\pi + 4}\right)}{2}$ $= \frac{12(\pi + 4) - (2 + \pi)(12)}{2(\pi + 4)}$ $= \frac{12\pi + 48 - 24 - 12\pi}{2(\pi + 4)}$ $= \frac{24}{2(\pi + 4)}$ $= \frac{12}{\pi + 4}$ $= x$ <p>Area Max when height equals the radius</p>	<p>Scale 15D (0, 4, 7, 11, 15) <i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> - $a'(x)$ used - States $\frac{dy}{dx} = 0$ <p><i>Mid Partial Credit</i></p> <ul style="list-style-type: none"> - Value of x at maximum found <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> - Value of y at maximum fully substituted

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2019

Marking Scheme

Mathematics

Higher Level

Paper 2

Marking Scheme – Paper 2, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 2, 5	0, 2, 3, 5		
10 mark scales			0, 4, 7, 10	0, 4, 5, 8, 10	
15 mark scales			0, 5, 10, 15	0, 5, 7, 11, 15	
20 mark scales					
25 mark scales					

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

Note: In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, in *scale 10C*, 9 marks may be awarded.

Rounding and units penalty to be applied only once in each section (a), (b), (c) etc.

Throughout the scheme indicate by use of # where an arithmetic error occurs.

Summary of mark allocations and scales to be applied

Section A

Question 1

- (a)(i) 10C
- (a)(ii) 10D
- (b) 5C

Question 2

- (a) 10C
- (b)(i) 5B
- (b)(ii) 10D

Question 3

- (a) 10C
- (b) 15D

Question 4

- (a) 10C
- (b)(i) 15D

Question 5

- (a) 15D
- (b)(i) 10D

Question 6

- (a)(i) 10C
- (b) 15D

Section B

Question 7 (50)

- (a)(i) 10C
- (a)(ii) 5C
- (a)(iii) 10D
- (a)(iv) 5C
- (b)(i) 15D
- (b)(ii) 5C

Question 8 (45)

- (a)(i) 10C
- (a)(ii) 15D
- (a)(iii) 10C
- (b)(i) 5B
- (b)(ii) 5C

Question 9 (55)

- (a) 10C
- (b) 10C
- (c) 15C
- (d)(i) 5B
- (d)(ii) 5C
- (d)(iii) 5C
- (e) 5C

Model Solutions & Detailed Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 25 Marks	Marking Notes
(a) (i)	$\frac{12}{20} \times \frac{8}{19} + \frac{8}{20} \times \frac{12}{19}$ $= \frac{96}{380} + \frac{96}{380} \text{ or } 2 \left(\frac{96}{380} \right)$ $\frac{12}{20} \times \frac{8}{19} + \frac{8}{20} \times \frac{12}{19} = \frac{192}{380} \text{ or } \frac{48}{95}$ <p>Or</p> $\frac{\binom{12}{1} \binom{8}{1}}{\binom{20}{2}} = \frac{96}{190} \text{ or } \frac{48}{95}$ <p>Or</p> $1 - \left[\frac{12}{20} \times \frac{11}{19} + \frac{8}{20} \times \frac{7}{19} \right] = 1 - \frac{188}{380}$ $= \frac{192}{380} \text{ or } \frac{48}{95}$	<p>Scale 10C (0, 4, 7, 10)</p> <p><i>Low Partial Credit:</i></p> <p>1 probability given e.g. $\frac{12}{20}$ or equivalent</p> <p>1 combination indicated e.g. $\binom{12}{1}$ or $\binom{8}{1}$ or $\binom{20}{2}$</p> <p>$\frac{12}{20} \times \frac{8}{19}$ or $\frac{8}{20} \times \frac{12}{19}$ or equivalent and stops</p> <p>$\frac{\binom{12}{1}}{\binom{20}{2}}$ or $\frac{\binom{8}{1}}{\binom{20}{2}}$ and stops</p> <p>$1 - \frac{12}{20} \times \frac{11}{19}$ or $1 - \frac{8}{20} \times \frac{7}{19}$ and stops</p> <p><i>High Partial Credit:</i></p> <p>$\frac{12}{20} \times \frac{8}{19}$ or $\frac{8}{20} \times \frac{12}{19}$ or equivalent and continues</p> <p>$\frac{\binom{12}{1}}{\binom{20}{2}}$ or $\frac{\binom{8}{1}}{\binom{20}{2}}$ and continues</p> <p>$1 - \frac{12}{20} \times \frac{11}{19}$ or $1 - \frac{8}{20} \times \frac{7}{19}$ and continues</p>
(a) (ii)	$\frac{12}{20} \times \frac{11}{19} \times \frac{10}{18} \times \frac{8}{17} = \frac{10560}{116280} \text{ or } \frac{88}{969}$ <p>Or</p> $\frac{\binom{12}{1}}{\binom{20}{1}} \times \frac{\binom{11}{1}}{\binom{19}{1}} \times \frac{\binom{10}{1}}{\binom{18}{1}} \times \frac{\binom{8}{1}}{\binom{17}{1}}$ $= \frac{10560}{116280} \text{ or } \frac{88}{969}$	<p>Scale 10D (0, 4, 5, 8, 10)</p> <p><i>Low Partial Credit:</i></p> <p>1 probability given</p> <p>1 combination indicated</p> <p><i>Mid Partial Credit</i></p> <p>3 or 4 correct probabilities indicated</p> <p><i>High Partial Credit:</i></p> <p>3 correct probabilities with multiplication completed</p> <p>4 probabilities with correct operator</p>

