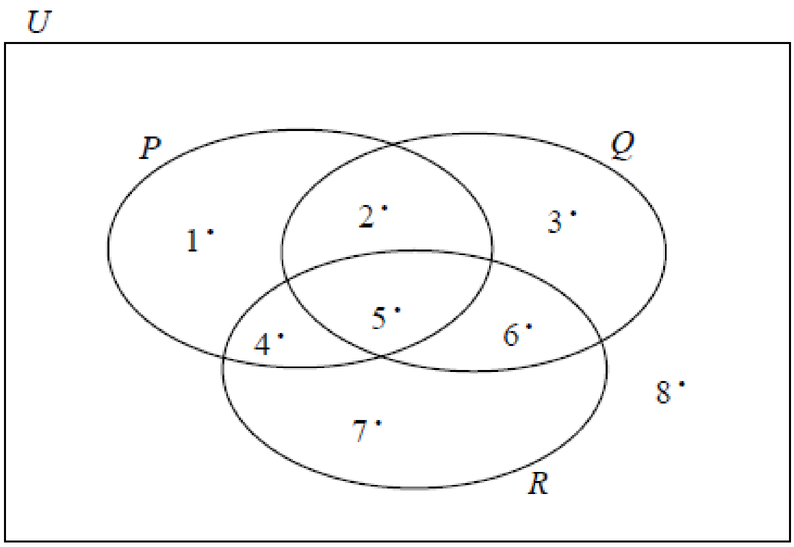
2015 Junior Certificate Higher Level
Official Sample Paper 1

Question 1 **(Suggested maximum time: 5 minutes)**

The sets U, P, Q, and R are shown in the Venn diagram below.



(a) Use the Venn diagram to list the elements of:

(i) $P \cup Q$

$P \cup Q$ is the set containing everything in P and everything in Q, so $\{1,2,3,4,5,6\}$.

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(ii) $Q \cap R$

$Q \cap R$ is the set containing everything that is in both Q and R, so $\{5,6\}$.

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(iii) $P \cup (Q \cap R)$

$P \cup (Q \cap R)$ is the set containing everything in P, and also everything in both Q and R, so $\{1,2,4,5,6\}$.

(b) Miriam says: "For all sets, union is distributive over intersection." **Name** a set that you would use along with $P \cup (Q \cap R)$ to show that Miriam's claim is true for the sets P, Q, and R in the Venn diagram above.

Miriam's statement means that $P \cup (Q \cap R) = (P \cup Q) \cap (P \cup R)$. To show this, we would need to use the sets $P \cup Q$ and $P \cup R$. So either of these sets $P \cup Q$ or $P \cup R$ is a valid answer to this question.

Question 2 **(Suggested maximum time: 5 minutes)**

The sets U, A, and B are defined as follows, where U is the universal set:

$$U = \{2,3,4,5,\dots,30\}$$
$$A = \{\text{multiples of } 2\}$$
$$B = \{\text{multiples of } 3\}$$
$$C = \{\text{multiples of } 5\}$$

(a) Find $\#[(A \cup B \cup C)']$ the number of elements in the complement of the set $A \cup B \cup C$.

The elements in the set $(A \cup B \cup C)'$ will be the whole numbers between 2 and 30 inclusive which are **not** multiples of 2, 3 or 5. These numbers are $\{7,11,13,17,19,23,29\}$. This means that the number of elements $\#[(A \cup B \cup C)'] = 7$.

(b) How many divisors does each of the numbers in $(A \cup B \cup C)'$ have?

Each of these numbers has exactly two divisors: itself and 1.

(c) What name is given to numbers that have exactly this many divisors?

These numbers are called **prime** numbers.

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Question 3 **(Suggested maximum time: 10 minutes)**

A group of 100 students were surveyed to find out whether they drank tea (T), coffee (C), or a soft drink (D) at any time in the previous week. These are the results:

24 had not drunk any of the three
51 drank tea or coffee, but not a soft drink
41 drank tea
8 drank tea and a soft drink, but not coffee
9 drank a soft drink and coffee
20 drank at least two of the three
4 drank all three.

(a) Represent the above information on the Venn diagram.

We suggest you draw out the Venn diagram on paper, and complete it as you go through the following explanation, to make it easier to follow the solution.

We can fill in the following data from the above statements: 24 students are in $(T \cup C \cup D)'$, 8 students are in $(T \cap D) \setminus C$ and 4 students are in $T \cap C \cap D$.

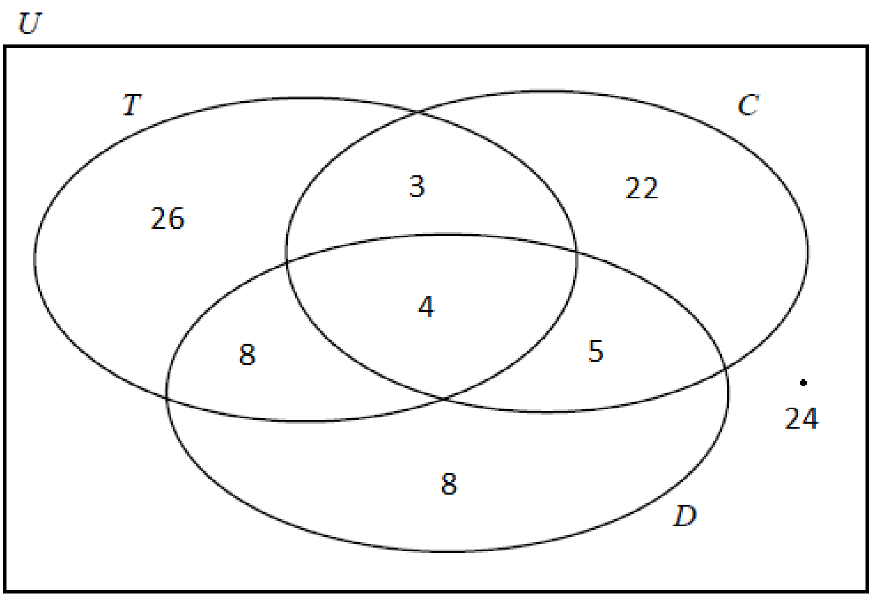
We are told that 9 students are in $D \cap C$. Since there are 4 in $T \cap D \cap C$, this means that there are 5 students in $(D \cap C) \setminus T$.

