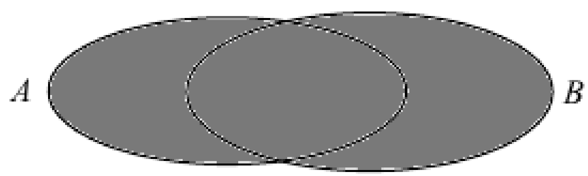
2015 Junior Cert Ordinary Level
Official Sample Paper 1

Question 1 **(Suggested maximum time: 5 minutes)**

(a) On the Venn diagram below, shade the region that represents $A \cup B$.

$A \cup B$ means "A union B" i.e. everything in the set A and everything in B.



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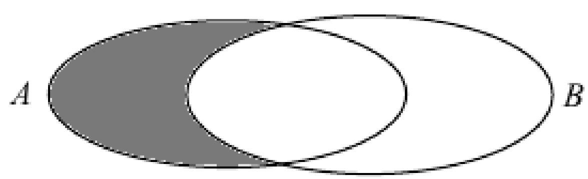
(b) On the Venn diagram below, shade the region that represents $A \backslash B$.

$A \backslash B$ means "A without B" i.e. everything in A without B.



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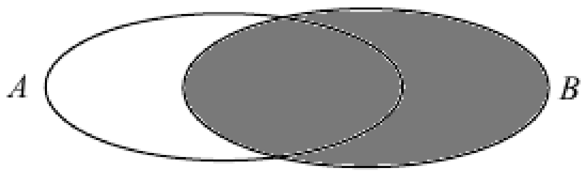
(c) Using your answers to **(a)** and **(b)** above, or otherwise, shade in the region $(A \cup B) \backslash (A \backslash B)$ on the Venn diagram below.

This set is the set in part **(a)** without the set in part **(b)**.



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(d) If A represents the students in a class who like fruit and B represents the students in the same class who like vegetables, write down what the set $A \backslash B$ represents.

$A \backslash B$ means A (those students who like fruit) without B (those students who like vegetables) so this set is

"The set of students who like fruit but don't like vegetables"

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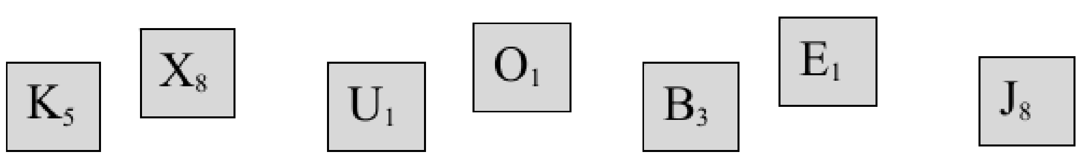
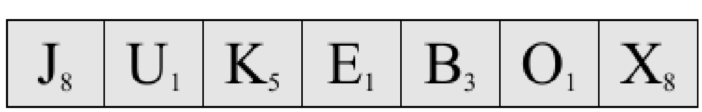
Question 2 **(Suggested maximum time: 5 minutes)**

In the game of *Scrabble*, players score points by making words from individual lettered tiles and placing them on a board. The points for each letter are written on the tile. To find the total score for a word, a player adds together the points for each tile used.

In a game, Maura selects these seven tiles.



She then arranges them to form the word below.



(a) Find the total number of points that Maura would score for the above word.

Total score $= 8+1+5+1+3+1+8 = 27$

(b) Certain squares on the board can be used to gain extra points for letters. Part of one line of the board is shown below. Maura places her word on this line with one letter in each adjacent box.

Maura places her word on the board below in a way that gives the maximum possible score. Write in her word to show how she does this.

			Double letter score J	U	K	E	B	O	Double letter score X

This placement will ensure she gets a double score for J and X, her highest value letters.

(c) Maura also gets a bonus of 50 points for using all her letters. Find the total number of points that Maura scores for this word.

Maura will score double points for J and X and then a 50 point bonus so

$$\text{Total score} = \underbrace{2(8)+1+5+1+3+1+2(8)}_{\text{word score}} + \underbrace{50}_{\text{bonus}} = 93$$

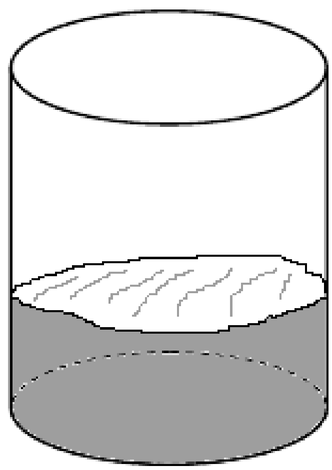
Question 3 **(Suggested maximum time: 5 minutes)**

(a) Write $\frac{3}{8}$ as a decimal.

