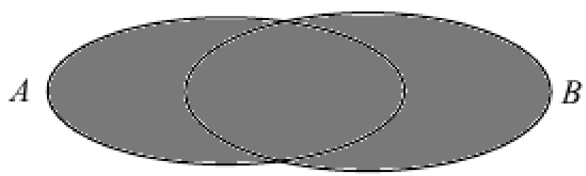
2014 Junior Cert Ordinary Level
Official Sample Paper 1

Question 1 **(Suggested maximum time: 5 minutes)**

(i) On the Venn diagram below, shade the region that represents $A \cup B$.

$A \cup B$ means "A union B" i.e. everything in the set A and everything in B.



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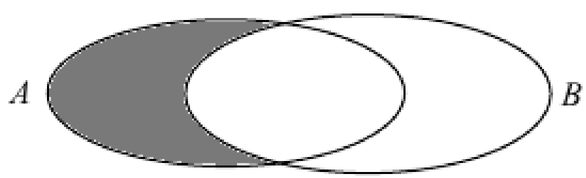
(ii) On the Venn diagram below, shade the region that represents $A \backslash B$.

$A \backslash B$ means "A without B" i.e. everything in A excluding anything in B.



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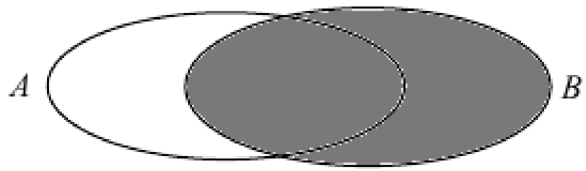
(iii) Using your answers to **(a)** and **(b)** above, or otherwise, shade in the region $(A \cup B) \setminus (A \setminus B)$ on the Venn diagram below.

This set is the set in part **(a)** without the set in part **(b)**.



(iv) If A represents the students in a class who like fruit and B represents the students in the same class who like vegetables, write down what the set $A \setminus B$ represents.

$A \setminus B$ means A (those students who like apples) without B (those students who like bananas) so this set is

"The set of students who like apples but don't like bananas"

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Question 2 **(Suggested maximum time: 5 minutes)**

In the game of *Scrabble*, players score points by making words from individual lettered tiles and placing them on a board. The points for each letter are written on the tile. To find the total score for a word, a player adds together the points for each tile used.

In a game, Maura selects these seven tiles.

K_5 X_8 U_1 O_1 B_3 E_1 J_8

She then arranges them to form the word below.

J_8	U_1	K_5	E_1	B_3	O_1	X_8

(i) Find the total number of points that Maura would score for the above word.

Total score $= 8+1+5+1+3+1+8 = 27$

(ii) Certain squares on the board can be used to gain extra points for letters. The scores for the letters are calculated first. Part of one line of the board is shown below. Maura places her word on this line with one letter in each adjacent box.

Maura places her word on the board below in a way that gives the maximum possible score. Write in her word to show how she does this.

There are two possible answers here. The key thing is to make sure to cover the Double word score, and also, if possible, place the highest scoring letter on the Double letter score. So:

			Double letter score J	U	K	E	Double word score B	O	X		Double letter score

Here Maura gets double points for J (16), giving a total of 35 points for the letters, and then a double word score, for a total of 70 points.

Alternatively:

			Double letter score		J	U	Double word score K	E	B	O	Double letter score X

Here she gets a double score for X (16), again giving a total of 35 points for the letters, and then a double word score, for a total of 70 points.

(iii) Maura also gets a bonus of 50 points for using all her letters. Find the total number of points that Maura scores for this word.

For the first placement shown above, Maura will score double points for J, followed by a double word score and then a 50 point bonus, so:

$$\text{Total score} = \underbrace{(2(8)+1+5+1+3+1+8)\times 2}_{\text{word score}} + \underbrace{50}_{\text{bonus}} = 35\times 2+50 = 120$$

In the second placement, Maura scores double for X first instead of J, so:

$$\text{Total score} = \underbrace{(8+1+5+1+3+1+2(8))\times 2}_{\text{word score}} + \underbrace{50}_{\text{bonus}} = 120$$

For either placement, the total score is 120.