We are told that 20 students are in $(T \cap C) \cup (C \cap D) \cup (D \cap T)$, so there must be $20 - 5 - 4 - 8 = 3$ students in $(T \cap C) \setminus D$.

We are told that 41 students are in T. This means that in $T \setminus (C \cup D)$, there must be $41 - 8 - 4 - 3 = 26$ students.

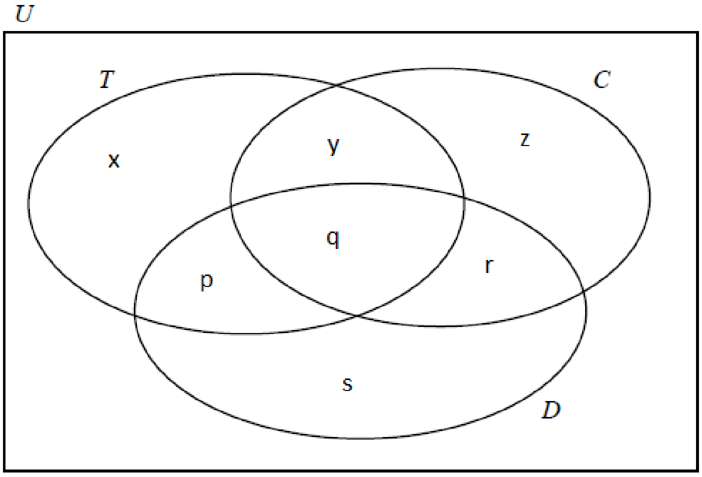
We are told that 51 students are in $(T \cup C) \setminus D$, so there must be $51 - 26 - 3 = 22$ students in $C \setminus (T \cup D)$.

Finally, the total number of students is 100, so the remaining section $D \setminus (T \cup C)$ must have $100 - 26 - 3 - 22 - 8 - 4 - 5 - 24 = 8$ students. Our completed Venn diagram:



ALTERNATE SOLUTION

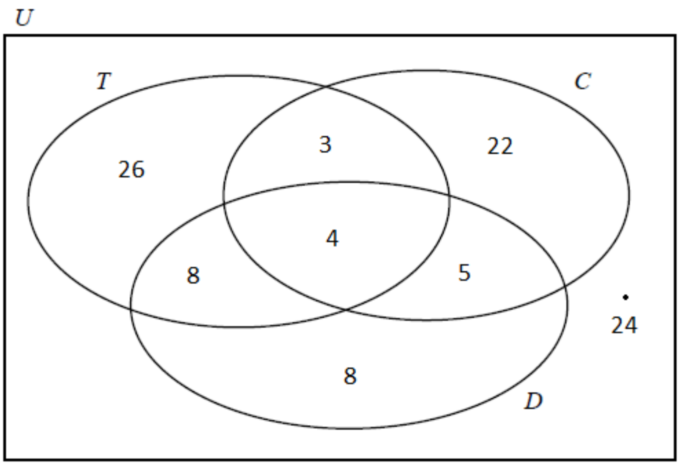
We will label the number of students in the separate sections of our Venn diagram as follows:



We know the following data directly from the above statements: 24 students are not in any of the three sets, $y = 8$ and $q = 4$. We are told that 9 students drank a soft drink and coffee, so $q + r = 9$. This means that $r = 5$. We are told that 20 students drank at least two of the three, so $20 = y + p + q + r$. This means that $p = 20 - 5 - 4 - 8 = 3$.

We are told that 41 drank tea, so $x + y + p + q = 41$. This means that $x = 41 - 8 - 4 - 3 = 26$. We are told that 51 students drank tea or coffee, but no a soft drink, so $x + y + z = 51$. This means that $z = 51 - 26 - 3 = 22$. Finally, the total number of students is 100, so $x + y + z + p + q + r + s + 24 = 100$. This means that $s = 100 - 26 - 3 - 22 - 8 - 4 - 5 - 24 = 8$.

Our completed Venn diagram:



(b) Find the probability that a student chosen at random from the group had drunk tea or coffee.

> The total number of students who had drunk tea or coffee, i.e. the number of students in $T \cup C$, is $26+8+4+3+5+22=68$. Thus, there is a probability of $\frac{68}{100}=0.68$ of choosing such a student.
>
>

(c) Find the probability that a student chosen at random from the group had drunk tea and coffee but **not** a soft drink.

> The total number of students who had drunk tea and coffee but not a soft drink, i.e. the number of students in $(T \cap C) \setminus D$, is 3. Thus, there is a probability of $\frac{3}{100}=0.03$ of choosing such a student.
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Question 4 **(Suggested maximum time: 10 minutes)**

Dermot has €5,000 and would like to invest it for two years. A special savings account is offering a rate of 3% for the first year and a higher rate for the second year, if the money is retained in the account. Tax of 41% will be deducted each year from the interest earned.

(a) How much will the investment be worth at the end of one year, after tax is deducted?

> At the end of the first year, the gross interest earned will be 3% of the sum invested, so $5000 \times \frac{3}{100} = €150$. We must deduct 41% tax from the gross interest, so $150 - \left(150 \times \frac{41}{100}\right) = 150 - 61.50 = €88.50$ is the net interest earned. Thus at the end of the first year, the investment will contain the original amount plus the net interest earned: $5000 + 88.50 = €5{,}088.50$
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(b) Dermot calculates that, after tax has been deducted, his investment will be worth €5,223.60 at the end of the second year. Calculate the rate of interest for the second year. Give your answer as a percentage, correct to one decimal place.

Firstly, note that instead of calculating 41% of the interest and then subtracting the tax, to find the net interest we can just find 59% of the gross interest.