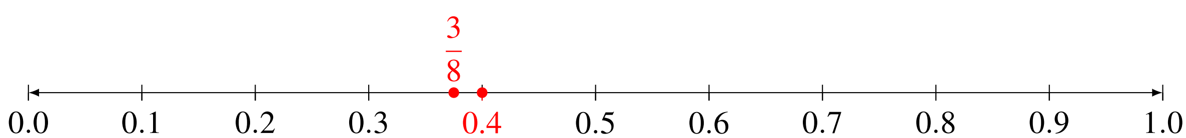
0.375

(b) Sketch the approximate height of water in the glass is the glass is $\frac{3}{8}$ full.

Take the height of the cylinder to be 1, then measure $\frac{3}{8} = 0.375$ on the side of the cylinder.



(c) Represent the numbers $\frac{3}{8}$ and 0.4 on the number line below.

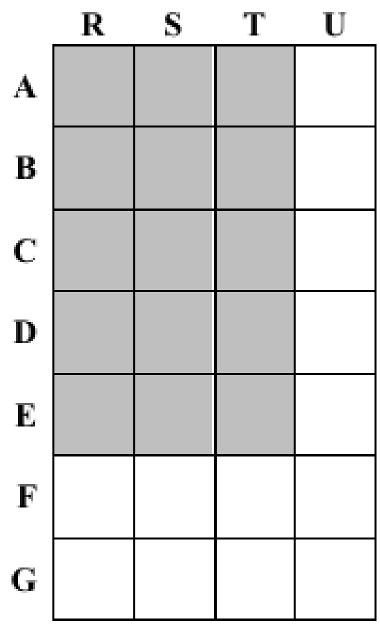


(d) How could the number line in **(c)** above help you decide which is the bigger of the two numbers.

Since both numbers are drawn on the same number line, the number which is furthest to the right is the bigger number.

Question 4 **(Suggested maximum time: 5 minutes)**

(a) **(i)** In the diagram below, what fraction of row **A** is shaded?



There are 4 boxes in row **A** and 3 of those are shaded which means $\frac{3}{4}$ of row **A** is shaded.

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(ii) In the same diagram, what fraction of column **R** is shaded?

There are 7 boxes in column **R** and 5 of those are shaded, so $\frac{5}{7}$ of column **R** is shaded.

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(iii) Using the diagram, or otherwise, calculate the result when the fractions in part **(i)** and part **(ii)** are multiplied.

The answer when $\frac{3}{4}$ and $\frac{5}{7}$ are multiplied is represented by the shaded area of the entire box.

There are 28 boxes in total and 15 of those are shaded so the total shaded area is $\frac{15}{28}$.

Alternatively, multiply the fractions by hand or by calculator to get the same answer.

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(b) Tim claims that the two fractions shown by the shading of the strips **A** and **B** below are the same. Is Tim correct? Give a reason for your answer.

The shaded areas are equal in size, but are different fractions of different sized strips.

$\frac{3}{5}$ of strip A is shaded, whereas $\frac{2}{3}$ of Strip B is shaded.

So, Tim is not correct. The fractions are not the same.

Question 5 **(Suggested maximum time: 10 minutes)**

Dermot has €5000 and would like to invest it for two years. A special savings account is offering an annual compound interest rate of 4% if the money remains in the account for the two years.

(a) Find the interest he would earn in the first year.

$$€5000 \times \frac{4}{100} = €200$$

(b) Tax of 41% will be deducted each year from the **interest** earned. Find the tax on the interest from part **(a)**.

$$€200 \times \frac{41}{100} = €82$$

(c) Find the amount of the investment at the start of the second year.

$$€5000 + \underbrace{€200}_{\text{interest}} - \underbrace{€82}_{\text{tax}} = €5118$$

(d) Find the total amount of Dermot's investment at the end of the second year, after the tax has been deducted from the interest.