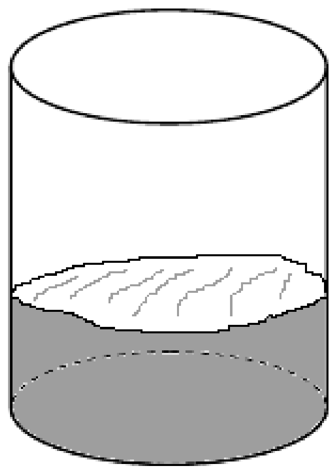
Question 3 **(Suggested maximum time: 5 minutes)**

(i) Write $\frac{3}{8}$ as a decimal.

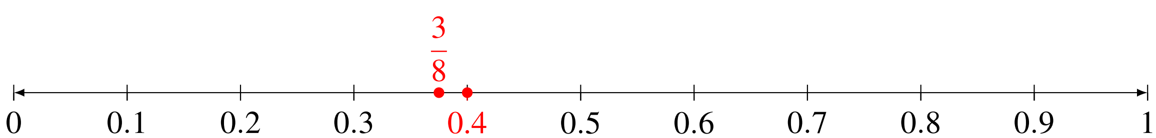
0.375

(ii) Sketch the approximate height of water in the glass is the glass is $\frac{3}{8}$ full.

Taking the height of the cylinder to be 1, then measure $\frac{3}{8} = 0.375$ on the side of the cylinder.



(iii) Represent the numbers $\frac{3}{8}$ and 0.4 on the number line below.



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(iv) How could the number line in **(c)** above help you decide which is the bigger of the two numbers.

Since both numbers are drawn on the same number line, the number which is furthest to the right is the bigger number.

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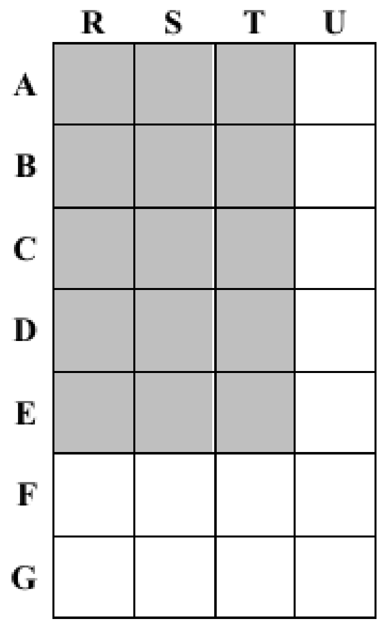
Question 4 **(Suggested maximum time: 5 minutes)**

(a) **(i)** In the diagram below, what fraction of row **A** is shaded?



There are 4 boxes in row **A** and 3 of those are shaded which means $\frac{3}{4}$ of row **A** is shaded.

(ii) In the same diagram, what fraction of column **R** is shaded?

There are 7 boxes in column **R** and 5 of those are shaded, so $\frac{5}{7}$ of column **R** is shaded.

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(iii) Using the diagram, or otherwise, calculate the result when the fractions in part **(i)** and part **(ii)** are multiplied.

The answer when $\frac{3}{4}$ and $\frac{5}{7}$ are multiplied is represented by the shaded area of the entire box. There are 28 boxes in total and 15 of those are shaded so the total shaded area is $\frac{15}{28}$.

Alternatively, multiply the fractions by hand or by calculator to get the same answer.

(b) Tim claims that the two fractions shown by the shading of the strips **A** and **B** below are the same. Is Tim correct? Give a reason for your answer.

The shaded areas are equal in size, but are different fractions of different sized strips.

$\frac{3}{5}$ of strip A is shaded, whereas $\frac{2}{3}$ of Strip B is shaded.

So, Tim is not correct. The fractions are not the same.

Question 5 **(Suggested maximum time: 10 minutes)**

Dermot has €5000 and would like to invest it for two years. A special savings account is offering an annual compound interest rate of 4% if the money remains in the account for the two years.

