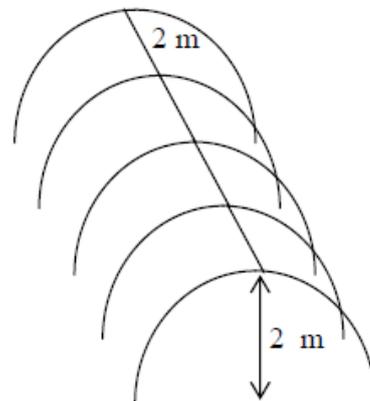


# 2012 Junior Certificate Higher Level Official Sample Paper 2

## Question 1

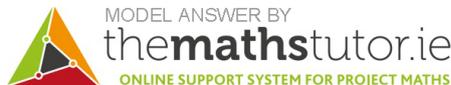
(Suggested maximum time: 20 minutes)

Deirdre constructs a “polytunnel” on a level part of her back garden. Five vertical, semicircular metal hoops, each of radius 2 m, are attached to brackets at ground level and covered with a polythene sheet. The hoops are 2 m apart.



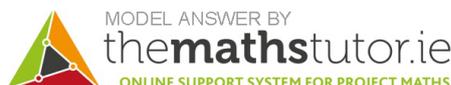
- (a) Find the area of ground covered by the tunnel.

The radius of the hoops is 2 m, so the width of the tunnel is the diameter of the hoops, which is 4 m. The five hoops are 2 m apart, so the length of the tunnel is 8 m. Thus, the area of the ground covered by the tunnel is  $4 \times 8 = 32 \text{ m}^2$ .



- (b) The hoops are also held in place by a straight piece of metal attached at the top of each hoop. Find the total length of metal needed to construct the tunnel.

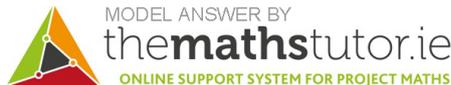
The length of metal which connects the five hoops will be 8 m long. The metal needed for each hoop will be the length of the perimeter of a semicircle. Given that the circumference of a circle is  $2\pi r$ , the length of each hoop will be  $\pi r$ , so  $2\pi$ . Thus the total length of metal needed is  $8 + 5(2\pi) = 8 + 10\pi = 39.42 \text{ m}$  correct to two decimal places.



- (c) The polythene is buried in the ground to a depth of 25 cm all around the tunnel (including both ends). Find the dimensions and area of the smallest rectangular sheet of polythene that can be used.

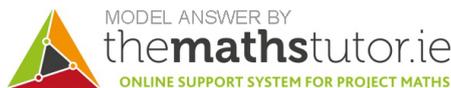
The width of the polythene sheet will be 0.25 m on either side plus the length of the curved area around the hoops. The length of the hoops is  $2\pi$  m, so the total width of the sheet is  $2\pi + 2(0.25) = 2\pi + 0.5 = 6.78$  m correct to two decimal places.

The length of the polythene sheet will be 0.25 m on either side plus the length of the tunnel plus the height of either end. The length of the tunnel is 8 m and each end is 2 m high, so the total length of the sheet is  $8 + 2(2) + 2(0.25) = 12.5$  m. Thus, the total area of the sheet is  $(6.78) \times (12.5) = 84.75$  m<sup>2</sup>

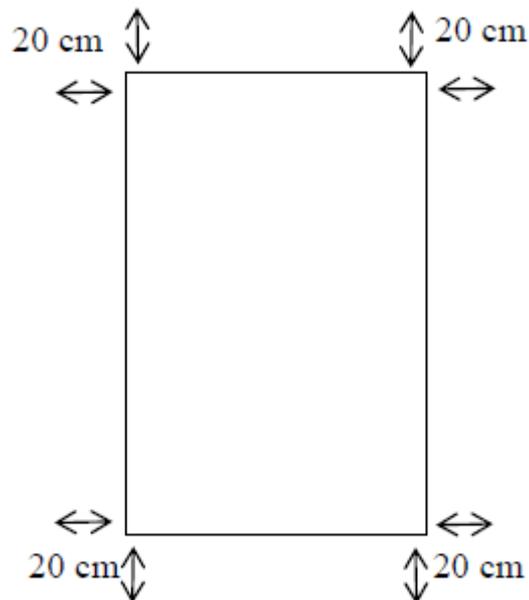


- (d) Find the volume of air in the tunnel.

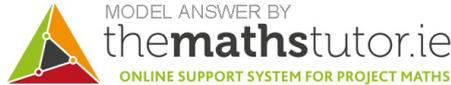
The tunnel is a half cylinder above the ground, so the volume will be half the volume of a cylinder, so  $\frac{1}{2}\pi r^2 h = \frac{1}{2}\pi(2)^2(8) = 16\pi = 50.27$  m<sup>3</sup> correct to two decimal places.



- (e) To finish, Deirdre constructs a rectangular raised bed of height 25 cm inside the tunnel. There is a space of 20 cm between the bed and each side of the tunnel. The bed is then filled with topsoil. Soil costs €80 per tonne and 1 m<sup>3</sup> of soil weighs 0.75 tonnes. Find the cost of filling the bed with soil.



The rectangular bed has length  $8 - 2(0.2) = 7.6$  m and width  $4 - 2(0.2) = 3.6$  m since the bed is 20 cm from each side of the tunnel. Thus, the volume of soil required will be  $7.6 \times 3.6 \times 0.25 = 6.84$  cubic metres. This amount of soil will weigh  $6.84 \times 0.75 = 5.13$  tonnes, and so the cost of filling this bed will be  $5.13 \times 80 = \text{€}410.40$



## Question 2

(Suggested maximum time: 10 minutes)

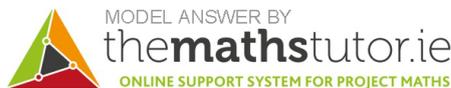
In an experiment, a standard die is tossed 600 times. The results are partially recorded in the table below.

Number of die	1	2	3	4	5	6
Frequency	92	101	115	98		105

- (a) Calculate the number of times that a 5 appeared. Write your answer in the table above.

Let  $x$  the number of times that a 5 appeared. We know that the total number of die tosses is 600, so

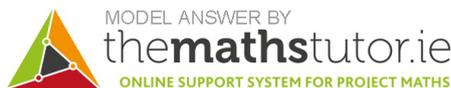
$$92 + 101 + 115 + 98 + x + 105 = 600 \quad \Leftrightarrow \quad 511 + x = 600 \quad \Leftrightarrow \quad x = 89$$



- (b) After looking at the results, Anne claims that the die is unbiased (fair). Do you agree with her? Give a reason for your answer.

*Answer:* Yes

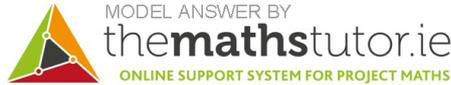
*Reason:* If we tossed a fair dice 600 times, we would expect that each number occurs approximately 100 times. Given that all of these frequencies are close to 100, we can safely assume that the die is unbiased.



- (c) If this die is tossed 300 times, how many times would you expect to get an even number, based on the data in the table? Give a reason for your answer.

*Answer:* 150

*Reason:* From the above table, we have determined that the die is fair. Therefore, we would expect that in 300 tosses an even number would occur half the total number of times, so 150 times.



### Question 3

(Suggested maximum time: 5 minutes)

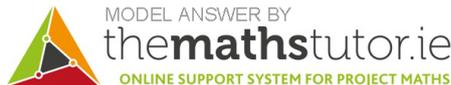
John is going to a festival for the weekend. Each outfit he will wear consists of a pair of jeans, a shirt, a jumper and a pair of shoes. He has packed:

- 3 pairs of jeans (black, navy and blue)
- 4 shirts (white, green, yellow and red)
- 2 jumpers (black and brown)
- 3 pairs of shoes (boots, sandals and flip-flops).

(a) Write down two examples of different outfits John could select to wear.

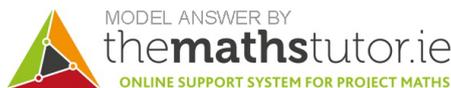
*Example 1:* Black jeans, white shirt, black jumper and boots.

*Example 2:* Navy jeans, green shirt, brown jumper and sandals



(b) How many different possible outfits can John wear over the weekend?

The number of different outfits John could wear is given by  $3 \times 4 \times 2 \times 3 = 72$ .