The interest he earns in the second year is

$$€5118 \times \frac{4}{100} = €204.72$$

Then he pays tax on this interest of

$$€204.72 \times \frac{41}{100} = €83.94$$

correct to the nearest cent. Which means the total after the second year is

$$€5118 + \underbrace{€204.72}_{\text{interest}} - \underbrace{€83.94}_{\text{tax}} = €5238.78$$

Question 6 **(Suggested maximum time: 5 minutes)**

Samantha is estimating the number of people at a concert.
There are people sitting and people standing at the concert.

(a) Samantha counts 52 rows of seats. She estimates that there are 19 people in each row. By rounding each number to the nearest 10, estimate the total number of people sitting at the concert.

Rounded to the nearest 10 we get $50 \times 20 = 1000$ people

(b) Samantha estimates the standing space is 600 m^2. She estimates that, on average, there are 2 people standing in each square metre. Use this to estimate the total number of people who are standing at the concert.

$600 \times 2 = 1200$ people

(c) A standing ticket for the concert cost €10 and a sitting ticket cost €15. Use your answers from parts **(a)** and **(b)** to estimate the total amount of money paid for tickets for the concert.

$$\begin{aligned} (1000 \times €15) + (1200 \times €10) &= €15000 + €12000 \\ &= €27000 \end{aligned}$$

Question 7 **(Suggested maximum time: 5 minutes)**

(a) Write $2 \times 2 \times 2 \times 2 \times 2 \times 2$ in the form 2^x, where $x \in \mathbb{N}$.

2^6

(b) If $2^p \times 2^3 = 2^8$, write down the value of p.

$2^p \times 2^3 = 2^{p+3}$ which means $p + 3 = 8$ and thus $p = 5$.

We can check the answer by putting $p = 5$ into the equation to get

$$2^5 \times 2^3 = 2^8$$

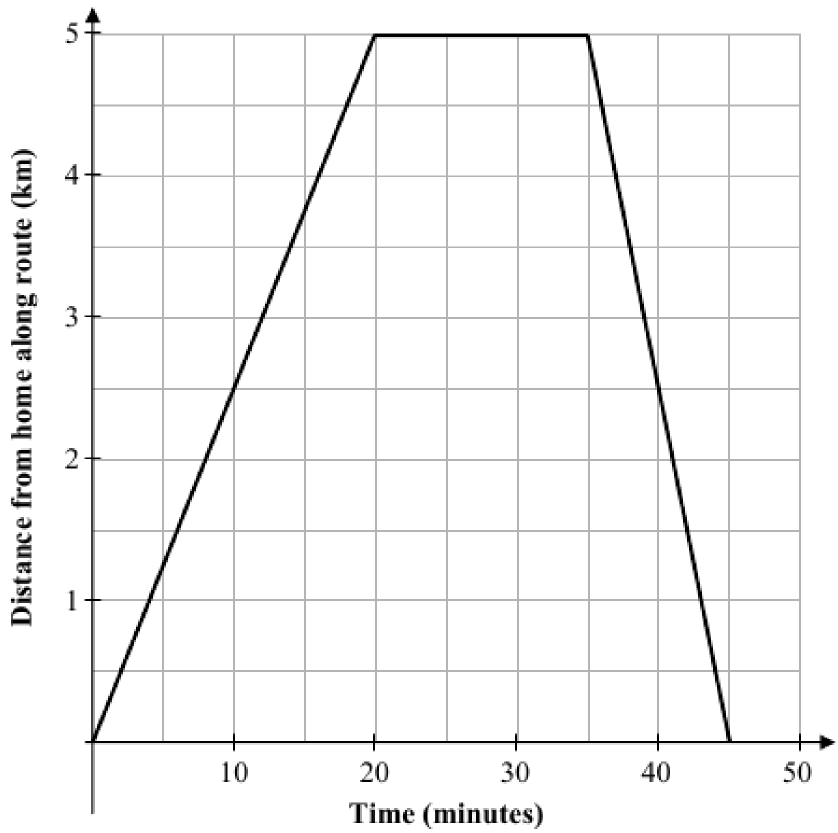
which is correct.