(i) Find the interest he would earn in the first year.

$$€5000 \times \frac{4}{100} = €200$$

(ii) Tax of 33% will be deducted each year from the **interest** earned. Find the tax (33%) on the interest from part **(i)**.

$$€200 \times \frac{33}{100} = €66$$

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(iii) Find the amount of the investment at the start of the second year.

$$€5000 + \underbrace{€200}_{\text{interest}} - \underbrace{€66}_{\text{tax}} = €5134$$

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(iv) Find the total amount of Dermot's investment at the end of the second year, after the tax has been deducted from the interest. Give your answer correct to the nearest euro.

The interest he earns in the second year is

$$€5134 \times \frac{4}{100} = €205.36$$

Then he pays tax on this interest of

$$€205.36 \times \frac{33}{100} = €67.77$$

correct to the nearest cent. Which means the total after the second year is

$$€5134 + \underbrace{€205.36}_{\text{interest}} - \underbrace{€67.77}_{\text{tax}} = €5271.59$$

which is €5272 correct to the nearest euro.

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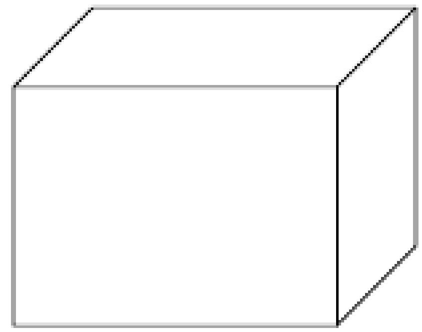
Question 6 **(Suggested maximum time: 5 minutes)**



(i) A rectangular solid is 24.9 cm long, 20.3 cm wide, and 19.6 cm high. By rounding off appropriately, select which of the values **A,B,C** or **D** is the best estimate for the volume of the solid.

A	**B**	**C**	**D**
1000 cm^3	100 cm^3	10,000 cm^3	65 cm^3

By rounding off to the nearest centimetre we get the volume of the box to be

$$Volume = length \times width \times height = 25 \times 20 \times 20 = 10,000 \text{ cm}^3$$

So **C** is the best estimate.

(ii) Using a calculator, or otherwise, calculate the exact volume of the solid in cm^3.

$$Volume = length \times width \times height = 24.9 \times 20.3 \times 19.6 = 9,907.212 \text{ cm}^3$$

Question 7 **(Suggested maximum time: 5 minutes)**

(a) Write $2 \times 2 \times 2 \times 2 \times 2 \times 2$ in the form 2^x, where $x \in \mathbb{N}$.

2^6

(b) If $a^p \times a^3 = a^8$, write down the value of p.

$a^p \times a^3 = a^{p+3}$ which means $p + 3 = 8$ and thus $p = 5$.