### Question 4

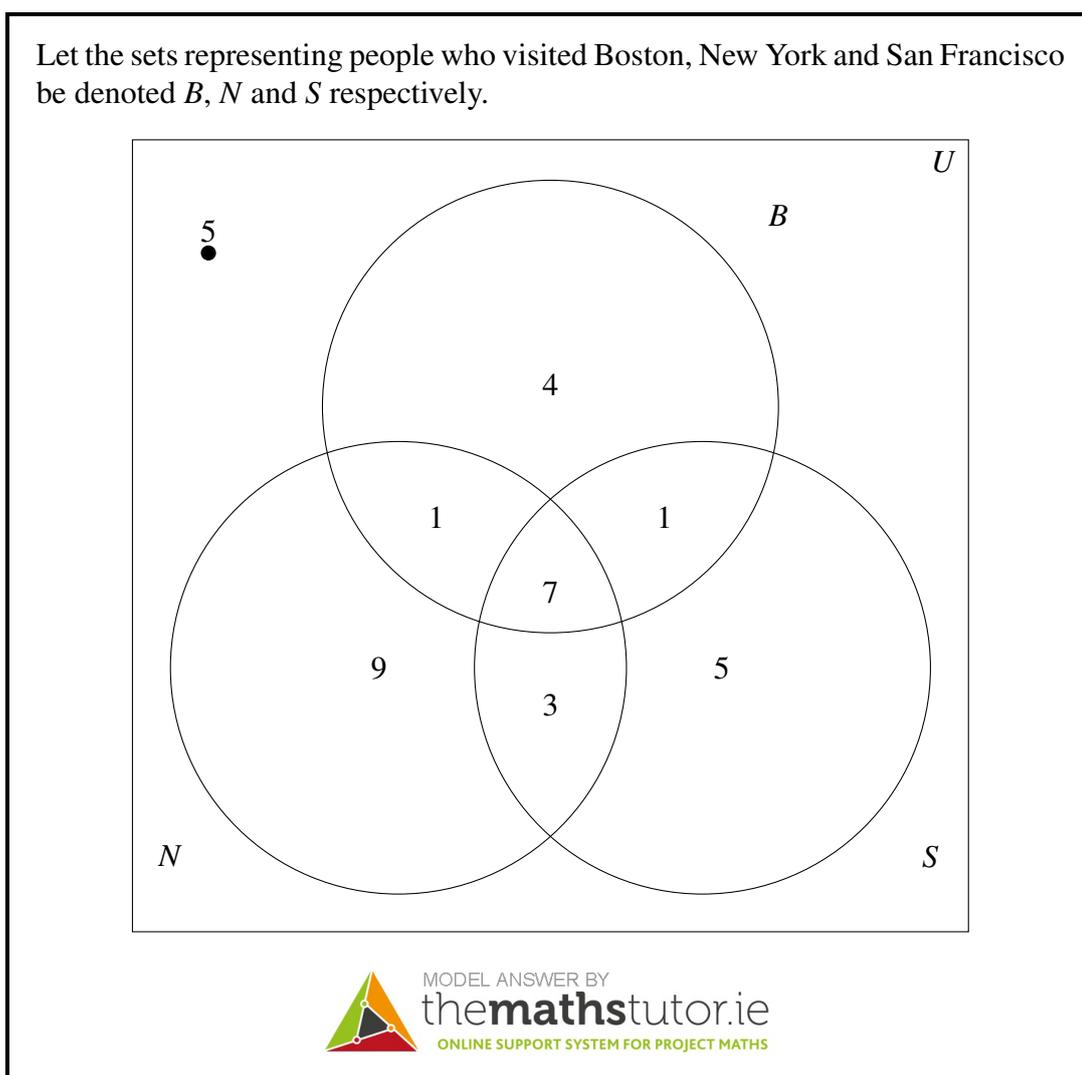
(Suggested maximum time: 10 minutes)

35 people coming back from America were asked if they had visited New York, Boston or San Francisco. The results were as follows:

- 20 had visited New York

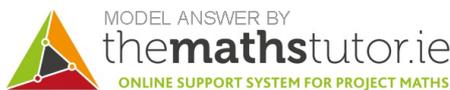
- 13 had visited Boston
- 16 had visited San Francisco
- 7 had been to all three cities
- 3 had been to both New York and San Francisco, but not Boston
- 1 had been to both New York and Boston, but not San Francisco
- 8 had been to Boston and San Francisco.

(a) Display this information in a Venn diagram



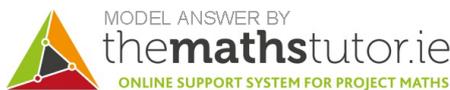
(b) If one person is chosen at random from the group, what is the probability that the person had not visited any of the three cities?

There were 5 people who had not visited any of the three cities. Therefore, the probability of choosing one of these people is  $\frac{5}{35}$ .



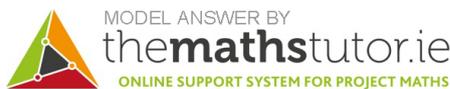
- (c) If one person is chosen at random, what is the probability that the person had visited New York only?

There were 9 people who visited New York only. Therefore, the probability of choosing one of these people is  $\frac{9}{35}$ .



- (d) If one person is chosen at random, what is the probability that the person had visited Boston or New York?

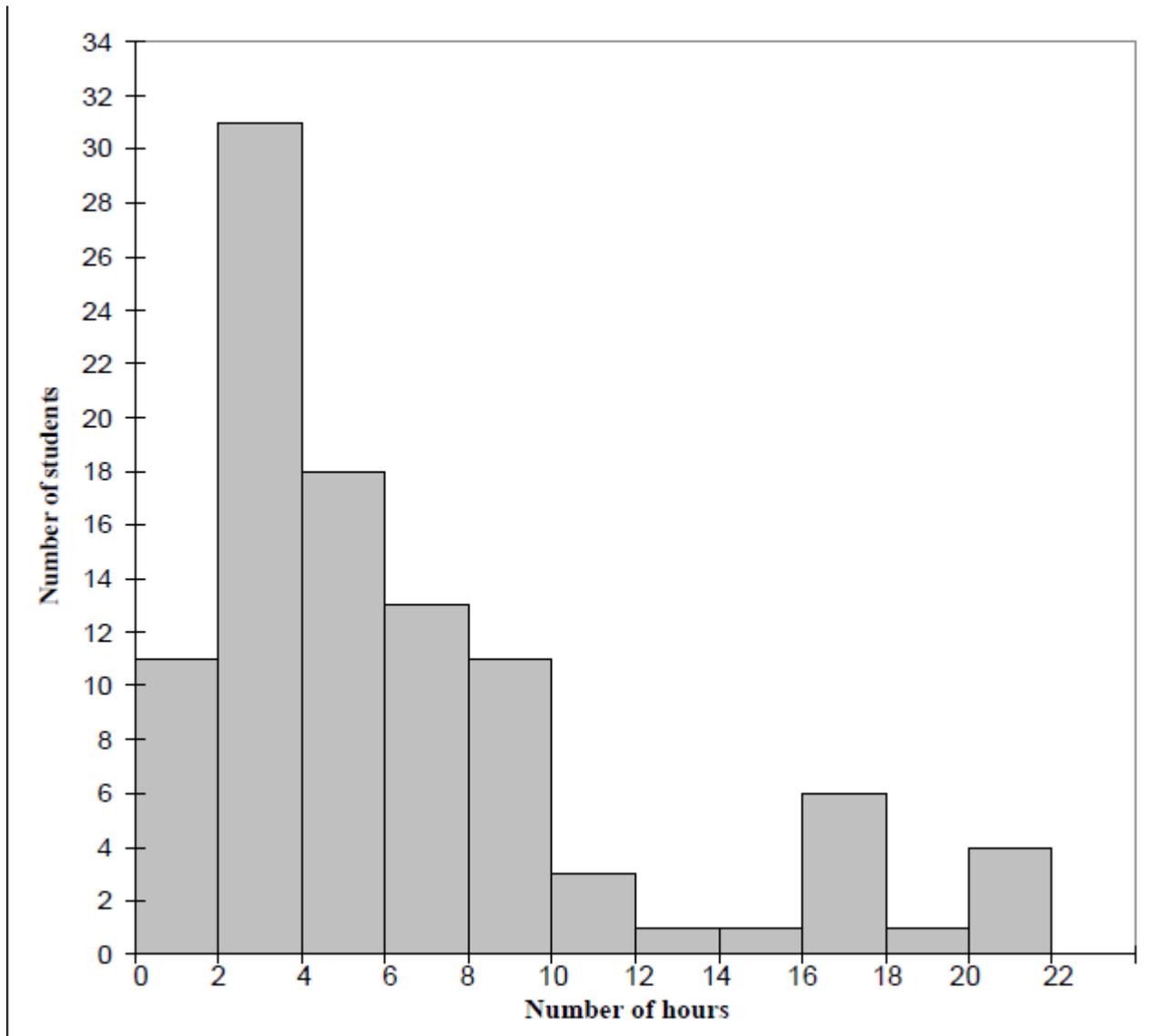
The total number of people who visited either Boston or New York was  $4 + 1 + 9 + 1 + 7 + 3 = 25$ . Therefore, the probability of choosing one of these people is  $\frac{25}{35}$ .