(c) Write $\frac{2^5 \times 2^6}{2^4 \times 2^3}$ in the form 2^x, where $x \in \mathbb{N}$.

$$\begin{aligned} \frac{2^5 \times 2^6}{2^4 \times 2^3} &= \frac{2^{5+6}}{2^{4+3}} \\ &= \frac{2^{11}}{2^7} \\ &= 2^{11-7} \\ &= 2^4 \end{aligned}$$

Question 8 **(Suggested maximum time: 10 minutes)**

Olive cycled from her home to the shop. She cycled along a particular route, and returned by the same route. The graph below shows her distance from home along the route travelled, from the time she left until the time the returned.



(a) What is the distance from Olive's home to the shop?

We can see from the graph that the first leg of the journey extended for a distance of 5km. So the shop is 5 km away.

(b) How long did Olive stay in the shop?

From the 20th minute until the 35th minute Olive's disance from home didn't change which means she was in the shop at that time. So she was there for (35 - 20) = 15 minutes.

(c) Compare her speed on her trip to the shop with her trip on the way home.

The trip to the shop takes 20 minutes and the return trip takes 10 minutes which means Olive's speed was twice as fast returning from the shop as it was cycling to the shop.

(d) Write a paragraph to describe her journey.

Olive takes 20 minutes to cycle 5km to the shop. She stays there for 15 minutes and then cycles 5km home in 10 minutes. Her total trip took $20+15+10=45$ minutes.

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Question 9 **(Suggested maximum time: 15 minutes)**

Tina is standing beside a race-track. A red car and a blue car are travelling at steady speeds on the track. At a particular time the red car has gone 70 m beyond Tina and its speed is 20 m/s. At the same instant the blue car has gone 20 m beyond Tina and its speed is 30 m/s.

(a) Complete the table below to show the distance between the red car and Tina, and the blue car and Tina, during the next 9 seconds.

The red car begins at a distance of 70 m from Tina and this distance increases at a rate of 20 m per second. Similarly, the blue car begins at a distance of 20 m from Tina and this distance increases at a rate of 30 m per second. Thus, our table becomes:

Time	Red Car Distance (m)	Blue Car Distance (m)
0	70	20
1	90	50
2	110	80
3	130	110
4	150	140
5	170	170
6	190	200
7	210	230
8	230	260
9	250	290
10	270	320

(b) After how many seconds will both cars be the same distance from Tina?

From the table, both the cars will be the same distance from Tina after 5 seconds.

(c) Write down a formula to represent the distance between the red car and Tina for any given time. State clearly the meaning of any letters used in your formula.

The red car starts off a distance of 70 m from Tina and increases by 20 m every second i.e.

$$distance = 70 + 20t$$

where t is the time in seconds.

(d) Write down a formula to represent the distance between the blue car and Tina for any given time.

The blue car starts off a distance of 20 m from Tina and increases by 30 m every second i.e.

$$distance = 20 + 30t$$

where t is the time in seconds.

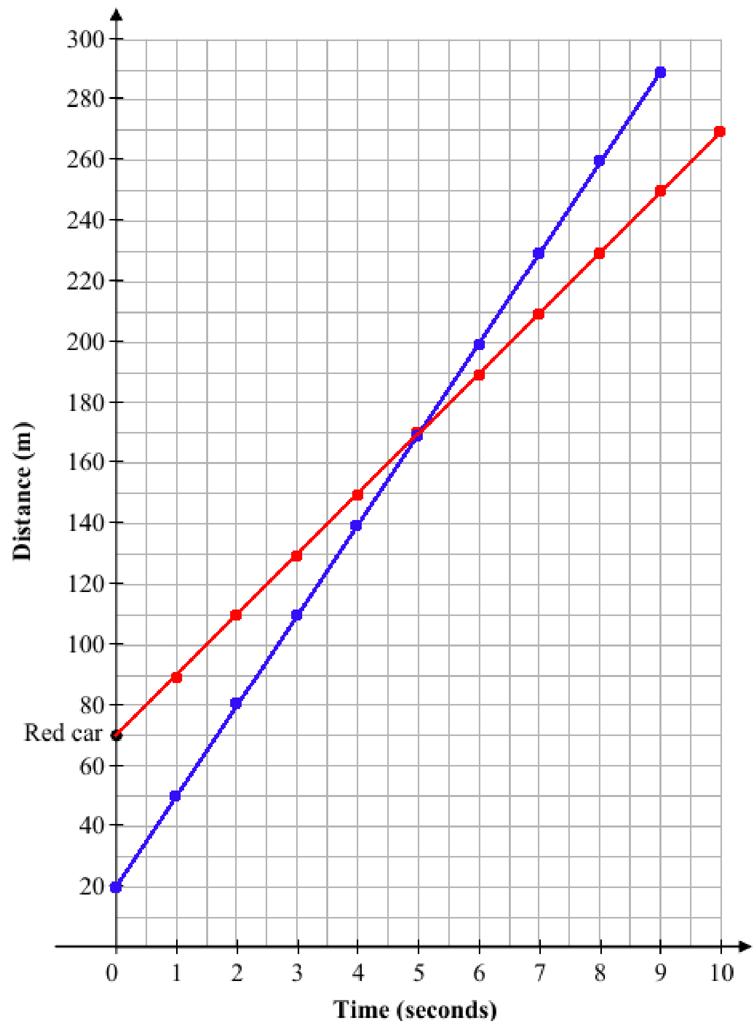
(e) Explain how you could use your formulas from **(c)** and **(d)** to verify the answer that you gave to part **(b)** above.