We can check the answer by putting $p = 5$ into the equation to get

$$a^5 \times a^3 = a^8$$

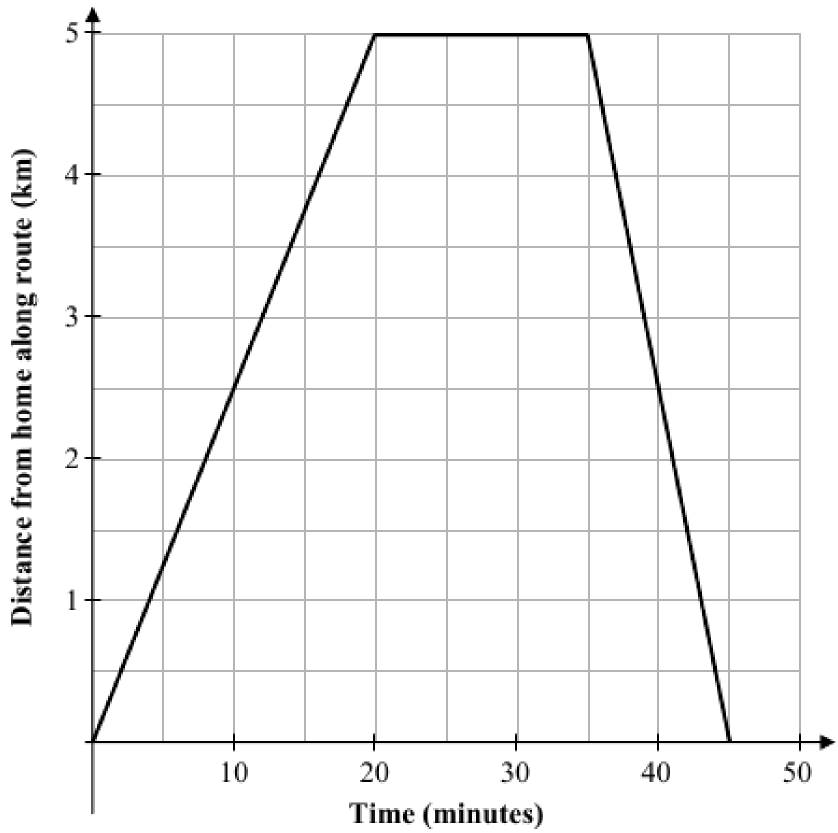
which is correct.

(c) Write $\dfrac{2^5 \times 2^6}{2^4 \times 2^3}$ in the form 2^x, where $x \in \mathbb{N}$.

$$\begin{aligned}
\frac{2^5 \times 2^6}{2^4 \times 2^3} &= \frac{2^{5+6}}{2^{4+3}} \\
&= \frac{2^{11}}{2^7} \\
&= 2^{11-7} \\
&= 2^4
\end{aligned}$$

Question 8 **(Suggested maximum time: 10 minutes)**

Olive cycled to the shop to get some milk for her tea. She cycled along a particular route, and returned by the same route. The graph below shows the different stages of her journey.



(i) How long did Olive stay in the shop?

From the 20th minute until the 35th minute, Olive's disance from home didn't change. This means she was in the shop during that time. So she was there for (35 - 20) = 15 minutes.

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(ii) how far from her home is the shop?

We can see from the graph that the first leg of the journey extended for a distance of 5km. So the shop is 5 km away.

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(iii) Compare the speed of her trip to the shop with her speed on the way home.

The trip to the shop takes 20 minutes and the return trip takes 10 minutes.
This means Olive's speed was twice as fast when she was returning from the shop as it was when she was cycling to the shop.

(iv) Write a paragraph to describe her journey.

Olive takes 20 minutes to cycle 5km to the shop. She stays there for 15 minutes and then cycles 5km home in 10 minutes. Her total trip took $20+15+10=45$ minutes.

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Question 9 **(Suggested maximum time: 10 minutes)**

Tina is standing beside a race-track. A red car and a blue car are travelling at steady speeds on the track. At a particular time the red car has gone 70 m beyond Tina and its speed is 20 m/s. At the same instant the blue car has gone 20 m beyond Tina and its speed is 30 m/s.

(i) Complete the table below to show the distance between the red car and Tina, and the blue car and Tina, during the next 9 seconds.

The red car begins at a distance of 70 m from Tina and this distance increases at a rate of 20 m per second. Similarly, the blue car begins at a distance of 20 m from Tina and this distance increases at a rate of 30 m per second. Thus, our table becomes:

Time	Red Car Distance (m)	Blue Car Distance (m)
0	70	20
1	90	50
2	110	80
3	130	110
4	150	140
5	170	170
6	190	200
7	210	230
8	230	260
9	250	290
10	270	320

(ii) After how many seconds will both cars be the same distance from Tina?