(b)	$\binom{6}{3} \times \binom{8}{4} = 1400$ <p style="text-align: center;">or</p> $\binom{1}{1} \times \binom{6}{3} \times \binom{8}{4} = 1400$	<p>Scale 5C (0, 2, 3, 5)</p> <p><i>Low Partial Credit:</i> $\binom{6}{3}$ or $\binom{8}{4}$ or $\binom{1}{1}$</p> <p><i>High Partial Credit:</i> $\binom{6}{3} \times \binom{8}{4}$ and stops</p>
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Q2	Model Solution – 25 Marks	Marking Notes
(a)	<p> $m = \frac{b-0}{0-a} = \frac{-b}{a}$ $y - 0 = \frac{-b}{a}(x - a)$ $ay = -bx + ab$ $bx + ay = ab$ Now divide across by ab $\frac{x}{a} + \frac{y}{b} = 1$ </p> <p>Or</p> <p> $m = \frac{b-0}{0-a} = \frac{-b}{a}$ $y = mx + c \Rightarrow y = \frac{-b}{a}x + c$ But $(0, b)$ is on this line, thus $b = \frac{-b}{a}(0) + c$ $\therefore b = c$ </p> <p>Equation $y = \frac{-b}{a}x + b$ $ay = -bx + ab$ $bx + ay = ab$ Now divide across by ab $\frac{x}{a} + \frac{y}{b} = 1$ </p> <p>Or</p> <p> $(a, 0) \in y = mx + c \Rightarrow 0 = ma + c$ $\Rightarrow -ma = c$ $(0, b) \in y = mx + c \Rightarrow b = c$ $\therefore -ma = b \Rightarrow m = \frac{-b}{a}$ </p> <p>Equation $y = \frac{-b}{a}x + b$ $ay = -bx + ab$ $bx + ay = ab$ Now divide across by ab $\frac{x}{a} + \frac{y}{b} = 1$ </p> <p>Or</p> <p> $\frac{x}{a} + \frac{y}{b} = 1$ </p> <p>LHS: $\frac{x}{a} + \frac{y}{b}$</p> <p>$(a, 0): \frac{a}{a} + \frac{0}{b} = 1=1$ or RHS</p> <p>$(0, b): \frac{0}{a} + \frac{b}{b} = 1=1$ or RHS</p>	<p>Scale 10C (0, 4, 7, 10)</p> <p><i>Low Partial Credit:</i> Slope formula with some substitution</p> <p><i>High Partial Credit:</i> Equation of line formula fully substituted</p> <p><i>Low Partial Credit:</i> Slope formula with some substitution</p> <p><i>High Partial Credit:</i> m expressed in terms of a and b, and c in terms of b</p> <p><i>Low Partial Credit:</i> $(a, 0)$ or $(0, b)$ correctly substituted e.g. $\frac{a}{a} + \frac{0}{b}$</p> <p><i>High Partial Credit:</i> $(a, 0)$ and $(0, b)$ correctly substituted</p>