### Question 5

(Suggested maximum time: 10 minutes)

The phase 9 *CensusAtSchool* questionnaire contained the question “Approximately how long do you spend on social networking sites each week?” The histogram below illustrates the answers given by 100 students, randomly selected from those who completed the survey.



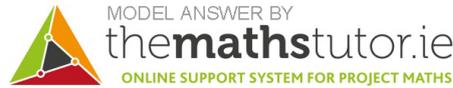
(a) Use the data from the histogram to complete the frequency table below.

Hours	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18	18-20	20-22
Students	11	31	18	13	11	3	1	1	6	1	4


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(b) What is the modal interval?

The modal interval is the interval with the largest frequency, so 2-4 hours.



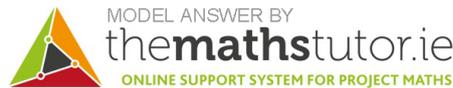
- (c) Taking mid-interval values, find the mean amount of time spent on social networking sites.

Consider the following:

Mid-Interval	1	3	5	7	9	11	13	15	17	19	21
Students	11	31	18	13	11	3	1	1	6	1	4
Product	11	93	90	91	99	33	13	15	102	19	84

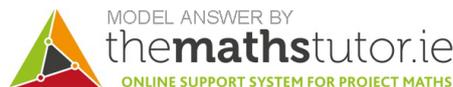
In the above table, the “Product” row is obtained by multiplying the number of students by the mid-interval values. The mean of the data is given by

$$\bar{x} = \frac{11 + 93 + 90 + 91 + 99 + 33 + 13 + 15 + 102 + 19 + 84}{11 + 31 + 18 + 13 + 11 + 3 + 1 + 1 + 6 + 1 + 4} = \frac{650}{100} = 6.5 \text{ hours}$$



- (d) John is conducting a survey on computer usage by students at his school. His questionnaire asks the same question. He plans to carry out his survey by asking the question to twenty first-year boys on the Monday after the mid-term break. Give two reasons why the results from John’s question might not be as representative as those in the histogram.

- Students may have spent more time than usual on social network sites in the previous week, as they were on holiday. This is likely to make their response unrepresentative of typical behaviour.
- The sample should include students from all years.
- The school may be mixed gender, so girls should be included in the sample.
- Twenty boys may represent too small a sample, if the school enrolment is big (many schools have more than a thousand students).



**Question 6****(Suggested maximum time: 10 minutes)**

Students in a third-year class were investigating how the number of jelly beans in a box varies for three different brands of jelly beans.

Each student counted the number of jelly beans in a bag of brand A, B and C. Their results are recorded in the tables below.

**Brand A**

23	25	25	26	26	26	26
27	27	27	27	28	29	29
29	30	30	31	31	31	32
32	32	33	34	35	35	39

**Brand B**

17	22	22	24	24	25	25
25	25	26	26	26	26	26
26	27	27	27	27	28	29
29	29	29	29	29	30	30

**Brand C**

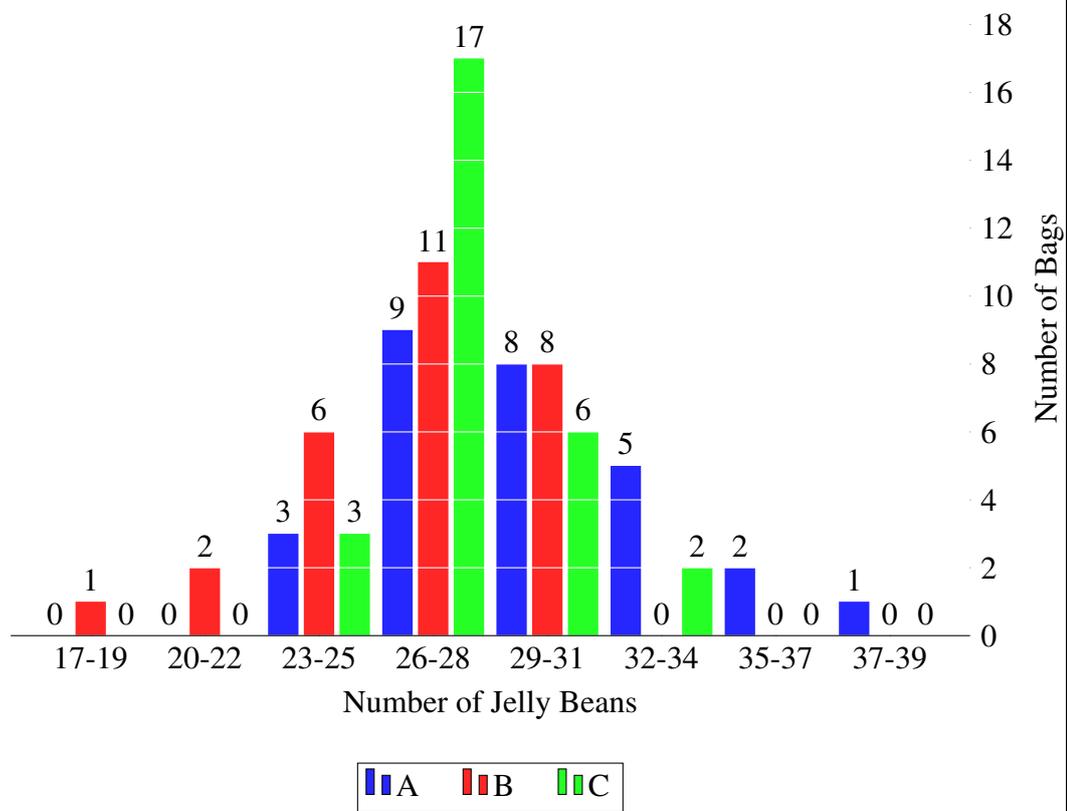
25	25	25	26	26	26	26
26	27	27	27	28	28	28
28	28	28	28	28	28	29
29	29	30	30	31	32	32

- (a) Display the data in a way that allows you to describe and compare the data for each brand.

We will consider a frequency table based on the data. We will consider sub-intervals of length 4 starting from 17-20 and ending with 33-36. Counting the number of bags which fall into each interval for brand A, B and C, we get the following table:

Brand	17-19	20-22	23-25	26-28	29-31	32-34	35-37	37-39
A	0	0	3	9	8	5	2	1
B	1	2	6	11	8	0	0	0
C	0	0	3	17	6	2	0	0

We can represent this data on a histogram as follows:



- (b) If you were to buy a bag of jelly beans which brand would you buy? Give a reason for your answer based on the data provided in the tables. In your explanation you should refer to the **mean** number of jelly beans per bag, and the **range** or **spread** of the number of jelly beans per bag for each brand.