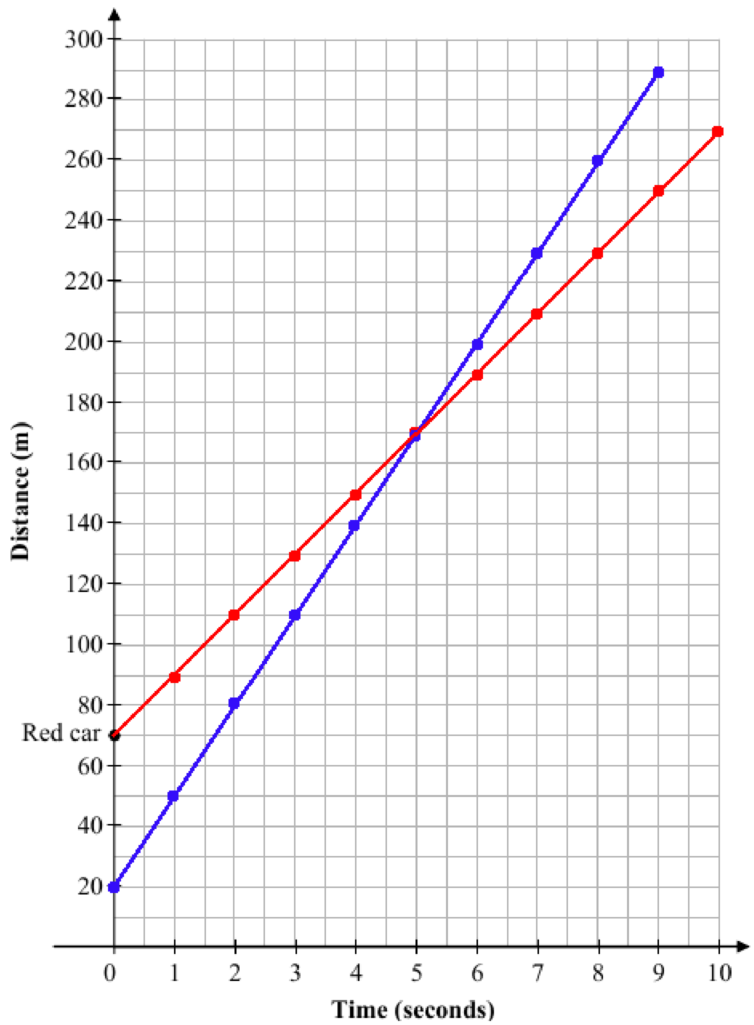
From the table, both cars will be the same distance from Tina after 5 seconds.

(iii) After 8s which car is furthest away from Tina and how far ahead of the other car is it?

From the table, after 8 seconds the red car is 230 m away and the blue car is 260 m away.

Furthest from Tina =	Blue Car
Distance between cars =	30 m

(iv) On the diagram below, draw graphs of the distance between the red car and Tina, and the distance between the blue car and Tina, over 9 seconds.



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(v) Write down a formula to represent the distance between the red car and Tina for any given time. If you use any letters in your formula you should clearly state what each one means.

The red car starts off a distance of 70 m from Tina and increases by 20 m every second i.e.

$$distance = 70 + 20t$$

where t is the time in seconds.

(vi) Write down a formula to represent the distance between the blue car and Tina for any given time.

The blue car starts off a distance of 20 m from Tina and increases by 30 m every second i.e.

$$distance = 20 + 30t$$

where t is the time in seconds.

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(vii) Use your formulas from **(v)** and **(vi)** to verify the answer that you gave to part **(ii)** above.

Let the distance from **(v)** and **(vi)** be equal. Then

$$\begin{aligned} 70 + 20t &= 20 + 30t \\ 70 - 20 &= 30t - 20t \\ 50 &= 10t \\ 5 &= t \end{aligned}$$

In other words, the distances are equal when $t = 5$ seconds, as shown in part (ii) above.

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Question 10 **(Suggested maximum time: 5 minutes)**

(i) John is three times as old as Mary. Mary's age in years is represented by x. Select one expression from **A,B,C,D** and **E** below which represents John's age in years.

A	**B**	**C**	**D**	**E**
$x+3$	$3x$	$3x+3$	$3x+9$	$x-3$

Mary's age is x and John's age is three times that i.e. $3x$. So **B** is the correct answer.

(ii) Select one expression from **A,B,C,D** and **E** above which represents John's age in 3 years time.