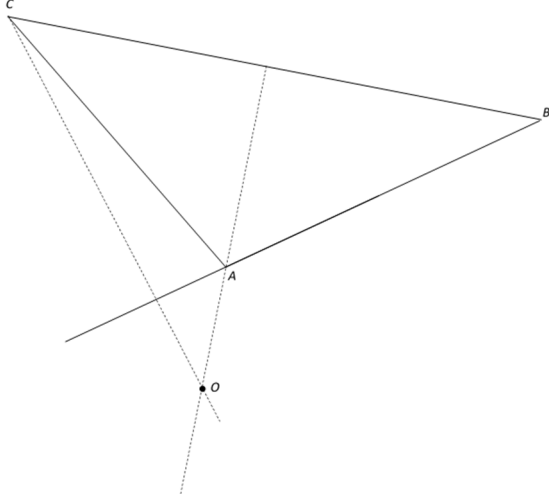
<p>(b) (i)</p>	$y - 0 = m(x - 6) \text{ or } y = m(x - 6)$ <p>Or</p> $y = mx - 6m$ <p>Or</p> $y = mx + c$ $\therefore 0 = 6m + c \Rightarrow c = -6m$	<p>Scale 5B (0, 2, 5) <i>Mid Partial Credit:</i> Equation of line formula with some relevant substitution</p>
<p>(b) (ii)</p>	$y = m(x - 6)$ $4x + 3y = 25$ $\Rightarrow 4x + 3m(x - 6) = 25$ $\Rightarrow x = \frac{25+18m}{3m+4}$ <p>Substitute this into $y = m(x - 6)$</p> $y = m\left(\frac{25 + 18m}{3m + 4}\right) - 6m$ $= \frac{25m + 18m^2 - 18m^2 - 24m}{3m + 4}$ $= \frac{m}{3m + 4}$ <p>Or</p> $4x + 3y = 25 \cap mx - y = 6m$ $4x + 3y = 25$ $\underline{3mx - 3y = 18m}$ $4x + 3mx = 18m + 25$ $x = \frac{25+18m}{3m+4}$ $4mx + 3my = 25m$ $\underline{4mx - 4y = 24m}$ $(3m + 4)y = m$ $\therefore y = \frac{m}{3m + 4}$	<p>Scale 10D (0, 4, 5, 8, 10) <i>Low Partial Credit:</i> Indication of use of simultaneous equations</p> <p><i>Mid Partial Credit</i> One relevant substitution</p> <p><i>High Partial Credit:</i> x or y value found</p> <p><i>Low Partial Credit:</i> Indication of use of simultaneous equations</p> <p><i>Mid Partial Credit</i> One successful elimination in equations</p> <p><i>High Partial Credit:</i> x or y value found</p>

Q3	Model Solution – 25 Marks	Marking Notes
(a)	$(-2 - 2)^2 + (k - 3)^2 = 65$ $16 + (k - 3)^2 = 65$ $(k - 3)^2 = 49$ $k - 3 = \pm\sqrt{49} = \pm 7$ $k = 10 \text{ and } k = -4$ <p>Or</p> $k^2 - 6k + 9 = 49$ $k^2 - 6k - 40 = 0$ $(k - 10)(k + 4) = 0$ $k = 10 \text{ and } k = -4$ <p>Or</p> $x^2 - 4x + 4 + y^2 - 6y + 9 = 65$ $x^2 + y^2 - 4x - 6y = 52$ $4 + k^2 + 8 - 6k = 52$ $k^2 - 6k - 40 = 0$ $(k - 10)(k + 4) = 0, \therefore k = 10, k = -4$ <p>Or</p> <p>Centre (2, 3), radius $\sqrt{65}$</p> $\sqrt{(2 + 2)^2 + (3 - k)^2} = \sqrt{65}$ <p>and proceed as above</p>	<p>Scale 10C (0, 4, 7, 10)</p> <p><i>Low Partial Credit:</i> Some relevant substitution Centre or radius</p> <p><i>High Partial Credit:</i> Equation in k^2</p>