Before we answer, we will calculate the mean number of jelly beans and range for each brand. First we need to get the total for each brand:

$$\begin{aligned} A : & \quad 23 + 25 + 25 + 26 + 26 + 26 + 26 + 27 + 27 + 27 + 27 + 28 + 29 + 29 \\ & \quad + 29 + 30 + 30 + 31 + 31 + 31 + 32 + 32 + 32 + 33 + 34 + 35 + 35 + 39 \\ & = 825 \end{aligned}$$

$$\bar{x}_A = \frac{825}{28} = 29.46$$

$$\begin{aligned} B : & \quad 17 + 22 + 22 + 24 + 24 + 25 + 25 + 25 + 25 + 26 + 26 + 26 + 26 + 26 \\ & \quad + 26 + 27 + 27 + 27 + 27 + 28 + 29 + 29 + 29 + 29 + 29 + 29 + 30 + 30 \\ & = 735 \end{aligned}$$

$$\bar{x}_B = \frac{735}{28} = 26.25$$

$$\begin{aligned} C : & \quad 25 + 25 + 25 + 26 + 26 + 26 + 26 + 26 + 27 + 27 + 27 + 28 + 28 + 28 \\ & \quad + 28 + 28 + 28 + 28 + 28 + 28 + 29 + 29 + 29 + 30 + 30 + 31 + 32 + 32 \\ & = 780 \end{aligned}$$

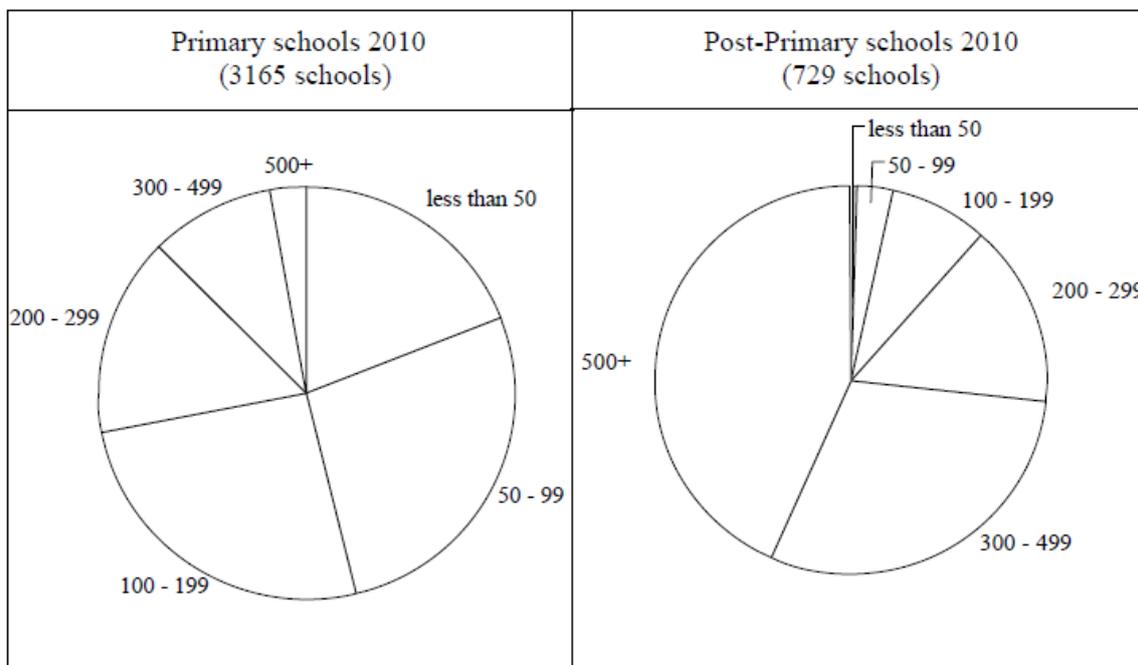
$$\bar{x}_C = \frac{780}{28} = 27.86$$

Thus, brand A has on average the greatest number of jelly beans in each bag. The range of jellies in brand A is [23, 39]. The range of jellies in brand B is [17, 30]. The range of jellies in brand C is [25, 32]. Based on this data, I would pick brand A as it has the highest mean and potentially largest number of jellies per bag.

**Question 7**

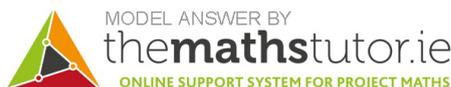
**(Suggested maximum time: 5 minutes)**

The number of students attending primary and post-primary schools in Ireland in 2010 is illustrated in the pie-charts below.



- (a) The angle in the slice for Primary schools with between 100 and 199 pupils is  $93.725^\circ$ . Calculate the number of schools in this category.

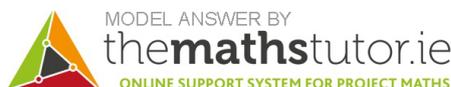
If we let  $x$  denote the number of schools in the 100 – 199 category, we know that  $x = \frac{93.725}{360} \times 3165 = 823.99896$  which gives us 824 schools.



- (b) Mary claims that the charts show that there is roughly the same number of post-primary schools as primary schools in the 200-299 range. Do you agree with Mary? Give a reason for your answer based on the data in the charts.

*Answer:* No.

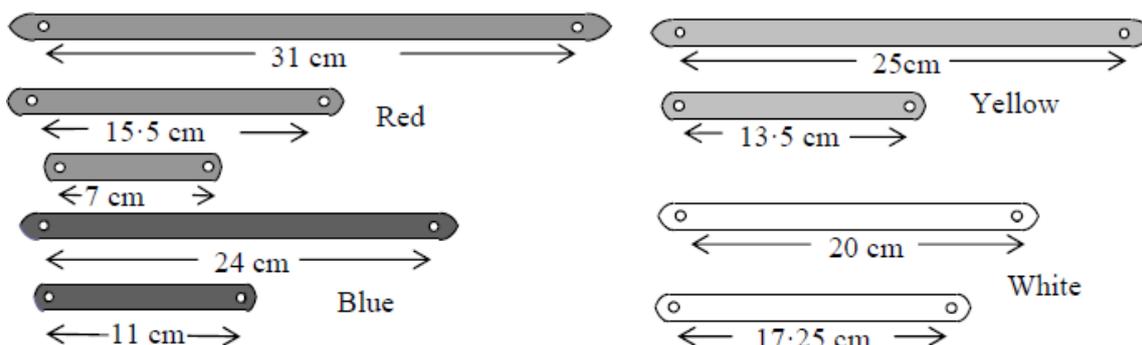
*Reason:* If we look at the slices corresponding to the 200 – 299 range, we see that the angles are approximately the same. This means that there is the same proportion of schools in this range for Primary and Post-Primary. However, since there are 3,165 Primary school and 729 Post-Primary schools, there will not be the same number of schools in this category.



### Question 8

(Suggested maximum time: 10 minutes)

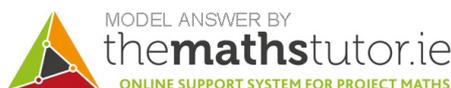
Monica has a set of nine coloured plastic strips (long red, middle red, short red, etc.) as shown below. The strips can be joined together, to form geometrical objects, by pins through small holes at their ends.



- (a) Is it possible to make an object in the shape of an isosceles triangle using any three of the nine strips? Explain your answer.

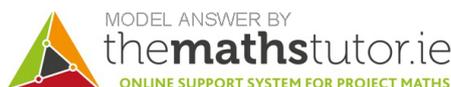
*Answer:* No.

*Reason:* If we were able to make an isosceles triangle, then two of the sides would have the same length. However, none of the strips have the same length. Therefore an isosceles triangle is impossible to make.



- (b) Monica would like to join four strips together to form an object in the shape of a parallelogram. Explain why it is not possible to do this.

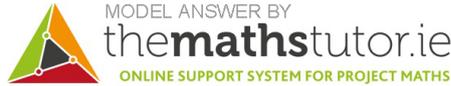
In a parallelogram, opposite sides have equal length. However, none of the strips have the same length, and we are only able to pick four strips. Therefore a parallelogram is impossible to make.



- (c) The long yellow, long blue and short red strips are used to form an object in the shape of a triangle. Monica thinks that this might be a right-angled triangle. Investigate if she is correct.

The long yellow strip has length 25 cm. The long blue strip has length 24 cm. The short red strip has length 7 cm. For this to be a right-angled triangle, it must satisfy Pythagoras' Theorem, i.e.  $a^2 + b^2 = c^2$  where  $c$  is the length of the hypotenuse and  $a$  and  $b$  are the lengths of the other two sides.

The hypotenuse is the longest side of a right-angled triangle, so in our example, it has length 25 cm. Thus, we need to check if  $25^2 = 24^2 + 7^2$ . Now,  $25^2 = 625$  and  $24^2 + 7^2 = 576 + 49 = 625$ . Thus, we have shown that the triangle satisfies Pythagoras' Theorem, and so must be right-angled.

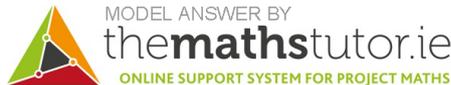


- (d) Monica uses the long blue and the long white strips to form the arms of a right angle. Find the length of a strip that would be needed to complete this triangular shape. Give your answer correct to two decimal places.