The value of the investment at the start of the second year is the same as the value at the end of the first year. Let our new rate of interest be denoted by $i\%$. The gross interest earned at the end of the second year will be $5088.50 \times \frac{i}{100}$. The net interest earned at the end of the second year will be $5088.50 \times \frac{i}{100} \times \frac{59}{100}$. Thus the value of the investment at the end of the second year will be the net interest plus the value at the start of the second year:

$$5088.50 + \left(5088.50 \times \frac{i}{100} \times \frac{59}{100}\right) = 5223.60$$

$$\Leftrightarrow \quad 5088.50\left(1 + \frac{i}{100} \times \frac{59}{100}\right) = 5223.60$$

$$\Leftrightarrow \quad 1 + \frac{i}{100} \times \frac{59}{100} = \frac{5223.60}{5088.50}$$

$$\Leftrightarrow \quad \frac{i}{100} \times \frac{59}{100} = \frac{5223.60}{5088.50} - 1$$

$$\Leftrightarrow \quad i = \frac{10000}{59}\left(\frac{5223.60}{5088.50} - 1\right)$$

Thus, the rate of interest is $i = 4.5\%$ correct to one decimal place.

Question 5 **(Suggested maximum time: 10 minutes)**

A meal in a restaurant cost Jerry €30.52. The price included VAT at 9%. Jerry wanted to know the price of the meal before the VAT was included. He calculated 9% of €30.52 and subtracted it from the cost of the meal.

(a) Explain why Jerry will not get the correct answer using this method.

€30.52 represents the cost plus 9% VAT, so it is 109% of the price before VAT. 9% of €30.52 will therefore be $109 \times \frac{9}{100} = 9.81\%$ of the price before VAT. When this value is subtracted from €30.52, Jerry will be left with $109 - 9.81 = 99.19\%$ of the cost of the meal before VAT.

(b) Suppose that the rate of VAT was 13·5% instead of 9%. How much would Jerry have paid for the meal in that case?

We divide €30.52 by 109%, giving $30.52 \div \frac{109}{100} =$ €28 which is the cost of the meal before VAT. We wish to add 13.5% to this figure, so we want 113.5% of €28 which is $28 \times \frac{113.5}{100} =$ €31.78

Question 6 **(Suggested maximum time: 5 minutes)**

Niamh is in a clothes shop and has a voucher which she **must** use. The voucher gives a €10 reduction when buying goods to the value of at least €35. She also has €50 cash.

(a) Write down an inequality in x to show the range of cash that she could spend in the shop.

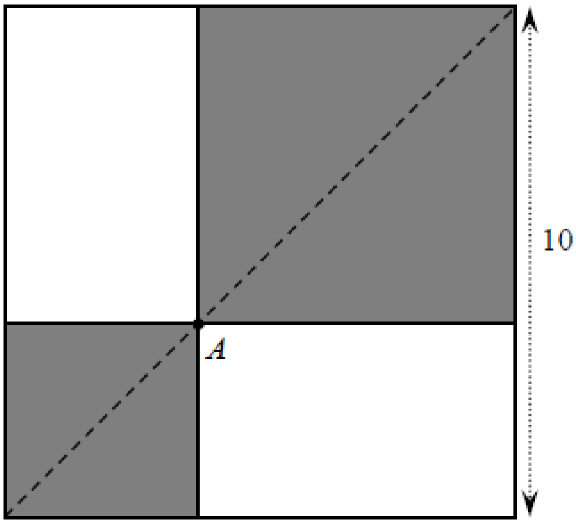
The voucher requires a minimum of €35 to be used. Since Niamh must use her voucher, she must spend a minimum of €35. If she spends this amount, she gets a €10 discount, meaning the minimum she can spend is €25. The maximum amount of cash she can spend is €50, so the range will be $25 \leqslant x \leqslant 50$.

(b) Niamh buys one item of clothing in the shop, using the voucher as she does so. Write an inequality in y to show the range of possible prices that this item could have, before the €10 reduction is applied.

As before, the minimum value Niamh can spend is €35. The maximum amount of cash she can spend is €50, and including the discount, the item of clothing could cost up to €60. Thus the range will be $35 \leqslant y \leqslant 60$.

Question 7 **(Suggested maximum time: 15 minutes)**

A square with sides of length 10 units is shown in the diagram. A point A is chosen on a diagonal of the square, and two shaded squares are constructed as shown. By choosing different positions for A, it is possible to change the value of the total area of the two shaded squares.



(a) Find the **minimum** possible value of the total area of the two shaded squares. Justify your answer fully.

The total area of the shaded region will be the sum of the areas of the two squares. The square to the left has sides of length A, and the square to the right has sides of length $10-A$. Thus the total area will be:

$$\text{Area } = A^2 + (10-A)^2 = A^2 + A^2 - 20A + 100 = 2A^2 - 20A + 100$$

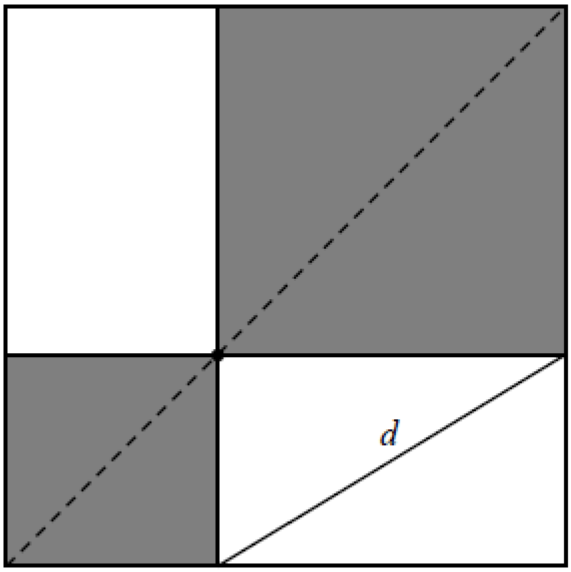
We begin by substituting some values for A:

A	$2A^2 - 20A + 100$	Area
0	$2(0)^2 - 20(0) + 100$	100
1	$2(1)^2 - 20(1) + 100$	82
2	$2(2)^2 - 20(2) + 100$	68
3	$2(3)^2 - 20(3) + 100$	58
4	$2(4)^2 - 20(4) + 100$	52
5	$2(5)^2 - 20(5) + 100$	50
6	$2(6)^2 - 20(6) + 100$	52
7	$2(7)^2 - 20(7) + 100$	58
8	$2(8)^2 - 20(8) + 100$	68
9	$2(9)^2 - 20(9) + 100$	82
10	$2(10)^2 - 20(10) + 100$	100

This function looks like:

By inspecting the graph, we can see that the function is at its minimum when $A = 5$. Substituting this into our function for the area, we get the minimum possible value of the area is 50 units^2

(b) The diagram below shows the same square. The diagonal of one of the rectangles is also marked. The length of this diagonal is d. Show that the value of the total area of the two shaded squares is equal to d^2 .