<p>(b)</p>	<p>Both axes are tangents to the circle. centre $(-g, -g)$ and radius is g Perpendicular distance $(-g, -g)$ to $3x - 4y + 6 = 0$ is equal to the radius</p> $\frac{-3g + 4g + 6}{5} = -g$ $g + 6 = \pm 5g$ $g + 6 = -5g, \therefore -g = 1$ <p>Centre $(1, 1)$ and radius 1 Equation: $(x - 1)^2 + (y - 1)^2 = 1$ or $x^2 + y^2 - 2x - 2y + 1 = 0$</p> <p>Or</p> <p>$s$ is a tangent to both axes therefore $c = g^2 = f^2$</p> <p>So equation is in the form $x^2 + y^2 + 2gx + 2gy + g^2 = 0$</p> $3x - 4y + 6 = 0 \Rightarrow y = \frac{3x+6}{4}$ <p>Substitute into circle: $x^2 + \left(\frac{3x+6}{4}\right)^2 + 2gx + \frac{2g(3x+6)}{4} + g^2 = 0$</p> $\Rightarrow 25x^2 + x(36 + 56g) + 36 + 48g + 16g^2 = 0$ <p>Tangent therefore $b^2 = 4ac$ $(36 + 56g)^2 = 4(25)(36 + 48g + 16g^2)$</p> $2g^2 - g - 3 = 0$ $g = -1 \text{ and } g = \frac{3}{2}$ <p>But can't have positive g as the co-ordinate $-g$ is in first quadrant. $\Rightarrow g = -1$.</p> <p>Therefore equation is $x^2 + y^2 - 2x - 2y + 1 = 0$ or $(x - 1)^2 + (y - 1)^2 = 1$</p>	<p>Scale 15D (0, 5, 7, 11, 15) <i>Low Partial Credit:</i> centre $(-g, -g)$ or equivalent</p> <p><i>Mid Partial Credit:</i> Substitution into perpendicular distance formula completed</p> <p>Perpendicular distance of centre to tangent equals radius with some substitution</p> <p><i>High Partial Credit:</i> equation in g or equivalent</p> <p><i>Low Partial Credit:</i> $c = g^2$ or f^2</p> <p>Effort to express x in terms of y or equivalent</p> <p><i>Mid Partial Credit:</i> Substitution into circle equation completed</p> <p><i>High Partial Credit:</i> Quadratic equation in g or f</p>
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Q4	Model Solution – 25 Marks	Marking Notes
(a)	$\cos(A + B) = \cos A \cos B - \sin A \sin B$ $\cos 2A = \cos^2 A - \sin^2 A$ $\cos 2A = (1 - \sin^2 A) - \sin^2 A$ $\cos 2A = 1 - 2\sin^2 A$ <p>Or</p> <p>Taking RHS</p> $1 - 2\sin^2 A = 1 - 2(1 - \cos^2 A)$ $= -1 + 2\cos^2 A$ $= -(\cos^2 A + \sin^2 A) + 2\cos^2 A$ $= \cos^2 A - \sin^2 A$ $= \cos A \cos A - \sin A \sin A = \cos 2A$ <p>Or</p> $(\cos A + i \sin A)^2 = \cos 2A + i \sin 2A$ $(\cos A + i \sin A)^2$ $= \cos^2 A + 2i \sin A \cos A$ $+ (i \sin A)^2$ $\cos 2A = \cos^2 A - \sin^2 A$ $\cos 2A = (1 - \sin^2 A) - \sin^2 A$ $\cos 2A = 1 - 2\sin^2 A$	<p>Scale 10C (0, 4, 7, 10)</p> <p><i>Low Partial Credit:</i> $\cos(A + B)$ formula with some substitution</p> <p>$\cos^2 A + \sin^2 A = 1$ indicated or clearly implied</p> <p><i>High Partial Credit:</i> $\cos 2A = \cos^2 A - \sin^2 A$</p> <p><i>Low Partial Credit:</i> $\cos^2 A + \sin^2 A = 1$ indicated or clearly implied</p> <p>$(\cos A + i \sin A)^2$ expanded</p> <p><i>High Partial Credit:</i> $\cos 2A = \cos^2 A - \sin^2 A$</p>