Let the distance from **(c)** and **(d)** be equal. Then

$$\begin{aligned} 70 + 20t &= 20 + 30t \\ 70 - 20 &= 30t - 20t \\ 50 &= 10t \\ 5 &= t \end{aligned}$$

In other words, the distances are equal when $t = 5$ seconds.

(f) On the diagram below, draw graphs of the distance between the red car and Tina, and the distance between the blue car and Tina, over 9 seconds.



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(g) Explain the connection between your answer to **(b)** and the graphs in **(f)** above.

Since both the lines in part **(f)** describe distance, the point where they interesect represents the point where both distances are equal. From the graph, we can see that this happens after 5 seconds when both cars are a distance of 170 m from Tina. This is consistent with out answer from part **(b)**.

Question 10 **(Suggested maximum time: 5 minutes)**

Mark works two jobs - he works in Bob's Bakery and in Ciara's Café.
He is paid €11 an hour for his work in Bob's Bakery, and €9 an hour for his work in Ciara's Café.
In one week he worked a total of 16 hours and was paid a total of €152.

Find out how many hours he worked in Bob's Bakery this week.

Let x be the number of hours Mark worked in Bob's Bakery and y be the number of hours Mark worked in Ciara's Café. We know the total hours worked is 16 which means

$$x + y = 16 \qquad (1)$$

We also know that the total pay is €152. Mark gets paid 11x euro for working in Bob's Bakery (€11 times the number of hours) and 9y euro for working in Ciara's Café. So

$$11x + 9y = 152 \qquad (2)$$

We now have two simultaneous equations (1) and (2) to solve.

$$\begin{aligned} x + y &= 16 \\ 11x + 9y &= 152 \end{aligned}$$

Multiply the first equation by -11 to get the following simultaneous equations

$$\begin{aligned} -11x - 11y &= -176 \\ 11x + 9y &= 152 \end{aligned}$$

Add these equations to get

$$\begin{aligned} -11x - 11y &= -176 \\ 11x + 9y &= 152 \\ \hline -11y + 9y &= 24 \\ -2y &= 24 \\ y &= 12 \end{aligned}$$

which means Mark worked 12 hours in Ciara's Café. Now we can use equation (1) to get

$$\begin{aligned} x + y &= 16 \\ x + 12 &= 16 \\ x &= 4 \end{aligned}$$

which means Mark worked 4 hours in Bob's Bakery.

Question 11 **(Suggested maximum time: 5 minutes)**

(a) Express $\frac{2x+1}{3}+\frac{3x-5}{2}$ as a single fraction. Give your answer in its simplest form.

$$\begin{aligned}\frac{2x+1}{3}+\frac{3x-5}{2} &= \frac{2(2x+1)+3(3x-5)}{6}\\ &= \frac{4x+2+9x-15}{6}\\ &= \frac{13x-13}{6}\\ &= \frac{13(x-1)}{6}\end{aligned}$$

(b) Solve the equation $\frac{2x+1}{3}+\frac{3x-5}{2}=\frac{13}{2}$.