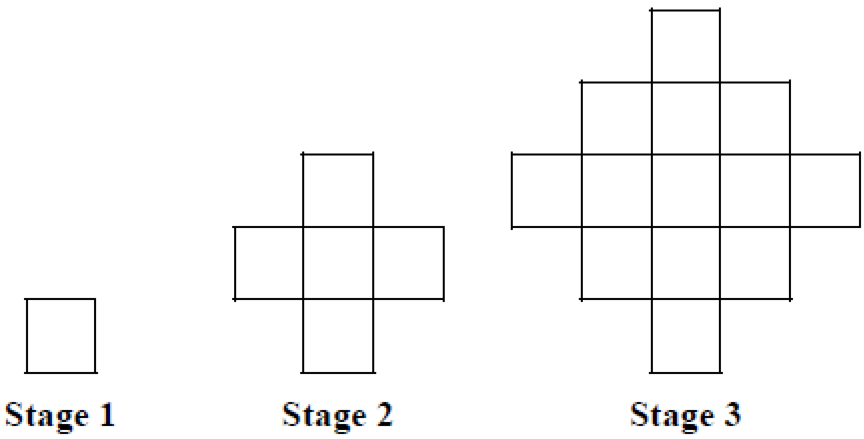
Given that the overall shape is a square, the triangle formed in the bottom right corner must be a right-angled triangle. This triangle has sides of length A and $10-A$, with hypotenuse d. Pythagoras' Theorem tells us that

$$d^2 = A^2 + (10-A)^2$$

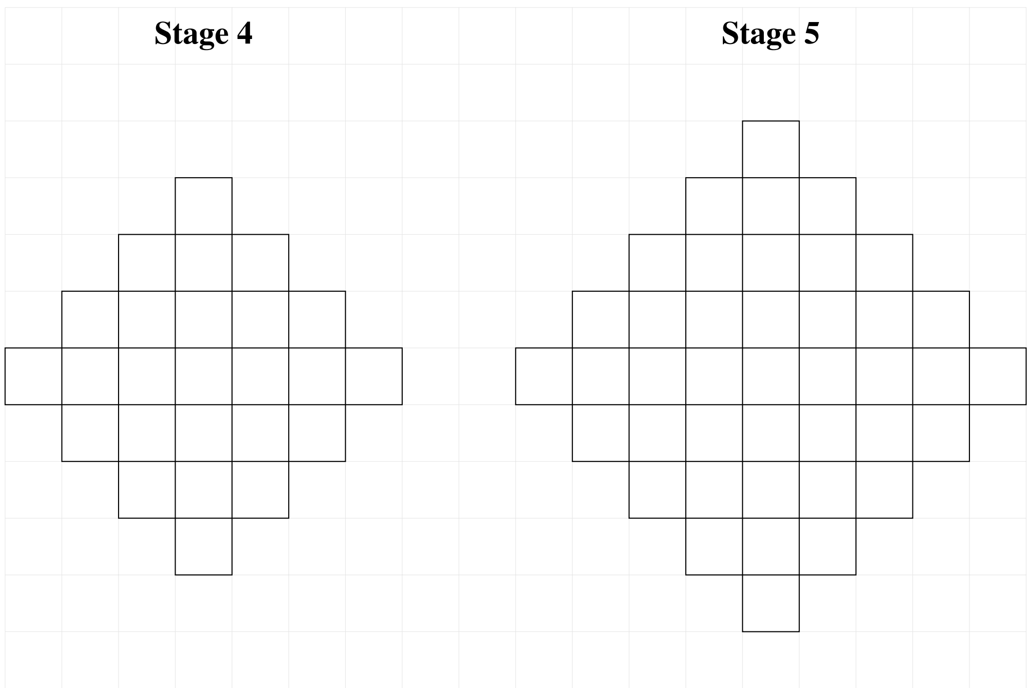
and so the area of the shaded squares is the same as d^2.

Question 8 **(Suggested maximum time: 20 minutes)**

The first three stages of a pattern are shown below. Each stage of the pattern is made up of small squares. Each small square has an area of one square unit.



(a) Draw the next two stages of the pattern.



(b) The perimeter of Stage 1 of the pattern is 4 units. The perimeter of Stage 2 of the pattern is 12 units. Find a general formula for the **perimeter** of Stage n of the pattern, where $n \in \mathbb{N}$

We'll consider the perimeters of the first five Stages:

Stage	1	2	3	4	5
Perimeter	4 units	12 units	20 units	28 units	36 units

From one stage to another, there is a difference of 8 in the perimeter. Thus, the perimeter is an arithmetic sequence with first element $a = 4$ and common difference $d = 8$. The general formula for the perimeter of Stage n is therefore

$$P_n = a + (n-1)d = 4 + 8(n-1) = 8n - 4$$

(c) Find a general formula for the **area** of Stage n of the pattern, where $n \in \mathbb{N}$

We will test to see if the sequence is linear by considering the first differences:

$$d_1 = A_2 - A_1 = 5 - 1 = 4$$
$$d_2 = A_3 - A_2 = 13 - 5 = 8$$
$$d_3 = A_4 - A_3 = 25 - 13 = 12$$
$$d_4 = A_5 - A_4 = 41 - 25 = 16$$

The first differences are not constant, so the sequence is not arithmetic. We will look at the second differences:

$$s_1 = d_2 - d_1 = 8 - 4 = 4$$
$$s_2 = d_3 - d_2 = 12 - 8 = 4$$
$$s_3 = d_4 - d_3 = 16 - 12 = 4$$

The second differences are constant, so the sequence is quadratic, and so has the general formula $A_n = an^2 + bn + c$ where a, b and c need to be determined. For a quadratic sequence of this form, the constant second difference is equal to $2a$, so we have $2a = 4$, or $a = 2$. This means our sequence has the form $A_n = 2n^2 + bn + c$