<p>(b)</p> <p>Let length of side be x Diagonal of any face = $\sqrt{x^2 + x^2} = \sqrt{2}x$ Internal diagonal = $x^2 + (\sqrt{2}x)^2 = \sqrt{3}x$</p> <p>By cosine rule: $x^2 = \left(\frac{\sqrt{3}x}{2}\right)^2 + \left(\frac{\sqrt{3}x}{2}\right)^2 - 2\frac{\sqrt{3}x}{2}\frac{\sqrt{3}x}{2}\cos A$</p> $\cos A = \frac{\left(\frac{\sqrt{3}x}{2}\right)^2 + \left(\frac{\sqrt{3}x}{2}\right)^2 - x^2}{2\left(\frac{\sqrt{3}x}{2}\right)\left(\frac{\sqrt{3}x}{2}\right)}$ $\cos A = \frac{1}{3}$ <p>Or</p> <p>Drop perpendicular from intersecting diagonals to side of cube, thereby creating angle $A/2$ at vertex in a right-angled triangle.</p> $\sin \frac{A}{2} = \frac{\frac{x}{2}}{\frac{\sqrt{3}x}{2}} = \frac{1}{\sqrt{3}}$ $\therefore \cos \frac{A}{2} = \frac{\sqrt{2}}{\sqrt{3}}$ $\cos A = 2\cos^2 \frac{A}{2} - 1 = 2\left(\frac{2}{3}\right) - 1 = \frac{1}{3}$ <p>Also: $\sin \frac{A}{2} = \frac{1}{\sqrt{3}} \rightarrow \frac{A}{2} = 35.2643896^\circ$</p> $A = 70.5287792^\circ$ $\cos A = 0.33236$	<p>Scale 15D (0, 5, 7, 11, 15)</p> <p><i>Low Partial Credit:</i> Length of any diagonal formulated</p> <p><i>Mid Partial Credit</i> Internal diagonal found</p> <p><i>High Partial Credit:</i> Fully substituted cosine rule</p> <p>Note: Accept and mark work where a consistent numerical value is assigned to one side of the cube.</p> <p><i>Low Partial Credit:</i> Length of any diagonal formulated</p> <p><i>Mid Partial Credit</i> Internal diagonal found</p> <p><i>High Partial Credit:</i> $\sin \frac{A}{2}$ fully substituted</p>
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Q5	Model Solution – 25 Marks	Marking Notes
(a)	<p>Standard Orthocentre Construction</p> 	<p>Scale 15D (0, 5, 7, 11,15)</p> <p><i>Low Partial Credit:</i> Some correct element of construction Some evidence of understanding of term orthocentre</p> <p><i>Mid Partial Credit</i> One correct altitude</p> <p><i>High Partial Credit:</i> Two correct altitudes but not intersecting.</p>
(b)	<p>$DC = OB$ Given $\Rightarrow DC = \text{Radius}$</p> <p>$\Rightarrow \triangle ODC$ is equilateral $\Rightarrow \angle ODC = 60$</p> <p>$\Rightarrow \angle AOD = 60$ Alternate</p> <p>$\triangle AOD$ is isosceles as $OA = OD$ $\angle OAD = \angle ODA = \frac{120}{2} = 60$</p> <p>$\angle ABE = 90^\circ$ as BE tangent</p> <p>$\angle BEA = 180 - 90 - 60 = 30^\circ$</p>	<p>Scale 10D (0, 4, 5, 8, 10)</p> <p><i>Low Partial Credit:</i> 1 relevant step listed or shown on diagram</p> <p><i>Mid Partial Credit</i> 3 relevant steps listed or shown on diagram</p> <p><i>High Partial Credit:</i> All valid steps included but with no justification</p>

Q6	Model Solution – 25 Marks	Marking Notes
(a)	<p>$P(F \cap S) = P(F) \times P(S)$ since the events are independent.</p> $\frac{1}{5} = \frac{9}{20} \times P(S)$ $\Rightarrow P(S) = \frac{4}{9}$ <p>So $P(S \setminus F) = \frac{4}{9} - \frac{1}{5} = \frac{11}{45} = x$</p> <p>Or</p> $P(S) = \frac{1}{5} + x$ $\frac{1}{5} = \frac{9}{20} \left(\frac{1}{5} + x \right) \Rightarrow \frac{11}{45} = x$ <p>Or</p> $P(F S) = \frac{P(F \cap S)}{P(S)} = P(F)$ $\frac{\frac{1}{5}}{\frac{1}{5} + x} = \frac{9}{20} \Rightarrow x = \frac{11}{45}$ <p>Or</p> $P(S F) = \frac{P(S \cap F)}{P(F)} = P(S)$ $\frac{\frac{1}{5}}{\frac{9}{20}} = \frac{1}{5} + x \Rightarrow x = \frac{11}{45}$ $y = 1 - \frac{11}{45} - \frac{1}{5} - \frac{1}{4} = \frac{11}{36}$	<p>Scale 10C (0, 4, 7, 10)</p> <p><i>Low Partial Credit:</i> $P(F \cap S) = P(F) \times P(S)$ or equivalent</p> $P(F) = \frac{1}{4} + \frac{1}{5}$ $P(S) = x + \frac{1}{5}$ $\frac{1}{4} + \frac{1}{5} + x + y = 1$ <p><i>High Partial Credit</i> x found</p>