Using part **(a)** we can re-write this equation as

$$\frac{13(x-1)}{6}=\frac{13}{2}$$

Now cross-multiply to get

$$\begin{aligned}2(13)(x-1) &= 6(13)\\ 26(x-1) &= 78\\ 26x-26 &= 78\\ 26x &= 104\\ x &= 4\end{aligned}$$

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Question 12 **(Suggested maximum time: 15 minutes)**

The expressions $3x+5$, $x+1$, and $2x-10$ are examples of **linear** expressions in x.
Some students are asked to write down linear and quadratic expressions that have $(x+2)$ as a factor.

(a) Write down a linear expression in x, other than $x+2$, that has $x+2$ as a factor.

Examples:

- $2x+4=2(x+2)$
- $5x+10=5(x+2)$
- $12x+24=12(x+2)$
- $100x+200=100(x+2)$

(b) To get her quadratic expression, Denise multiplies $x+2$ by $2x+3$. Find Denise's expression. Give your answer in the form ax^2+bx+c, where $a,b,c \in \mathbb{Z}$.

$$\begin{aligned}(x+2)(2x+3) &= x(2x+3)+2(2x+3)\\ &= 2x^2+3x+4x+6\\ &= 2x^2+7x+6\end{aligned}$$

(c) Anton writes down a quadratic expression of the form x^2-k, where k is a number. For what value of k will Anton's expression have $x+2$ as a factor?

Using the difference of two squares we get

$$x^2-k=(x-\sqrt{k})(x+\sqrt{k})$$

In order to have $x+2$ as a factor we must have $x+2=x+\sqrt{k}$. In other words, $2=\sqrt{k}$ which means $k=4$.

(d) **(i)** Fiona's expression is $3x^2+11x+10$. She uses division to check if $x+2$ is a factor of it. Explain how division will allow her to check this.

If $x+2$ is a factor of $3x^2+11x+10$ then it should divide in evenly i.e. with remainder = 0. Fiona can do this check to see if $x+2$ is a factor.

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(ii) Divide $3x^2+11x+10$ by $x+2$.

$$
\begin{array}{r|rrrrrr}
 & 3x & + & 5 & & \\
\hline
x+2\ ) & 3x^2 & + & 11x & + & 10 \\
 & 3x^2 & + & 6x & & \\
\hline
 & & + & 5x & + & 10 \\
 & & + & 5x & + & 10 \\
\hline
 & & & & & 0
\end{array}
$$

The remainder is 0, therefore $(3x+5)$ is the answer.

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(e) Write down one quadratic expression, other than those already given above, that has $x+2$ as a factor. Give your answer in the form ax^2+bx+c, where $a,b,c\in\mathbb{Z}$.

We can multiply $x+2$ by any linear expression in x to create a quadratic expression with $x+2$ as a factor. For example, we can multiply by $2x+1$ to get

$$
\begin{aligned}
(x+2)(3x+1) &= x(3x+1)+2(3x+1) \\
&= 3x^2+x+6x+2 \\
&= 3x^2+7x+2
\end{aligned}
$$

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Question 13 **(Suggested maximum time: 5 minutes)**

(a) Solve the equation $x^2 + 7x + 12 = 0$.

We can use the quadratic formula. For an equation of the form $ax^2 + bx + c$, the roots are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here we have $a = 1, b = 7, c = 12$. So

$$\begin{aligned} x &= \frac{-7 \pm \sqrt{7^2 - 4(1)(12)}}{2(1)} \\ &= \frac{-7 \pm \sqrt{49 - 48}}{2} \\ &= \frac{-7 \pm \sqrt{1}}{2} \\ &= \frac{-7 \pm 1}{2} \end{aligned}$$

which means

$$x = \frac{-7-1}{2} = \frac{-8}{2} = -4 \qquad \text{and} \qquad x = \frac{-7+1}{2} = \frac{-6}{2} = -3$$

(b) Verify **one** of your answers from **(a)**.

Let's verify $x = -3$. Substitute this into the quadratic equation to get

$$(-3)^2 + 7(-3) + 12 = 9 - 21 + 12 = 0$$

which verifies that $x = -3$ is a solution.