Currently, John's age is $3x$, so in three years his age will be $3x+3$. This is expression **C**.

(iii) Select one expression from **A,B,C,D** and **E** which represents Mary's age in 3 years time.

Currently, Mary's age is x, so in three years her age will be $x+3$. This is expression **A**.

(iv) In three years time, John's age added to Mary's age will give a total of 26 years. Write down an equation in x to represent this statement.

$$\begin{aligned}(\text{John's age in 3 years}) + (\text{Mary's age in 3 years}) &= 26\\ (3x+3)+(x+3) &= 26\\ 4x+6 &= 26\end{aligned}$$

(v) Solve your equation to find Mary's present age.

$$\begin{aligned} 4x+6 &= 26 \\ 4x &= 20 \\ x &= 5 \end{aligned}$$

So Mary is 5 years old. We can verify this against the problem statement in part (iv).

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Question 11 **(Suggested maximum time: 5 minutes)**

(i) 1000 people attended a concert. Of those, x were adults and y were children. Use this information to write an equation in x and y.

$x+y=1000$

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(ii) An adult's ticket cost €10 and a child's ticket cost €5. The total amount collected through ticket sales was €8,750. Use this information to write another equation in x and y.

The total collected from adults is €10 times the number of people i.e. $10x$ and the total collected from children is $5y$ which means

$$10x+5y=8750$$

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(iii) Solve your two equations to find the number of adults and the number of children who attended the concert.

The two equations are

$$\begin{aligned} x+y &= 1000 \\ 10x+5y &= 8750 \end{aligned}$$

Multiply the first equation by -5 and then add the two equations together to get

$$\begin{array}{rcl} -5x-5y &=& -5000 \\ 10x+5y &=& 8750 \\ \hline 10x-5x &=& 3750 \\ 5x &=& 3750 \\ x &=& 750 \end{array}$$

So the number of adults is 750. Now use the equation from part **(i)** to find y:

$$\begin{aligned} x+y &= 1000 \\ 750+y &= 1000 \\ y &= 250 \end{aligned}$$

Which means the number of children at the concert was 250.

We can verify our alues for x and y by checking against the original problem statement in part (ii).

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Question 12 **(Suggested maximum time: 10 minutes)**

(a) Find the value of $\dfrac{2x+1}{3}+\dfrac{3x-5}{2}$ when $x=7$.

$$\begin{aligned}\frac{2(7)+1}{3}+\frac{3(7)-5}{2} &= \frac{14+1}{3}+\frac{21-5}{2}\\ &= \frac{15}{3}+\frac{16}{2}\\ &= 5+8\\ &= 13\end{aligned}$$

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(b) Find the value of $\dfrac{2x+1}{3}+\dfrac{3x-5}{2}$ as a single fraction. Give your answer in its simplest form.

$$\begin{aligned}\frac{2x+1}{3}+\frac{3x-5}{2} &= \frac{2(2x+1)+3(3x-5)}{6}\\ &= \frac{4x+2+9x-15}{6}\\ &= \frac{13x-13}{6}\\ &= \frac{13(x-1)}{6}\end{aligned}$$

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(c) Suggest a method to check that your answer to part **(b)** above is correct. Perform this check.

Since we now know that $\dfrac{2x+1}{3}+\dfrac{3x-5}{2}=\dfrac{13(x-1)}{6}$

we can check our answer by substituting $x=7$ into the expression on the right to get

$$\frac{13(7-1)}{6}=\frac{78}{6}=13$$

which is the same as the answer which we found in part **(b)**.

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(d) Solve the equation $\dfrac{2x+1}{3}+\dfrac{3x-5}{2}=\dfrac{13}{2}$.

Using part **(b)** we can re-write this equation as

$$\frac{13(x-1)}{6}=\frac{13}{2}$$

Now cross-multiply to get

$$\begin{aligned} 2(13)(x-1) &= 6(13) \\ 26(x-1) &= 78 \\ 26x-26 &= 78 \\ 26x &= 104 \\ x &= 4 \end{aligned}$$

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Question 13 **(Suggested maximum time: 10 minutes)**

Some students are asked to write down linear and quadratic expressions that have $(x+2)$ as a factor.

