2015 Junior Certificate Higher Level
Official Supplementary Sample Questions

Question 1 **(Suggested maximum time: 10 minutes)**

Below is a photograph of an island. The highest point on the island is 916 metres above sea level. Using this information, and the photograph, estimate as accurately as possible the volume of the island that is above sea level. Give your answer in the form $a \times 10^n$, where $n \in \mathbb{N}$ and $1 \leqslant a < 10$. State clearly any assumptions that you make in finding your answer.

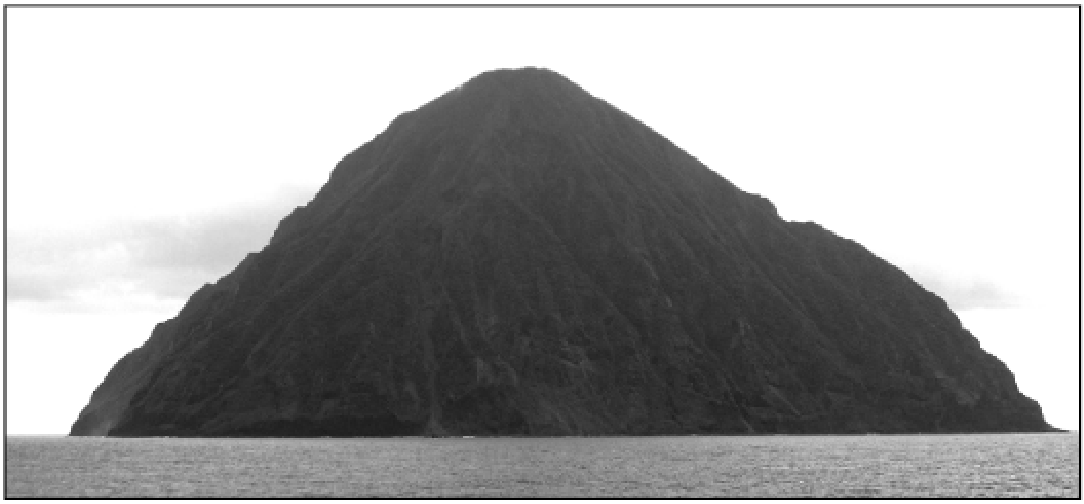

Photo: author: *Karakara* from *ja*. Wikimedia Commons. CC BY-SA 3.0. (Altered)

Measuring from the top of the island to sea-level, the height of the island in the picture is approximately 4.5 cm (this may vary depending on your printer, but we will work from this estimate). This corresponds to 916 m, 1cm in the picture is equivalent to $\frac{916}{4.5} = 203.56$ m correct to two decimal places. So our scale is 1cm : 203.56 m

The island has a triangular profile in the picture, so to estimate the volume, we will assume that the island is cone shaped. The volume of a cone with radius r and height h is $\frac{1}{3}\pi r^2 h$. Here, $h = 916$ so we need to find the radius. The width of the island at sea level is approximately 12 cm, which corresponds to $12 \times 203.56 = 2442.72$ m. This will be the diameter of the cone, so the radius (half of the diameter) is $r = 1221.36$ m. Finally, the volume will be

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (1221.36)^2 (916) = 1.430907225 \times 10^9 \text{m}^3$$

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Question 2 **(Suggested maximum time: 10 minutes)**

n is a natural number.

(a) Write down the next 3 natural numbers, in terms of n.

The next 3 natural numbers are $n+1$, $n+2$ and $n+3$.

Hence, or otherwise, complete the following.

(b) Show that the sum of any 3 consecutive natural numbers is divisible by 3.

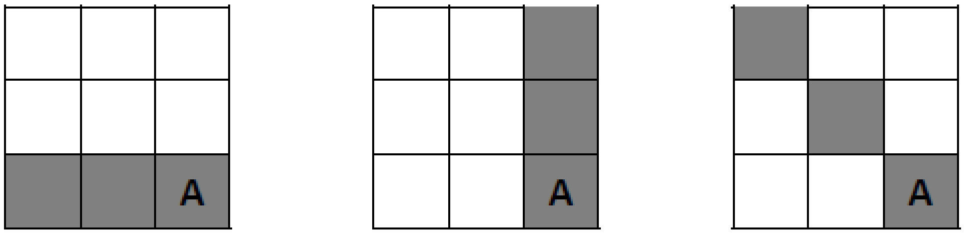
Consider 3 consecutive natural numbers n, $n+1$ and $n+2$. If we sum these numbers, we get $n+(n+1)+(n+2)=3n+3$. We can factorise this sum into $3(n+1)$, which is divisible by 3, concluding the proof.