<p>(b)</p> <p>If n Germans then $2n$ Irish and $3n+10$ children in total</p> $\frac{n}{3n+10} \times \frac{2n+10}{3n+9} = \frac{1}{6}$ $\frac{2n^2+10n}{9n^2+57n+90} = \frac{1}{6}$ $3n^2 + 3n - 90 = 0$ $n^2 + n - 30 = 0$ $(n+6)(n-5) = 0$ <p>$n = 5$ German children.</p> <p>There are 10 Irish (and 10 Spanish) so 25 children in the club.</p> <p>Or</p> <p>25 by trial and improvement method:</p> <p>5 German, 10 Irish, 10 Spanish and verified to indicate $\frac{5}{25} \times \frac{20}{24} = \frac{1}{6}$</p>	<p>Scale 15D (0, 5, 7, 11, 15)</p> <p><i>Low Partial Credit:</i> $2n$ $3n+10$ One correct probability e.g. $\frac{n}{3n+10}$</p> <p><i>Mid Partial Credit:</i> $\frac{n}{3n+10}$ and $\left(\frac{2n+10}{\blacksquare} \text{ or } \frac{\blacksquare}{3n+9}\right)$</p> <p><i>High Partial Credit:</i> $\frac{n}{3n+10} \times \frac{2n+10}{3n+9} = \frac{1}{6}$</p> <p><i>Low Partial Credit:</i> Some correct element in approach</p> <p><i>Mid Partial Credit</i> Tests more than one value</p> <p><i>High Partial Credit:</i> Correct number of each nationality but not verified that probability is $\frac{1}{6}$</p> <p>Correct answer (25) with no supporting work</p>
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Section B

Q7	Model Solution – 50 Marks	Marking Notes
(a) (i)	$ AD ^2 = 90^2 - 60^2$ $90^2 = 60^2 + AD ^2$ $ AD = \sqrt{8100 - 3600} = \sqrt{4500} = 30\sqrt{5}$	<p>Scale 10C (0, 4, 7, 10) <i>Low Partial Credit:</i> $OD = 60$ Pythagoras formulated Effort to find angle other than $\angle ODA$</p> <p><i>High Partial Credit:</i> $\sqrt{8100 - 3600}$ or equivalent</p>
(a) (ii)	$\cos(\angle DOA) = \frac{60}{90}$ $\cos^{-1}\left(\frac{6}{9}\right) = 0.84$ <p>Or</p> $\sin(\angle DOA) = \frac{30\sqrt{5}}{90} = \frac{\sqrt{5}}{3} = 0.745356$ $ \angle DOA = 48.189^\circ$ $ \angle DOA = 0.84139 = 0.84$	<p>Scale 5C (0, 2, 3, 5) <i>Low Partial Credit:</i> Relevant trigonometric ratio formulated</p> <p><i>High Partial Credit:</i> Relevant trigonometric ratio fully substituted</p>
(a) (iii)	<p>Area of sector: $\frac{1}{2}r^2\theta$</p> $\frac{1}{2}(0.9)^2 \times 2(0.84) = 0.6804 \text{ m}^2$ <p>Area $\triangle ACO$: $\frac{1}{2} AC OD = \frac{1}{2}(60\sqrt{5})60 \text{ cm}^2$</p> $\frac{1}{2}(1.34164)(0.6) = 0.40 \text{ m}^2$ <p>Or</p> <p>Area $\triangle ACO$: $\frac{1}{2} AO OC \sin(\angle AOC) =$</p> $\frac{1}{2}(90)(90) \sin 2(48.189^\circ)$ $= 4024.9174 \text{ cm}^2 = 0.40 \text{ m}^2$ <p>Area of segment = $0.6804 - 0.40 = 0.28$</p>	<p>Scale 10D (0, 4, 5, 8, 10) <i>Low Partial Credit:</i> Formula for area of sector with some substitution Formula for area of $\triangle ACO$ with some substitution</p> <p><i>Mid Partial Credit:</i> One relevant area fully substituted</p> <p><i>High Partial Credit:</i> Both relevant areas fully substituted Mishandling conversion of units</p>
(a) (iv)	<p>Volume = $0.28 \times 2.5 = 0.7$</p>	<p>Scale 5C (0, 2, 3, 5) <i>Low Partial Credit:</i> Formula for volume of trough with some substitution Indicates some relevant use of 2.5</p> <p><i>High Partial Credit:</i> Formula fully substituted</p>