(a) The expressions $3x+5$, $x+1$, and $2x-10$ are examples of **linear** expressions in x.Write down a linear expression in x, other than $x+2$, that has $x+2$ as a factor.

Examples:

- $2x+4 = 2(x+2)$
- $5x+10 = 5(x+2)$
- $12x+24 = 12(x+2)$
- $100x+200 = 100(x+2)$

(b) Anton writes down a quadratic expression of the form x^2-k, where k is a number. For what value of k will Anton's expression have $x+2$ as a factor?

Using the difference of two squares we get

$$x^2-k = (x-\sqrt{k})(x+\sqrt{k})$$

In order to have $x+2$ as a factor we must have $x+2 = x+\sqrt{k}$. In other words, $2 = \sqrt{k}$ which means $k=4$.

(c) To get her quadratic expression, Denise multiplies $x+2$ by $2x+3$. Find Denise's expression.

$$\begin{aligned}(x+2)(2x+3) &= x(2x+3)+2(2x+3)\\ &= 2x^2+3x+4x+6\\ &= 2x^2+7x+6\end{aligned}$$

(d) **(i)** Fiona's expression is $3x^2+11x+10$. She uses division to check if $x+2$ is a factor of it. Explain how division will allow her to check this.

If $x+2$ is a factor of $3x^2+11x+10$ then it should divide in evenly i.e. with remainder = 0. Fiona can do this check to see if $x+2$ is a factor.

(ii) Divide $3x^2+11x+10$ by $x+2$.

$$
\begin{array}{r|rrrrr}
 & 3x & + & 5 & & \\
\hline
x+2 & 3x^2 & + & 11x & + & 10 \\
 & 3x^2 & + & 6x & & \\
\cline{2-4}
 & & + & 5x & + & 10 \\
 & & + & 5x & + & 10 \\
\cline{4-6}
 & & & & & 0
\end{array}
$$

The remainder is 0, therefore $(3x+5)$ is the answer.

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(e) Write down one quadratic expression, other than those already given above, that has $x+2$ as a factor.

We can multiply $x+2$ by any linear expression in x to create a quadratic expression with $x+2$ as a factor. For example, we can multiply by $3x+1$ to get

$$
\begin{aligned}
(x+2)(3x+1) &= x(3x+1)+2(3x+1) \\
&= 3x^2+x+6x+2 \\
&= 3x^2+7x+2
\end{aligned}
$$

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Question 14 **(Suggested maximum time: 20 minutes)**

(a) $f(x) = 3x - 5$. Find:

(i) f(7)

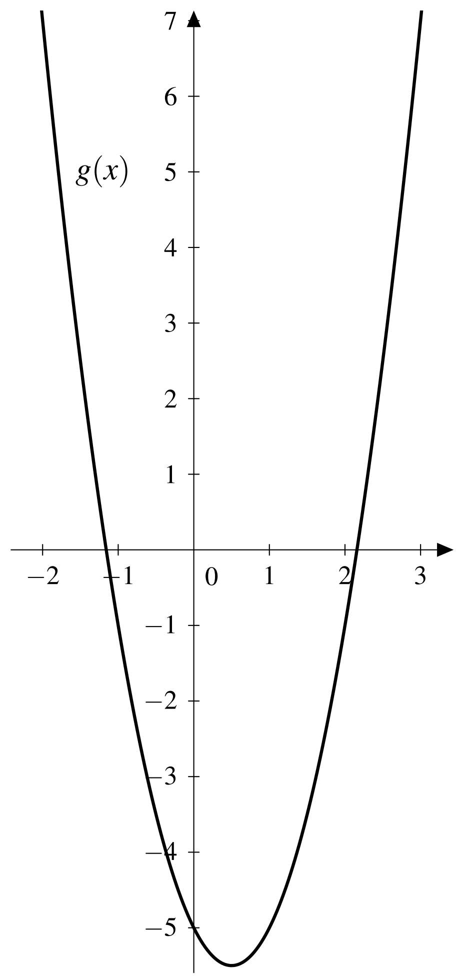
$f(7) = 3(7) - 5 = 21 - 5 = 16$

(ii) f(-1)