(c) Prove or disprove the following statement: "The sum of any 4 consecutive natural numbers is never divisible by 4."

Consider 4 consecutive natural numbers n, $n+1$, $n+2$ and $n+3$. If we sum these numbers, we get $n+(n+1)+(n+2)+(n+3)=4n+6$. Now 4 divides the $4n$ term, but 4 does not divide 6. Therefore, regardless of the value of n, 4 cannot divide the sum, proving the statement.

Question 3 **(Suggested maximum time: 15 minutes)**

Noughts and Crosses is a two-person game played on a 3×3 grid, made up of 9 small squares. We call each of the 3 rows, 3 columns, and 2 diagonals a *line*. An example of one type of line is shaded in each of the 3×3 grids below. The square marked **A** in each diagram belongs to all 3 of the lines.



Row Column Diagonal

(a) In the 3×3 grid below, write in each small square the number of different lines to which it belongs. One small square is already filled in for you - it belongs to 3 different lines.

		3

Firstly, we note that every cell is contained in one row and one column, so each cell will belong to at least 2 lines. The four corners are on one of the diagonals, so they belong to 3 lines. The centre cell is on both diagonals, so it belongs to 4 lines. Thus, our grid becomes:

3	2	3
2	4	2
3	2	3

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Imagine *Noughts and Crosses* played on an $n \times n$ grid, made up of n^2 small squares, where $n \geqslant 3, n \in \mathbb{N}$.

A *line* of this grid is one of its rows, one of its columns, or one of its 2 diagonals.

(b) What is the **minimum** number of lines to which each small square of the $n \times n$ grid must belong? Justify your answer.

As noted above, each cell is contained in one row and one column, but not necessarily a diagonal. Thus, each cell in an $n \times n$ grid will belong to at least two lines.

(c) For certain values of n, the maximum number of different lines to which a small square can belong is 4, while for other values of n this **maximum** number is 3. State for which values of n this maximum number is 4, and for which values of n this maximum number is 3. Justify your answer.

Let us use the notation (i, j) to mean the cell in row number i and column number j. The 3×3 and 4×4 grids become:

1,1	1,2	1,3
2,1	2,2	2,3
3,1	3,2	3,3

1,1	1,2	1,3	1,4
2,1	2,2	2,3	2,4
3,1	3,2	3,3	3,4
4,1	4,2	4,3	4,4

Now, note that diagonal from the top left to bottom right is when $i = j$ and that the diagonal from the bottom left to the top right is when $i + j = n + 1$. Every cell on a diagonal will belong to at least 3 lines. If a cell is on both diagonals, however, it will belong to 4 lines. This happens when both $i = j$ and $i + j = n + 1$, i.e. when $i + i = n + 1$ or $n = 2i - 1$.

Thus, if n is an odd number, the centre cell will be on both diagonals and the maximum number will be 4. If n is an even number, there will be no cell on both diagonals, and so the maximum number will be 3.

Question 4 **(Suggested maximum time: 10 minutes)**

In a shop, the selling price of each item includes VAT at a fixed rate.
Show how the shopkeeper could calculate the amount of VAT charged on an item, if she knows the selling price of the item and the rate of VAT. Give an example if necessary.

Firstly, we will calculate the original selling price. If VAT is charged at a rate of $r\%$, then the selling price after VAT will represent $1+\frac{r}{100}$ of the selling price before VAT. So, to get the original selling price, we divide the selling price after VAT by the above factor. Then, we can subtract this value from the selling price to get the VAT charged.

For example, suppose that an item is being sold for €121 after VAT of 10% has been charged. This figure represents $1+\frac{10}{100}=1.1$ of the original selling price. Thus, the selling price before VAT will be $\frac{121}{1.1}=€110$. Thus, VAT charged will be $121-110=€11$

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Question 5 **(Suggested maximum time: 5 minutes)**

Maisy writes down the following theorem:

"If a triangle has sides of length 3 cm, 4 cm, and 5 cm, then it is a right-angled triangle."

(a) State the converse of Maisy's theorem.

The converse of Maisy's theorem states:

"If a triangle is a right-angled triangle, then it has sides of length 3 cm, 4 cm, and 5 cm."

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(b) Is the converse of Maisy's theorem true or false? Justify your answer.

The converse is false. For example, a triangle with sides 5 cm, 12 cm and 13 cm is right-angled.