The long blue strip has length 24 cm. The long white strip has length 20 cm. If these are to be the length of two sides of a right angled triangle, the third side (the hypotenuse) must satisfy Pythagoras Theorem. If  $x$  is the length of this side, then we have

$$x^2 = 24^2 + 20^2 = 576 + 400 = 976 \quad \Rightarrow \quad x = \sqrt{976}$$

and so  $x = 31.24$  cm correct to two decimal places.

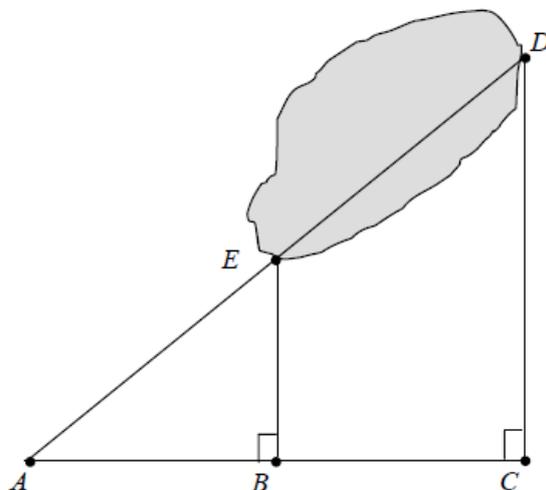


### Question 9

(Suggested maximum time: 5 minutes)

Three paths, [AE], [BE] and [CD], have been constructed to provide access to a lake from a road AC as shown in the diagram. The lengths of the paths from the road to the lake are as follows:

- $|AE| = 120$  m
- $|BE| = 80$  m
- $|CD| = 200$  m

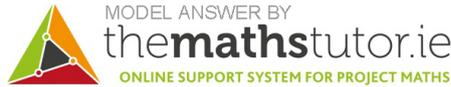


(a) Explain how these measurements can be used to find  $|ED|$ .

The two triangles  $\triangle AEB$  and  $\triangle ADC$  are similar since they have the same three angles. Thus, the corresponding sides are in proportion to one another. This means that

$$\frac{|AD|}{|AE|} = \frac{|CD|}{|BE|}$$

We can use this to find  $|AD|$  and since  $|AD| = |AE| + |ED|$ , we can find  $|ED|$ .

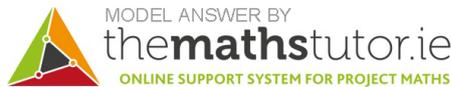


(b) Calculate  $|ED|$ , the distance across the lake.

From the above, we have

$$\frac{|AD|}{120} = \frac{200}{80} \quad \Rightarrow \quad |AD| = 120 \times \frac{200}{80} = 300$$

Thus,  $|ED| = |AD| - |AE| = 300 - 120 = 180$  m.



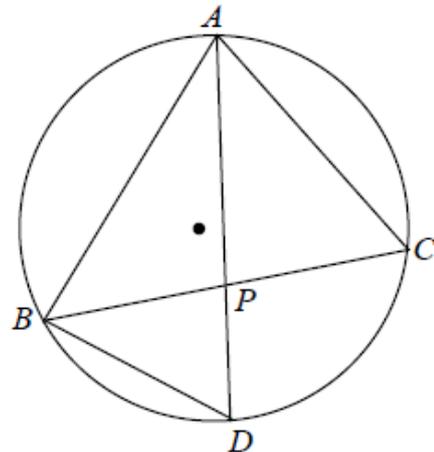
### Question 10

(Suggested maximum time: 5 minutes)

$A$ ,  $B$ ,  $C$ , and  $D$  are four points on a circle as shown.

$[AD]$  bisects  $\angle BAC$ .

$P$  is the point of intersection of  $AD$  and  $BC$ .

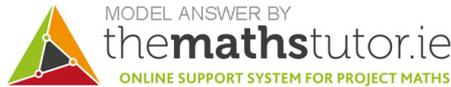


(a) Show that  $\triangle ADB$  and  $\triangle APC$  are similar.

We will do this by showing that the two triangles have the same angles. We are told in the question that  $[AD]$  bisects  $\angle BAC$ , so  $|\angle BAD| = |\angle PAC|$ . Furthermore, since the angles  $\angle ADB$  and  $\angle ACP$  are standing on the same arc from  $A$  to  $B$ , they must be the same, so  $|\angle ADB| = |\angle ACP|$ . Finally, we know that

$$|\angle BAD| + |\angle ADB| + |\angle DBA| = 180^\circ = |\angle CPA| + |\angle PAC| + |\angle ACP|$$

This implies that  $|\angle DBA| = |\angle CPA|$ , and so the two triangles must be similar.

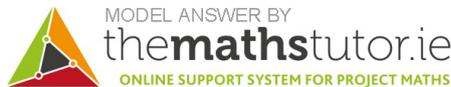


(b) Show that  $|AC| \cdot |BD| = |AD| \cdot |PC|$ .

Since the two triangles are similar, corresponding sides must be in proportion to one another. Thus, we must have

$$\frac{|AC|}{|AD|} = \frac{|PC|}{|BD|}$$

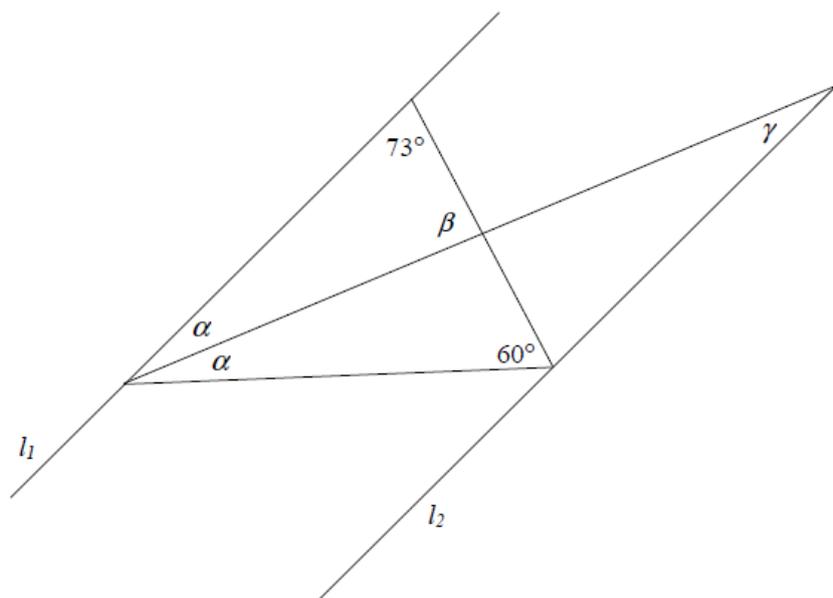
Cross multiplying, this equation gives us  $|AC| \times |BD| = |AD| \times |PC|$ .



### Question 11

(Suggested maximum time: 5 minutes)

If  $l_1 \parallel l_2$ , find the sizes of the angles  $\alpha$ ,  $\beta$  and  $\gamma$  in the following diagram.



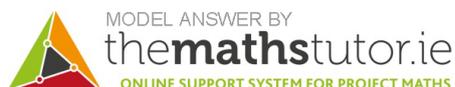
Firstly, we consider the triangle with angles  $73^\circ$ ,  $60^\circ$  and  $2\alpha$ . The sum of these three angles is  $180^\circ$ , so

$$2\alpha + 73 + 60 = 180 \quad \Leftrightarrow \quad 2\alpha = 47 \quad \Leftrightarrow \quad \alpha = 23.5^\circ$$

Secondly, we'll consider the triangle with angles  $\alpha$ ,  $\beta$  and  $73^\circ$ . Again, the sum of these three angles is  $180^\circ$ , so

$$73 + \alpha + \beta = 180 \quad \Leftrightarrow \quad \beta = 83.5^\circ$$

Thirdly, since the two lines are parallel, then corresponding angles  $\gamma = \alpha = 23.5^\circ$

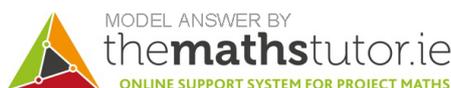
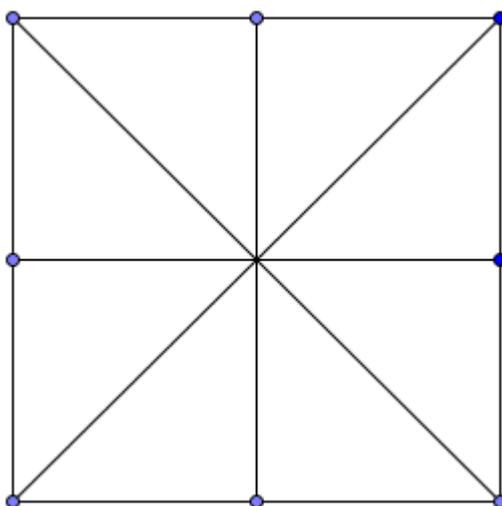