$f(-1) = 3(-1) - 5 = -3 - 5 = -8$

(b) **(i)** draw the graph of the function $g : x \mapsto 2x^2 - 2x - 5$ in the domain $-2 \leq x \leq 3$ where $x \in \mathbb{R}$.

x	-2	-1	0	1	2	3
$g(x)$	7	-1	-5	-5	-1	7



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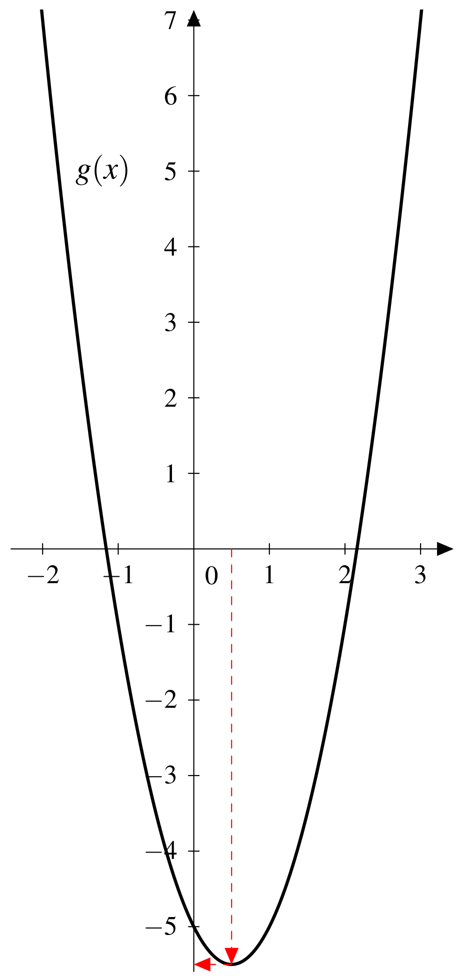
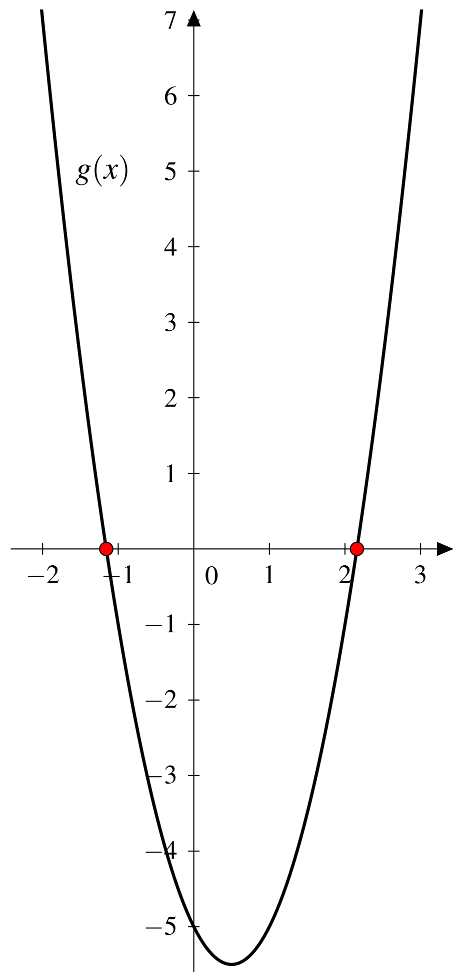
(ii) Use the graph to estimate the value of $2x^2 - 2x - 5$ when $x = 0.5$. (Show your work on the graph)



From the graph, it looks like $g(0.5)$ is approximately -5.5.

(iii) Use the graph to estimate the values of x for which $g(x) = 0$. (Show your work on the graph)



The red marks on the graph represent the points where $g(x) = 0$. These are approximately $x = -1.1$ and $x = 2.1$.