Now, we know that $A_1 = 1$ and $A_2 = 5$, so

$$1 = A_1 = 2(1)^2 + b(1) + c = 2 + b + c \quad \Rightarrow \quad b + c = -1$$
$$5 = A_2 = 2(2)^2 + b(2) + c = 8 + 2b + c \quad \Rightarrow \quad 2b + c = -3$$

We will solve these equations simultaneously:

$$\begin{array}{rcl} b + c & = & -1 \\ 2b + c & = & -3 \\ \hline b & = & -2 \end{array}$$

Now, from the first of our two simultaneous equations to see that $(-2) + c = -1$ or $c = -1 + 2 = 1$. This means that our sequence has the form $A_n = 2n^2 - 2n + 1$

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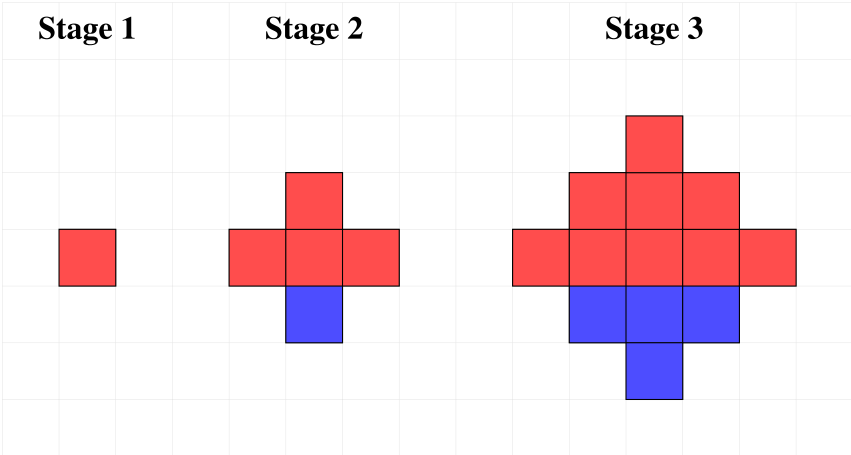
It will be easier to split the patterns into an upper and a lower section. The first three Stages are:



We will count the number of small squares in each section for the first five Stages

Stage	1	2	3	4	5
Upper	1 square	4 squares	9 squares	16 squares	25 squares
Lower	0 squares	1 squares	4 squares	9 squares	16 squares
Total Area	1 units^2	5 units^2	13 units^2	25 units^2	41 units^2

From this table, we can see that for stage n, there will be n^2 squares in the upper section and $(n-1)^2$ squares in the lower section. The general formula for the perimeter of Stage n is therefore

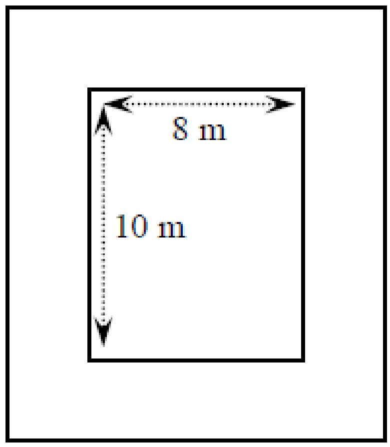
$$A_n = n^2 + (n-1)^2 = n^2 + n^2 - 2n + 1 = 2n^2 - 2n + 1 \text{ units}^2$$

(d) What kind of sequence (linear, quadratic, exponential, or none of these) do the areas follow? Justify your answer.

This sequence is quadratic because the highest power in the general formula is n^2. It cannot be exponential because there is no term of the form a^n.

Question 9 **(Suggested maximum time: 20 minutes)**

A plot consists of a rectangular garden measuring 8 m by 10 m, surrounded by a path of constant width, as shown in the diagram. The total area of the plot (garden and path) is 143 m^2.



Three students, Kevin, Elaine, and Tony, have been given the problem of trying to find the width of the path. Each of them is using a different method, but all of them are using x to represent the width of the path.

Kevin divides the path into eight pieces. He writes down the area of each piece in terms of x. He then forms an equation by setting the area of the path plus the area of the garden equal to the total area of the plot.

(a) Write, in terms of x, the area of each section into Kevin's diagram below.

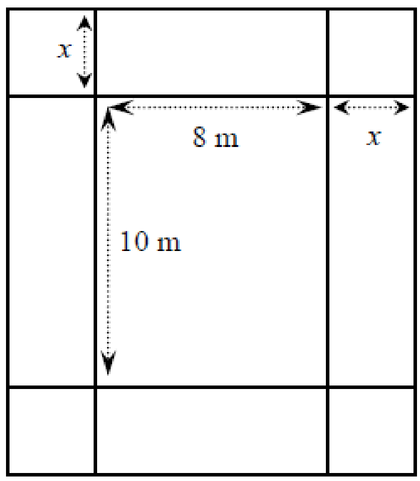
We have four different types of sections in Kevin's diagram: the four corner squares, the rectangles at the top and bottom, the rectangles at the sides and the rectangle in the middle. the areas are given as follows:

Corners: The area of these squares is $x \times x = x^2$ m^2

Top and Bottom: The area of these rectangles is $x \times 8 = 8x$ m^2

Sides: The area of these rectangles is $x \times 10 = 10x$ m^2

Centre: The area of this rectangle is $8 \times 10 = 80$ m^2



Kevin's Diagram

(b) Write down and simplify the equation that Kevin should get. Give your answer in the form $ax^2 + bx + c = 0$.

If we add the areas from all of Kevin's sections, we will get the total area, which is 143 m^2.

$$4(x^2)+2(8x)+2(10x)+1(80)=143 \quad \Leftrightarrow \quad 4x^2+16x+20x+80=143$$
$$\Leftrightarrow \quad 4x^2+36x-63=0$$

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Elaine writes down the length and width of the plot in terms of x. She multiplies these and sets the answer equal to the total area of the plot.

(c) Write, in terms of x, the length and the width of the plot in the spaces on Elaine's diagram.

The overall width of the plot consists of the width of the garden, plus the width of the path on both sides, so $8+2x$.

Similarly, the overall length of the plot consists of the length of the garden, plus the width of the path on both sides, so $10+2x$.