<p>(b) (i)</p>	<p>Volume =</p> $\pi \left[\left(\left(\frac{2}{3} \right) 1.25^3 \right) \right]$ $+ \pi [(1.25^2 \times 3.5)]$ $+ \pi \left[\left(\left(\frac{1}{3} \right) 1.25^2 \times 1.5 \right) \right]$ $= 4.0906 + 17.1805 + 2.4544$ $= 23.73$	<p>Scale 15D (0, 5, 7, 11, 15)</p> <p><i>Low Partial Credit:</i> 1 volume formula with some substitution</p> <p><i>Mid Partial Credit</i> 2 volumes fully substituted</p> <p><i>High Partial Credit:</i> 3 volumes fully substituted</p>
<p>(b) (ii)</p>	<p>$23.73 \times 0.02 = 0.4746 \text{ cm}^3$</p> $\frac{r}{h} = \frac{1.25}{1.5} = \frac{5}{6}$ $r = \frac{5h}{6}$ <p>Volume in cone = $\frac{1}{3} \pi \left(\frac{5h}{6} \right)^2 \times h = 0.4746$</p> $h^3 = \frac{0.4746 \cdot 3.6}{25\pi} = 0.65262$ $h = \sqrt[3]{0.65262} = 0.8674$ $h = 0.87$	<p>Scale 5C (0, 2, 3, 5)</p> <p><i>Low Partial Credit:</i> volume $\times 0.98$ or equivalent volume multiplied by 2% effort at $r : h$</p> <p><i>High Partial Credit:</i> Volume formula expressed in one variable</p>

Q8	Model Solution – 45 Marks	Marking Notes
(a) (i)	<p>Confidence interval</p> $0.2175 \pm 1.96 \sqrt{\frac{\left(\frac{174}{800}\right)\left(\frac{626}{800}\right)}{800}}$ $0.2175 - 1.96 \sqrt{\frac{(0.2175)(0.7825)}{800}} < p$ $< 0.2175 + 1.96 \sqrt{\frac{(0.2175)(0.7825)}{800}}$ $0.2175 - 1.96\sqrt{0.00021274} < p$ $< 0.2175 + 1.96\sqrt{0.00021274}$ $0.188913 < p < 0.246087$ $0.1889 < p < 0.2461$ <p>or</p> $18.89\% < p < 24.61\%$	<p>Scale 10C(0, 4, 7,10)</p> <p><i>Low Partial Credit:</i> 0.2175 or $\frac{174}{800}$ CI formulated with some substitution</p> <p><i>High Partial Credit:</i> CI fully substituted</p>
(a) (ii)	$\frac{x - \bar{x}}{\sigma}$ $\frac{95-87.3}{12} = 0.64167 \text{ (z score)}$ $\Rightarrow p(Z \leq 0.64167) = 0.7389$ $P(z \geq 0.64) = 1 - 0.7389$ $= 0.2611 \text{ or } 26.11\%$	<p>Scale 15D(0, 5, 7, 11, 15)</p> <p><i>Low Partial Credit:</i> μ or σ identified</p> <p><i>Mid Partial Credit:</i> z found</p> <p><i>High Partial Credit:</i> $P(z < 0.64)$ and stops or continues incorrectly</p>
(a) (iii)	$z = -0.52 = \frac{x - 87.3}{12}$ $\Rightarrow x = 81.06 \text{ km/h}$ $x = 81 \text{ km/h}$	<p>Scale 10C (0, 4, 7, 10)</p> <p><i>Low Partial Credit:</i> $x - 87.3$ $\frac{\quad}{12}$ $z \in [0.52, 0.53]$ or $z \in [-0.52, -0.53]$ and stops</p> <p><i>High Partial Credit:</i> Formula for x fully substituted</p>

<p>(b) (i)</p>	<p>Average speed has changed</p> <p>p-value < 0.05</p>	<p>Scale 5B (0, 2, 5) <i>Mid Partial Credit:</i> Answer or reason correct</p>
<p>(b) (ii)</p>	$0.024 = 2(1 - P(z \leq T))$ <p>$\Rightarrow P(z \leq T) = 0.988$</p> <p>Therefore $T = 2.26$ or -2.26.</p> <p>Because the mean has reduced $z = -2.26$</p> $-2.26 = \frac{x - 87.3}{\frac{12}{\sqrt{100}}}$ <p>$\Rightarrow x = 84.588$ km/h $\Rightarrow x = 84.6$</p>	<p>Scale 5C (0, 2, 3, 5) <i>Low Partial Credit:</i> $0.024 = 2(0.012)$ Value(s) of T found</p> <p><i>High Partial Credit:</i> Formula for x fully substituted</p>