### Question 12

(Suggested maximum time: 5 minutes)

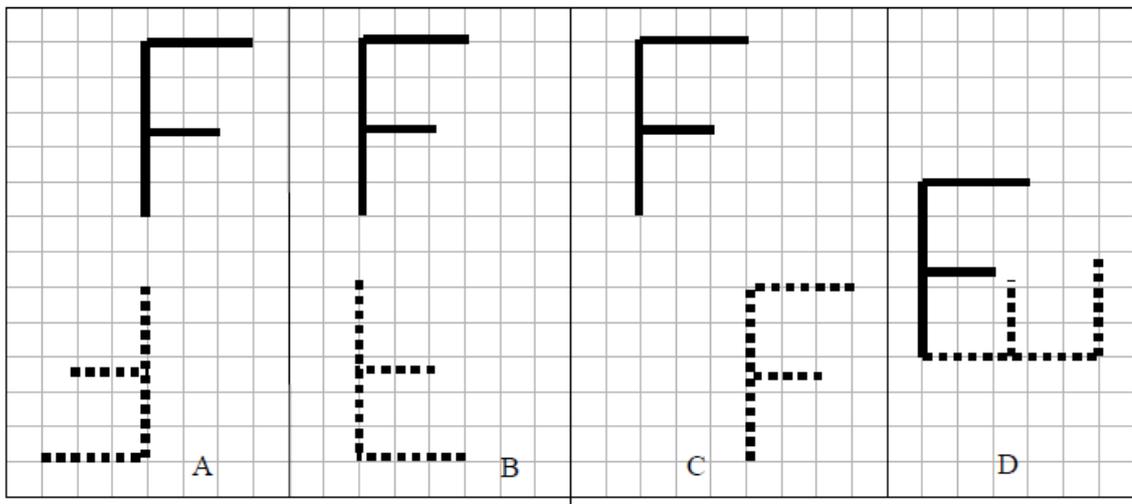
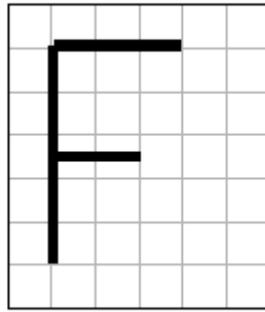
(a) The diagram shows a square. Draw in all its axes of symmetry.

The four axes of symmetry of a square are:



(b) Each of the four diagrams A, B, C and D shows the object in **Figure 1** and its image under a transformation. For each of A, B, C and D, state one transformation (translation, axial symmetry or central symmetry) that will map the object onto that image.

Figure 1



A - Central symmetry

B - Axial symmetry

C - Translation

D - Axial Symmetry

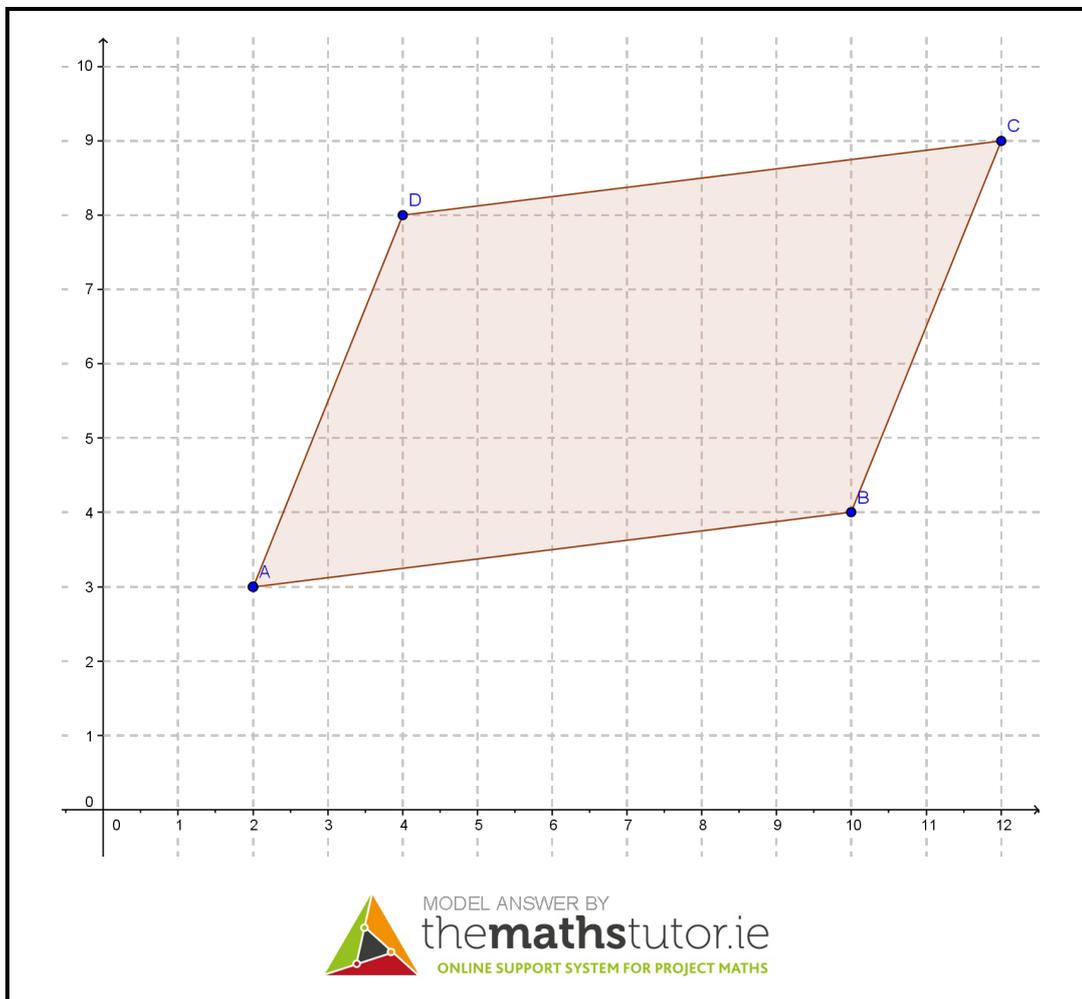


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**Question 13****(Suggested maximum time: 10 minutes)**

$A(2, 3)$ ,  $B(10, 4)$ ,  $C(12, 9)$ , and  $D(4, 8)$  are four points.

- (a) Plot the points on the co-ordinate plane below and join  $A$  to  $B$ ,  $B$  to  $C$ ,  $C$  to  $D$ , and  $D$  to  $A$  to form the quadrilateral  $ABCD$ .



- (b) Verify that **one pair** of opposite sides of  $ABCD$  are equal in length.

Given two points  $(x_1, y_1)$  and  $(x_2, y_2)$ , the distance between these points is given by  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ . Thus:

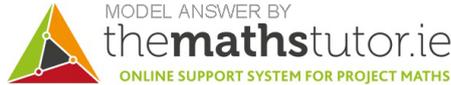
$$|AD| = \sqrt{(4 - 2)^2 + (8 - 3)^2} = \sqrt{2^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29}$$

$$|BC| = \sqrt{(12 - 10)^2 + (9 - 4)^2} = \sqrt{2^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29}$$

Hence  $|AD| = |BC|$

- (c) By finding  $E$  and  $F$ , the midpoints of  $[AC]$  and  $[BD]$  respectively, verify that the diagonals of  $ABCD$  bisect each other.

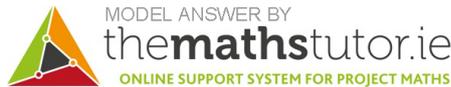
The midpoint two points  $(x_1, y_1)$  and  $(x_2, y_2)$  is given by  $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$ . Thus, the midpoint of  $[AC]$  is given by  $E = (\frac{2+12}{2}, \frac{3+9}{2}) = (7, 6)$ . Similarly, the midpoint of  $[BD]$  is given by  $F = (\frac{10+4}{2}, \frac{4+8}{2}) = (7, 6)$ . Since the diagonals have the same midpoint, they bisect each other.