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Question 6 **(Suggested maximum time: 10 minutes)**

(a) Factorise $5x-15$ and $6-2x$.

$$5x-15 = 5(x-3) \qquad \text{and} \qquad 6-2x = 2(3-x) = -2(x-3)$$

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If A and B are variable quantities, we say that A is proportional to B if the fraction $\frac{A}{B}$ is a constant.

(b) Using your answers to part **(a)** above, show that $5x-15$ is proportional to $6-2x$.

$$\frac{5x-15}{6-2x} = \frac{5(x-3)}{-2(x-3)} = -\frac{5}{2}$$

This fraction is a constant, so $5x-15$ is indeed proportional to $6-2x$.

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(c) Is x^2+3x+2 proportional to $2x+2$? Justify your answer.

Firstly, we need to factorise our two quantities:

$$x^2+3x+2 = (x+2)(x+1) \qquad \text{and} \qquad 2x+2 = 2(x+1)$$

Now, we divide one by the other:

$$\frac{x^2+3x+2}{2x+2} = \frac{(x+2)(x+1)}{2(x+1)} = \frac{x+2}{2}$$

This fraction is **not** constant since it depends on x. Thus, the two quantities are not proportional.

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Question 7 **(Suggested maximum time: 10 minutes)**

Mark works two jobs - he works in Bob's Bakery and in Ciara's Café. He is paid €11.50 an hour for his work in Bob's Bakery, and €9.30 an hour for his work in Ciara's Café.

In one week he worked a total of 34 hours and was paid a total of €362.40.

Find how many hours he worked in Bob's Bakery in this week.

Let x be the hours that Mark worked in Bob's Bakery, and let y be the hours that Mark worked in Ciara's Café. We know that the total hours worked in a week is 34, so $x+y=34$. Given the rates of pay from the two establishments and the total amount earned, we know that $(11.5)x+(9.3)y=362.4$

We can solve these equations simultaneously:

$$\begin{aligned}(11.5)x+(9.3)y &= 362.4\\ x+y &= 34\end{aligned}$$

Since we want to find the number of hours worked in Bob's Bakery, i.e. x, we will eliminate y. We multiply the second equation by 9.3 and subtract:

$$\begin{array}{rcl}(11.5)x+(9.3)y &=& 362.4\\ (9.3)x+(9.3)y &=& 316.2\\ \hline 2.2x &=& 46.2\end{array}$$

Thus $x=\frac{46.2}{2.2}=21$ and so Mark worked 21 hours in Bob's Bakery.

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Question 8 **(Suggested maximum time: 5 minutes)**

(a) Give an example of a data set where this statement is false:

"minimum $<$ **mean** $<$ maximum".

The set containing one number, for example $\{1\}$. Here, the minimum is the same as the maximum and also the mean, so we have

"minimum $=$ **mean** $=$ maximum".

Thus, the statement is false.

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(b) Describe for what kind of data sets this statement is false:

"minimum $<$ **mean** $<$ maximum".

For this kind of set, we need the mean to be equal to the maximum and the minimum. So either the set contains a single element, or if more than one element, all elements of the set have the same numerical value. This also means that the standard deviation for the data will be zero.

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Question 9 **(Suggested maximum time: 10 minutes)**

A rectangular television screen has a diagonal of length 42 inches. The sides of the television screen are in the ratio 16 : 9.

Find the area of the television screen, correct to the nearest whole number.

Let w be the width of the screen and let l be the length of the screen. We know that the ratio of w to l is 16 : 9, so

$$\frac{w}{l} = \frac{16}{9} \qquad \Rightarrow \qquad w = \frac{16}{9}l$$

Since the television screen is rectangular, we know that the width, length and diagonal of the television screen have to solve Pythagoras' Theorem, so:

$$\begin{aligned} w^2 + l^2 = 42^2 \qquad &\Leftrightarrow \qquad \left(\frac{16}{9}l\right)^2 + l^2 = 1764 \\ &\Leftrightarrow \qquad \frac{256}{81}l^2 + l^2 = 1764 \\ &\Leftrightarrow \qquad \frac{337}{81}l^2 = 1764 \\ &\Leftrightarrow \qquad l^2 = \frac{142884}{337} \end{aligned}$$

Now,

$$A = lw = l\left(\frac{16}{9}l\right) = \frac{16}{9}l^2 = \frac{16}{9} \times \frac{142884}{337} = \frac{254016}{337}$$

Thus, the area is 754 in^2 correct to the nearest whole number.

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