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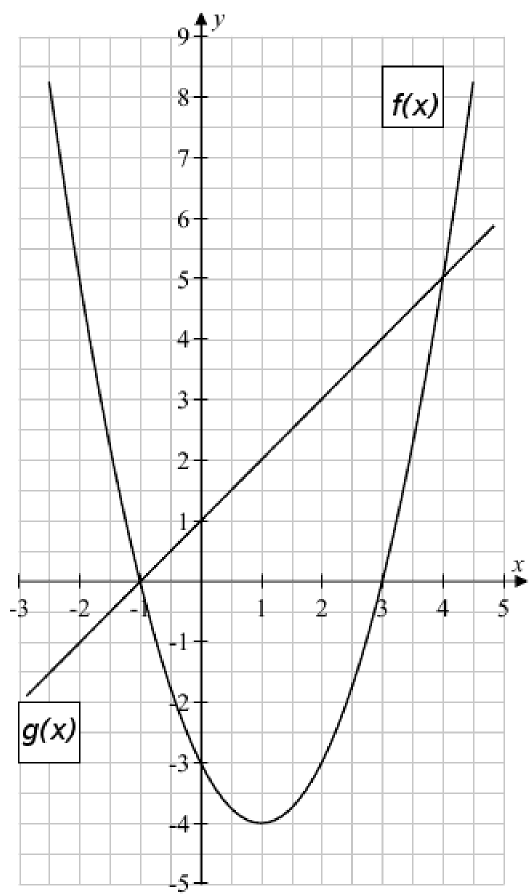
Question 14 **(Suggested maximum time: 10 minutes)**

The graphs of two functions, f and g, are shown on the grid below. The functions are:

$$f(x) = x^2 - 2x - 3$$
$$g(x) = x + 1$$

(a) Match the graphs to the functions by writing $f(x)$ or $g(x)$ in the boxes beside the corresponding graphs on the grid.



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(b) For one of the functions above, explain how you decided on your answer.

$f(x)$ is a quadratic function which means it's either $\cup$-shaped or $\cap$-shaped, while $g(x)$ is linear so it's a straight line.

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(c) Use the graph to find $f(2)$.

When $x = 2$ follow the line directly downwards to find the y-value, $y = -3$, thus $f(2) = -3$.

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(d) Verify your answer to **(c)** above by finishing the following calculation.

$$\begin{aligned} f(2) &= (2)^2 - 2(2) - 3 \\ &= \end{aligned}$$

$$\begin{aligned} f(2) &= (2)^2 - 2(2) - 3 \\ &= 4 - 4 - 3 \\ &= -3 \end{aligned}$$

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(e) Use the graph to find the value of x for which $g(x) = 3$.

Locate 3 on the y-axis and follow this line horizontally to the right until it hits the line $g(x)$ at the point $x = 2$.

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(f) Use the graphs to find the values of x for which $x^2 - 2x - 3 = x + 1$.

These two lines are equal at their points of intersection. These are the points $(-1, 0)$ and $(4, 5)$.

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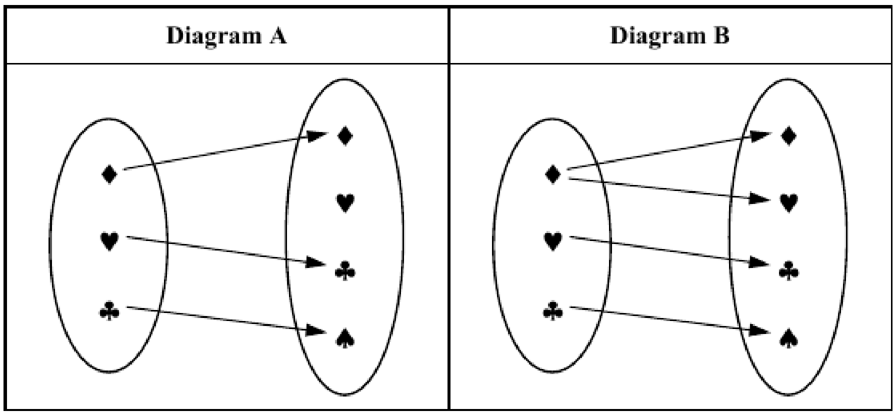
Question 15 **(Suggested maximum time: 5 minutes)**

Two diagrams, labelled **A** and **B** are shown below.
One diagram represents a function; the other does not.



State which diagram does **not** represent a function. Justify your answer.

Diagram **B** does not represent a function. A function relates each item in its domain (left set) to a single item in its range (right set). However, diagram **B** relates the diamond figure on the left to two items on the right instead of just one.

Diagram **A** is a function since each item on the left is mapped to one item on the right.

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