- (d) Can you now conclude that  $ABCD$  is a parallelogram? Give a reason for your answer.

*Answer:* Yes.

*Reason:* Given a quadrilateral, if the two diagonals bisect each other, then the quadrilateral must be a parallelogram.

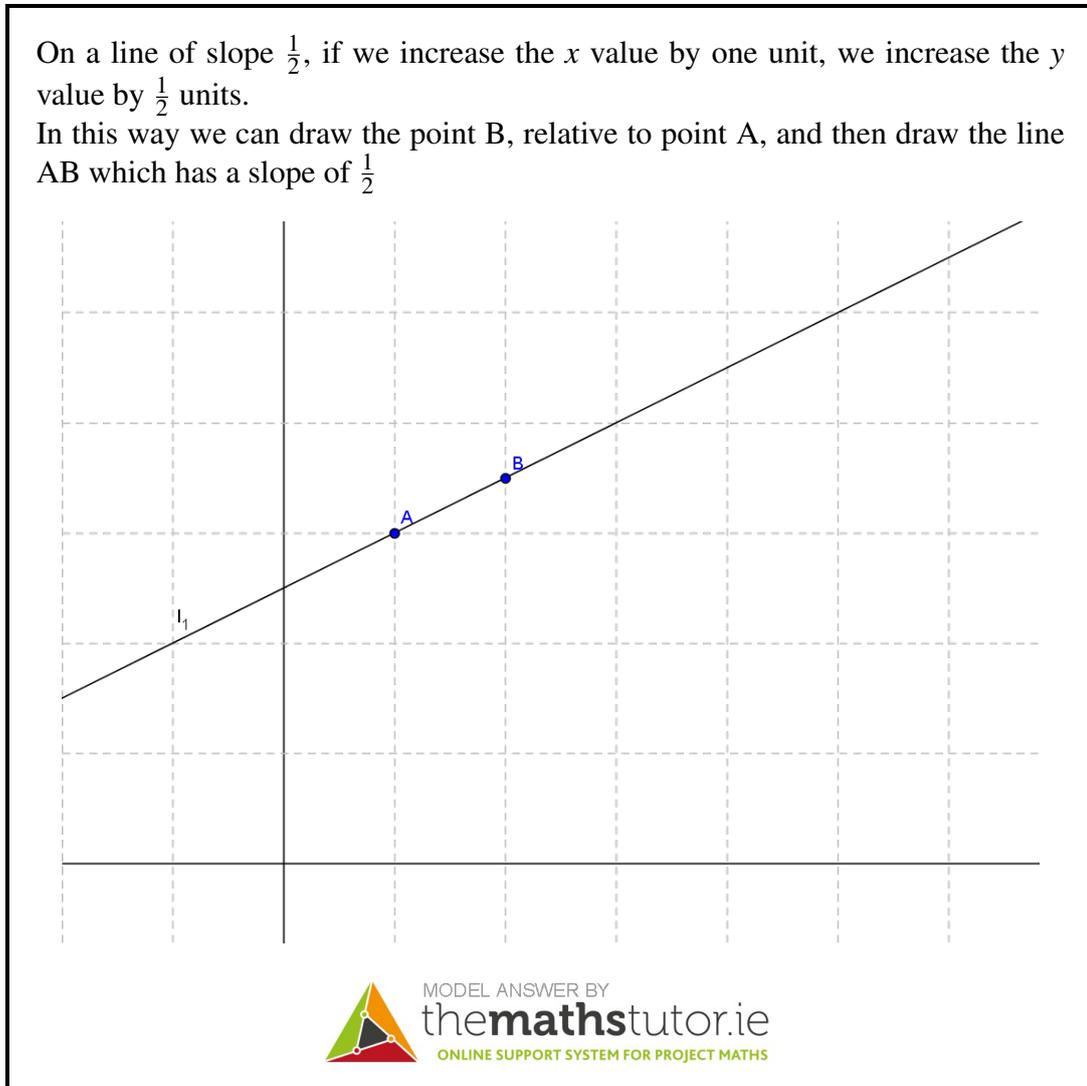


**Question 14**

**(Suggested maximum time: 5 minutes)**

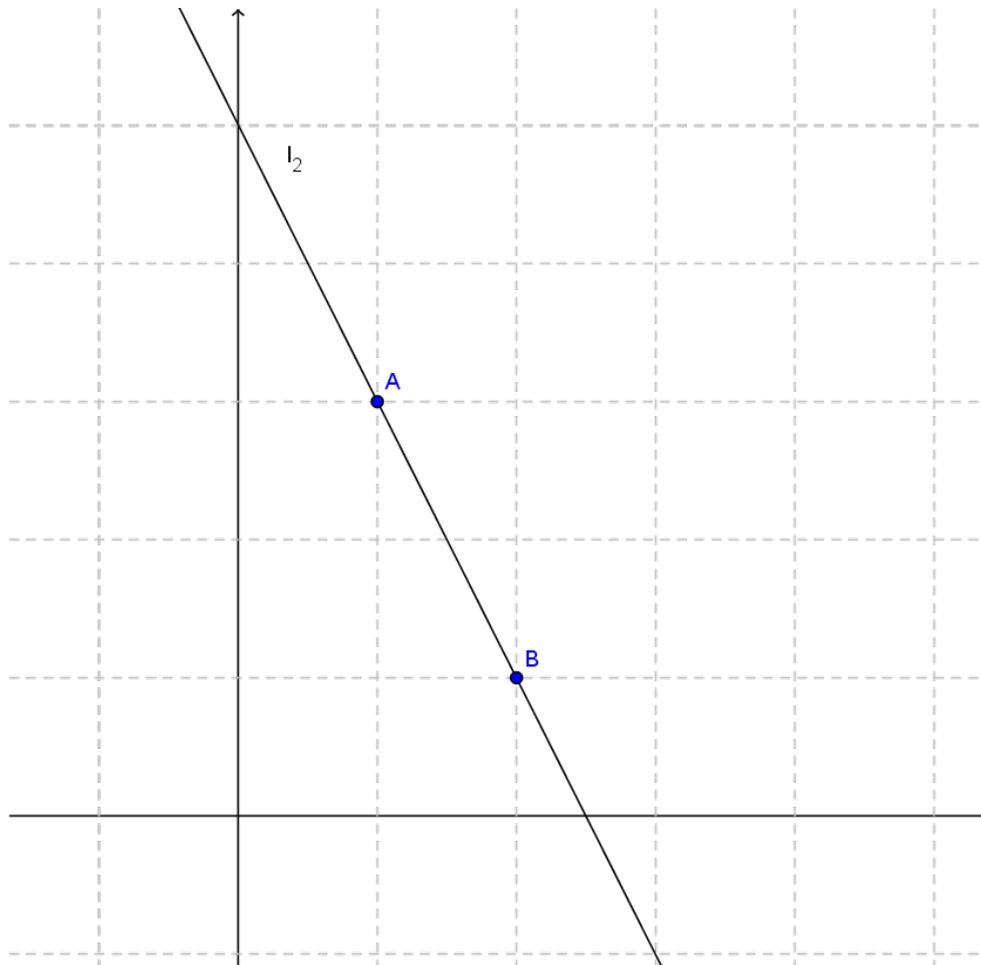
The point A is shown on the coordinate plane. The same scale is used on both axes.

- (a) Draw and label a line  $l_1$  through A which has a slope of  $\frac{1}{2}$ .



- (b) Draw and label a line  $l_2$  through A which has a slope of  $-2$ .

On a line of slope  $-2$ , if we increase the  $x$  value by one unit, we decrease the  $y$  value by 2 units.  
In this way we can draw the point B, relative to point A, and then draw the line AB which has a slope of  $-2$



**Question 15**

**(Suggested maximum time: 10 minutes)**

During a trigonometry lesson a group of students wrote down some statements about what they expected to happen when they looked at the values of trigonometric functions of some angles. Here are some of the things they wrote down.

- 1 The value from any of these trigonometric functions will always be less than 1.
- 2 If the size of the angle is doubled then the value from the trigonometric functions will not double.

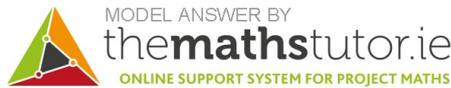
- 3 The value from all of the trigonometric functions will increase if the size of the angle is increased.
- 4 I do not need to use a calculator to find  $\sin 60^\circ$ . I can do it by drawing an equilateral triangle. The answer will be in surd form.