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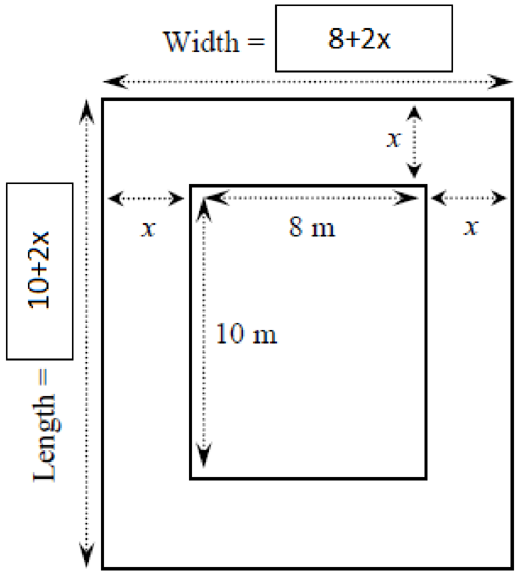

Elaine's Diagram

(d) Write down and simplify the equation that Elaine should get. Give your answer in the form $ax^2+bx+c=0$.

The overall width times the overall length will give the total area of the plot. So:

$$(8+2x)(10+2x) = 143 \quad \Leftrightarrow \quad 80+16x+20x+4x^2 = 143$$
$$\Leftrightarrow \quad 4x^2+36x-63 = 0$$

As we might have expected, this agrees with Kevin's calculations.

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(e) Solve an equation to find the width of the path.

We can factorise our equation as follows:

$$4x^2+36x-63 = 0 \quad \Leftrightarrow \quad (2x-3)(2x+21) = 0$$
$$\Leftrightarrow \quad 2x-3 = 0 \quad \text{or} \quad 2x+21 = 0$$

Thus, either $x = \frac{3}{2}$ or $x = -\frac{21}{2}$. Since our path has to have a positive width, we take $x = 1.5$ m as the width of the path.

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(f) Tony does not answer the problem by solving an equation. Instead, he does it by trying out different values for x. Show some calculations that Tony might have used to solve the problem.

We will test some values for x. The simplest approach is to take $x = 1, 2, 3, \ldots$ until we find two points close the correct area.

$$\begin{aligned} x = 1 \quad &\Rightarrow \quad \text{Area} = (8+2)(10+2) = 120 \text{ m}^2 \\ x = 2 \quad &\Rightarrow \quad \text{Area} = (8+4)(10+4) = 168 \text{ m}^2 \\ x = 3 \quad &\Rightarrow \quad \text{Area} = (8+6)(10+6) = 224 \text{ m}^2 \end{aligned}$$

Since the total area is actually 143, the correct value of x should be between 1 and 2 because $120 < 143 < 168$. We can try $x = 1.5$ as a guess (and in fact we already know this is the correct answer).

$$x = 1.5 \quad \Rightarrow \quad \text{Area} = (8+3)(10+3) = 143 \text{ m}^2$$

(g) Which of the three methods do you think is best? Give a reason for your answer.

Answer: Elaine's method is best.

Reason: Although all three methods give the correct answer, Elaine's method is the quickest and simplest. Tony's method involves estimating the correct answer, which could give rise to some errors. Kevin's method requires several calculations before we arrive at the equation to solve.

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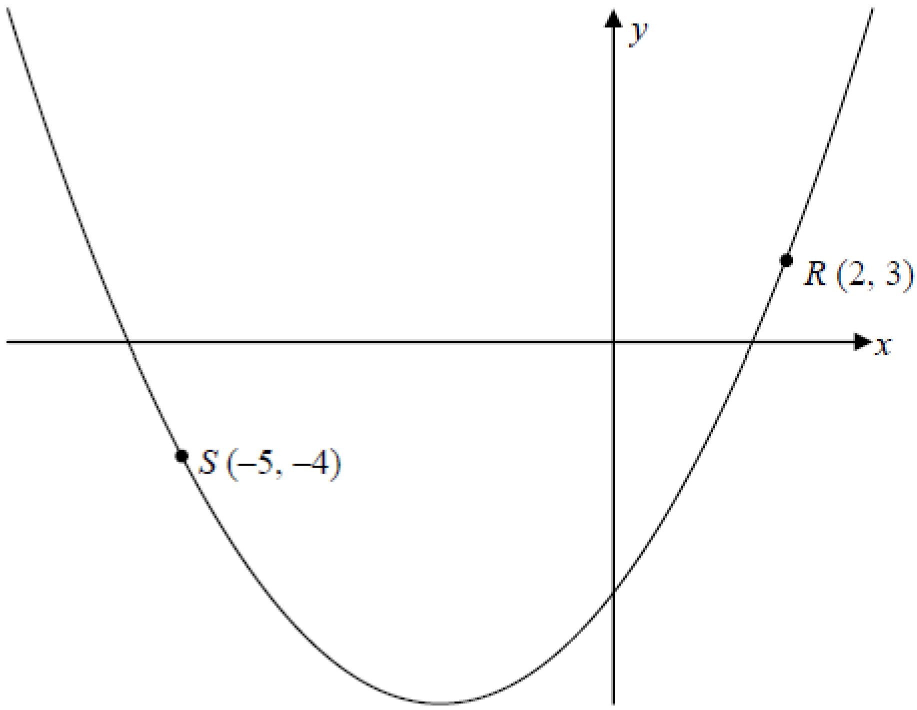
Question 10 **(Suggested maximum time: 20 minutes)**

Part of the graph of the function $y = x^2 + ax + b$, where $a, b \in \mathbb{Z}$, is shown below



The points $R(2,3)$ and $S(-5,-4)$ are on the curve.

(a) Use the given points to form two equations in a and b.

We know that if a point (x_1, y_1) is on a line, then x_1 and y_1 must satisfy the equation of that line. Thus:

$$3 = (2)^2 + a(2) + b \quad \Leftrightarrow \quad 3 = 4 + 2a + b \quad \Leftrightarrow \quad 2a + b = -1$$

Similarly,

$$-4 = (-5)^2 + a(-5) + b \quad \Leftrightarrow \quad -4 = 25 - 5a + b \quad \Leftrightarrow \quad 5a - b = 29$$

(b) Solve your equations to find the value of a and the value of b.