Q9	Model Solution – 55 Marks	Marking Notes
(a)	$ SG ^2 = 30^2 + 58^2 - 2(30)(58)(\cos 68)$ $= 2960.369$ $ SG = 54.409 \text{ m}$ $ SG = 54.4$	<p>Scale 10C (0, 4, 7, 10) <i>Low Partial Credit:</i> Some relevant substitution into correct cosine formula</p> <p><i>High Partial Credit:</i> Formula fully substituted</p>
(b)	$\frac{54.4}{\sin 68} = \frac{30}{\sin \angle HSG}$ $\sin \angle HSG = 0.51131$ <p>Or</p> $ \angle HSG = 30.75$ $\cos \angle HSG = \frac{54.4^2 + 58^2 - 30^2}{2(54.4)(58)}$ $= 0.859432$ $ \angle HSG = 30.747^\circ = 30.75$	<p>Scale 10C (0, 4, 7, 10) <i>Low Partial Credit:</i> Some relevant substitution into relevant formula</p> <p><i>High Partial Credit:</i> Formula fully substituted</p> <p>Note: Finds $\angle HGS \Rightarrow \checkmark \#$</p>
(c)	$\text{Area } \triangle GSH = \frac{1}{2}(30)(58) \sin 68 = 806.65$ <p>Also Area $\triangle GSH$:</p> $\frac{1}{2}(54.4)(58) \sin 30.75$ <p>and</p> $\frac{1}{2}(54.4)(30) \sin 81.25$	<p>Scale 15C (0, 5, 10, 15) <i>Low Partial Credit:</i> Some substitution into area formula</p> <p><i>High Partial Credit:</i> Formula fully substituted</p>
(d) (i)	$\frac{1}{2}(58)(r) \text{ or } 29r$	<p>Scale 5B (0, 2, 5) <i>Mid Partial Credit:</i> Right angle indicated Relevant triangle indicated on diagram Area of triangle formula with some substitution</p>

<p>(d) (ii)</p>	<p>Area ΔGHS</p> $= \frac{1}{2}(30)(r) + \frac{1}{2}(54 \cdot 4)(r) + \frac{1}{2}(58)(r)$ $= 15r + 27 \cdot 2r + 29r = 71 \cdot 2r$	<p>Scale 5C (0, 2, 3, 5)</p> <p><i>Low Partial Credit:</i> Relevant use of previous answer in this part Indication of 3 relevant triangle areas to be added Area of 1 additional triangle (in terms of r)</p> <p><i>High Partial Credit:</i> Addition of 2 areas (each written in terms of r)</p>
<p>(d) (iii)</p>	$71 \cdot 2r = 806 \cdot 62$ $r = \frac{806 \cdot 62}{71 \cdot 2}$ $= 11 \cdot 3289 = 11 \cdot 3$	<p>Scale 5C (0, 2, 3, 5)</p> <p><i>Low Partial Credit:</i> Both relevant answers presented</p> <p><i>High Partial Credit:</i> Areas equated</p>
<p>(e) (ii)</p>	$\tan 14 = \frac{ TS }{ PS }$ $\sin 15 \cdot 375 = \frac{11 \cdot 3}{ PS } = 42 \cdot 51$ $\Rightarrow PS = 42 \cdot 619$ $\tan 14 = \frac{ TS }{42 \cdot 619}$ $ TS = 10 \cdot 626 = 10 \cdot 6$ <p>Or</p> $ \angle HPS = 180 - 15 \cdot 375 - 34$ $= 130 \cdot 625^\circ$ $\frac{\sin 130 \cdot 625}{58} = \frac{\sin 34}{ PS }$ $ PS = 42 \cdot 73$ $\tan 14 = \frac{ TS }{42 \cdot 73}$ $ TS = 10 \cdot 653 = 10 \cdot 7$	<p>Scale 5C (0, 2, 3, 5)</p> <p><i>Low Partial Credit:</i> Some relevant substitution</p> <p><i>High Partial Credit:</i> Formula fully substituted</p>