They then found the sin, cos and tan of some angles, correct to three decimal places, to test their ideas.

- (a) Do you think that **1** is correct? Give an example to justify your answer.

*Answer:* No.

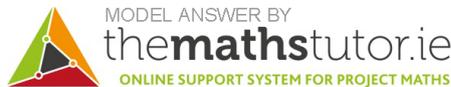
*Example:*  $\tan(250^\circ) = 2.727$



- (b) Do you think that **2** is correct? Give an example to justify your answer.

*Answer:* Yes.

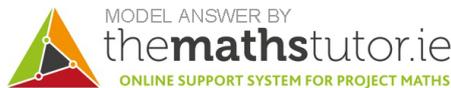
*Example:*  $\sin(90^\circ) = 0$ , but  $\sin(180^\circ) = -1$



- (c) Do you think that **3** is correct? Give an example to justify your answer.

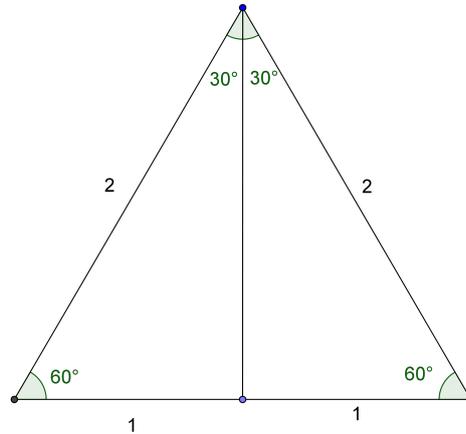
*Answer:* No.

*Example:*  $\cos(45^\circ) = 0.7071$ , but  $\cos(90^\circ) = 0$



- (d) Show how an equilateral triangle of side 2 cm can be used to find  $\sin 60^\circ$  in surd form.

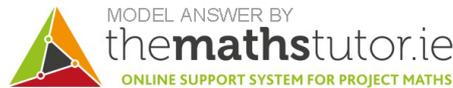
We construct an equilateral triangle with sides of length 2 cm, and then bisect the triangle to form two right angled triangles.



Let  $x$  denote the length of the vertical line. Since both triangles are right-angled, they must satisfy Pythagoras' Theorem. Thus,

$$1^2 + x^2 = 2^2 \quad \Leftrightarrow \quad x^2 = 4 - 1 = 3 \quad \Leftrightarrow \quad x = \sqrt{3}$$

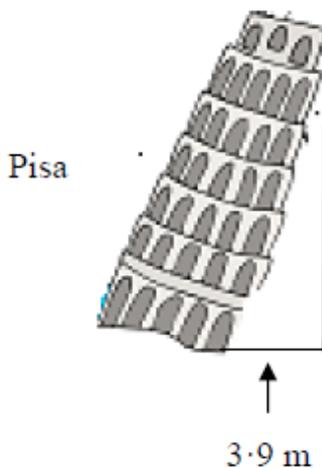
In a right angled triangle, we have  $\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$ , and so  $\sin(60^\circ) = \frac{\sqrt{3}}{2}$



### Question 16

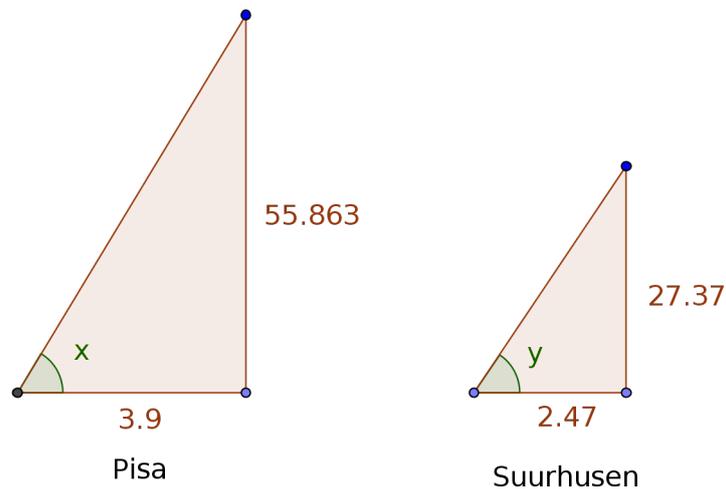
(Suggested maximum time: 10 minutes)

The Leaning Tower of Pisa is 55.863 m tall and leans 3.9 m from the perpendicular, as shown below. The tower of the Suurhusen Church in north-western Germany is 27.37 m tall and leans 2.47 m from the perpendicular.



By providing diagrams and suitable calculations and explanations, decide which tower should enter the *Guinness Book of Records* as the **Most Tilted Tower in the World**.

We can use this information to construct two right angled triangles (not to scale), which will represent the lean of the buildings. We are assuming that "tall" refers to vertical height, as opposed to slant height:



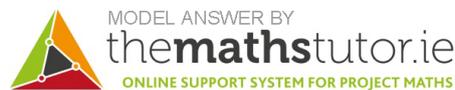
If the angles  $x$  and  $y$  are small, then the lean of the building will be large. Thus, whichever angle is smaller will represent the greatest lean. We know the following:

$$\tan(x) = \frac{\text{opposite}}{\text{adjacent}} = \frac{55.863}{3.9} = 14.3238 \quad \Rightarrow \quad x = \tan^{-1}(14.3238) = 86.0^\circ$$

and similarly,

$$\tan(y) = \frac{\text{opposite}}{\text{adjacent}} = \frac{27.37}{2.47} = 11.0810 \quad \Rightarrow \quad y = \tan^{-1}(11.0810) = 84.8^\circ$$

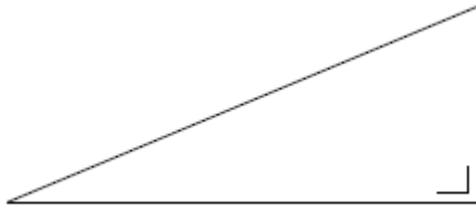
Since  $y$  is smaller than  $x$ , the Suurhusen Church is more tilted than the Leaning Tower of Pisa. If it is a choice between these two, then the Suurhusen Church tower should be listed as the Most Tilted Tower in the World.



### Question 17

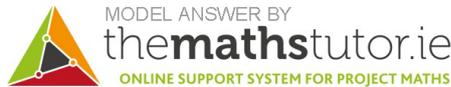
(Suggested maximum time: 5 minutes)

In the right-angled triangle shown in the diagram, one of the acute angles is four times as large as the other acute angle.

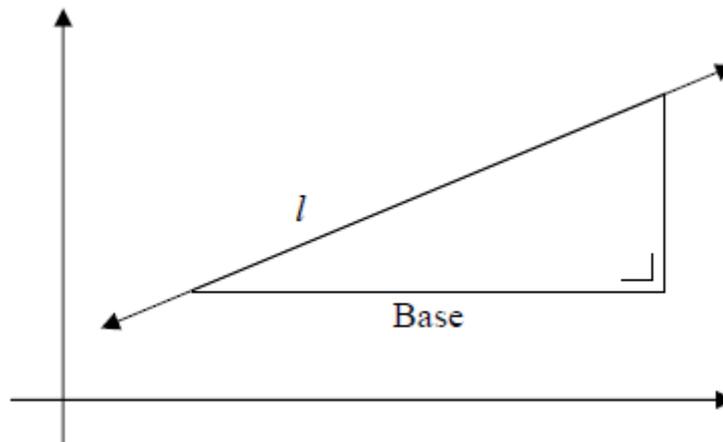


- (a) Find the measures of the two acute angles in the triangle.

If we let  $x$  denote the smallest angle in the triangle, then the other acute angle is equal to  $4x$ . Since all angles have to sum to  $180^\circ$ , we have  $x + 4x + 90^\circ = 180^\circ$ , and so  $5x = 90^\circ$  giving  $x = 18^\circ$ .



- (b) The triangle in part (a) is placed on a co-ordinate diagram. The base is parallel to the  $x$ -axis. Find the slope of the line  $l$  that contains the hypotenuse of the triangle. Give your answer correct to three decimal places.



The slope of the line will be the change in  $y$ -values divided by the change in  $x$ -values. In relation to the triangle, this will be  $\frac{\text{opposite}}{\text{adjacent}}$ , which we know is equal to  $\tan$  of the angle to the left of the triangle. We evaluated this angle in part (a) to be  $18^\circ$ , so the slope will be  $\tan(18^\circ) = 0.325$  correct to three decimal places.