We can solve the two above equations simultaneously. Adding both equations together we get:

$$\begin{array}{rcl} 2a+b &=& -1 \\ 5a-b &=& 29 \\ \hline 7a &=& 28 \end{array}$$

This means that $a = \frac{28}{7} = 4$. We can now go to the first equation: $2(4)+b = -1$, or $b = -1-8 = -9$. This means that the function is of the form $y = x^2 + 4x - 9$.

(c) Write down the co-ordinates of the point where the curve crosses the y-axis.

The curve will cross the y-axis when $x = 0$, so when $y = (0)^2 + 4(x) - 9 = -9$

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(d) By solving an equation, find the points where the curve crosses the x-axis. Give each answer correct to one decimal place.

The curve will cross the x-axis when $y = 0$, so when $x^2 + 4x - 9 = 0$. We will use the quadratic formula to solve for an equation of the form $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \Leftrightarrow \quad x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(-9)}}{2(1)}$$

$$\Leftrightarrow \quad x = \frac{-4 \pm \sqrt{16 + 36}}{2}$$

$$\Leftrightarrow \quad x = \frac{-4 \pm \sqrt{52}}{2}$$

$$\Leftrightarrow \quad x = \frac{-4 \pm \sqrt{4}\sqrt{13}}{2}$$

$$\Leftrightarrow \quad x = \frac{-4 \pm 2\sqrt{13}}{2}$$

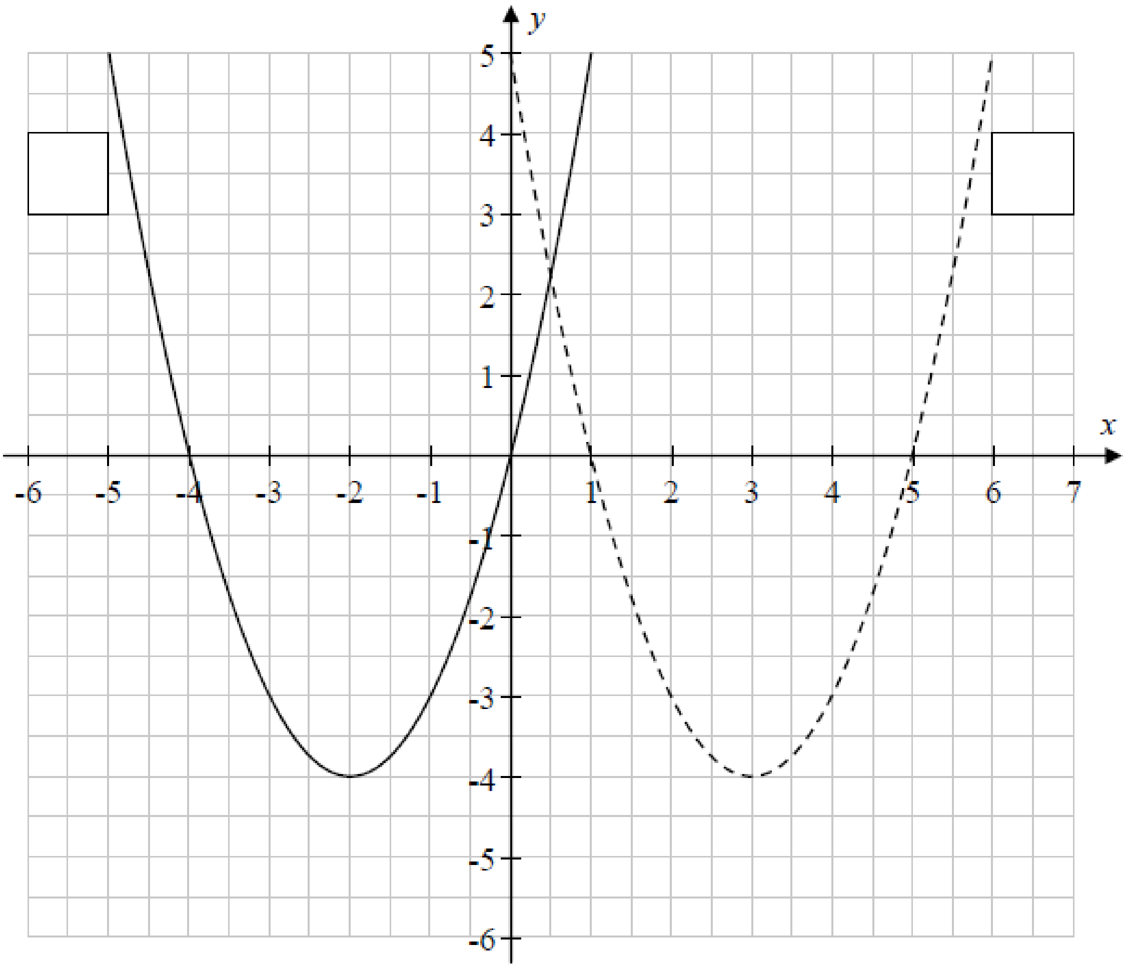
$$\Leftrightarrow \quad x = -2 \pm \sqrt{13}$$

Thus, the curve crosses the x-axis at the points $x = 1.6$ and $x = -5.6$ correct to one decimal place.

Question 11 **(Suggested maximum time: 15 minutes)**

The graphs of two functions, f and g, are shown on the co-ordinate grid below. The functions are:

$$f : x \mapsto (x+2)^2 - 4$$
$$g : x \mapsto (x-3)^2 - 4$$

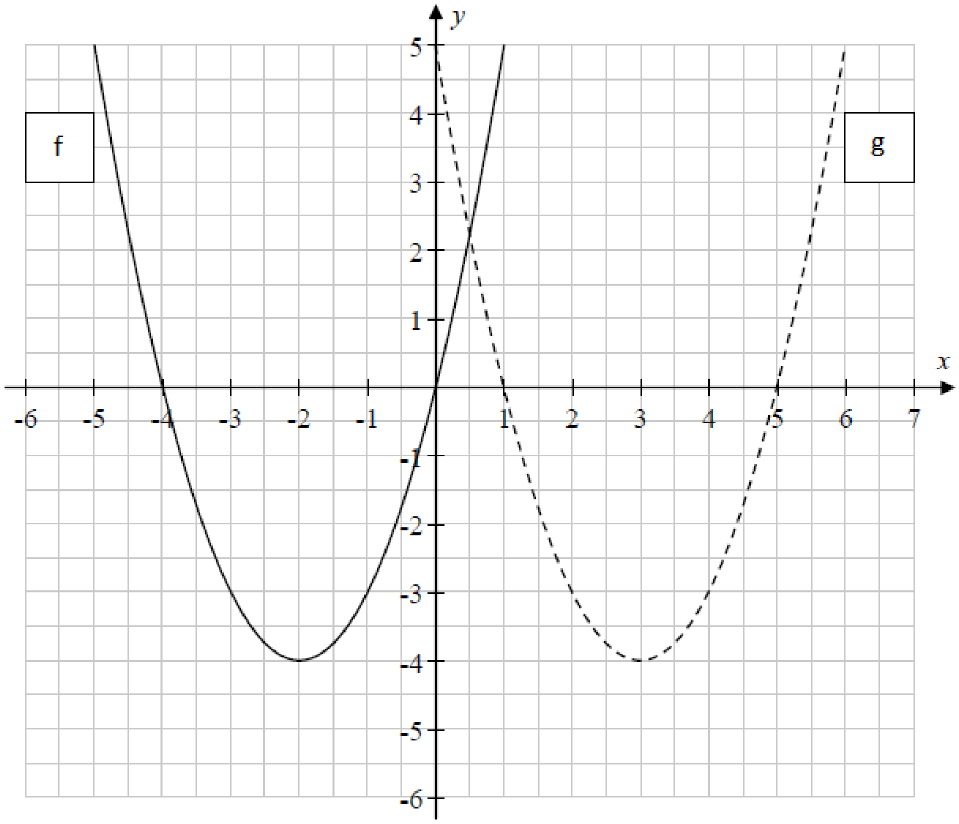


(a) Match the graphs to the functions by writing f or g beside the corresponding graph on the grid.

We will work out the values of $f(0)$ and $g(0)$ and use them to determine which graph is which.

$$f(0) = (0+2)^2 - 4 = 4 - 4 = 0 \qquad \text{and} \qquad g(0) = (0-3)^2 - 4 = 9 - 4 = 5$$

Thus, examining the graph we can see that f is the graph on the left and g is the dashed graph to the right.



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(b) Write down the roots of f and the roots of g.

By inspection of the graph, the roots of f are -4 and 0.

Similarly, the roots of g are 1 and 5.

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ALTERNATE SOLUTION

Roots of f: we will manipulate the equation of f to determine its roots:

$$(x+2)^2 - 4 = 0 \quad \Leftrightarrow \quad (x+2)^2 = 4 \quad \Leftrightarrow \quad x+2 = \pm\sqrt{4} = \pm 2$$

Thus, the roots of f are $x = 0$ and $x = -4$.

Roots of g: we will manipulate the equation of g to determine its roots:

$$(x-3)^2 - 4 = 0 \quad \Leftrightarrow \quad (x-3)^2 = 4 \quad \Leftrightarrow \quad x-3 = \pm\sqrt{4} = \pm 2$$

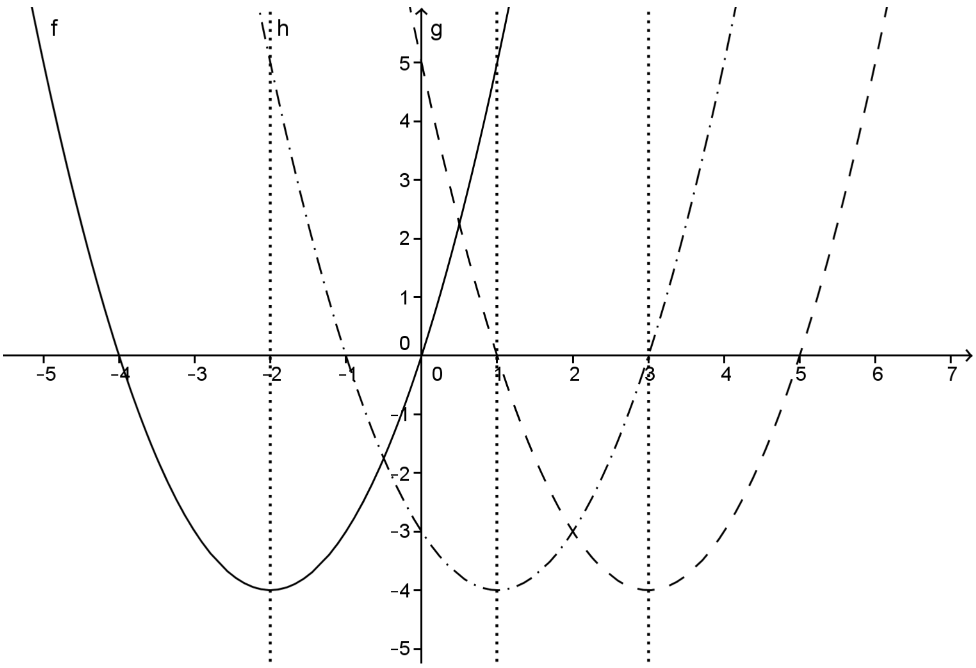
Thus, the roots of g are $x = 1$ and $x = 5$.

(c) Sketch the graph of $h : x \mapsto (x-1)^2 - 4$ on the co-ordinate grid above, where $x \in \mathbb{R}$.

Consider the functions f and g. Both of these functions are of the form $(x-c)^2-4$. For f we have $c=-2$ and for g we have $c=3$. The shapes of the two graphs are the same, but f is centred about the line $x=-2$, whereas g is centred about the line $x=3$.

If we consider the graph of f, to get the graph of g, we shift the axis of symmetry of f from $x=-2$ to $x=3$, a shift of $3-(-2)=5$ units to the right. Thus, changing the value of c shifts the axis of symmetry along the x-axis.

Now, since $h(x)=(x-1)^2-4$, so $c=1$. Again, if we consider the graph of f, to get the graph of h, we shift the axis of symmetry of f by $1-(-2)=3$ units to the right. This gives:

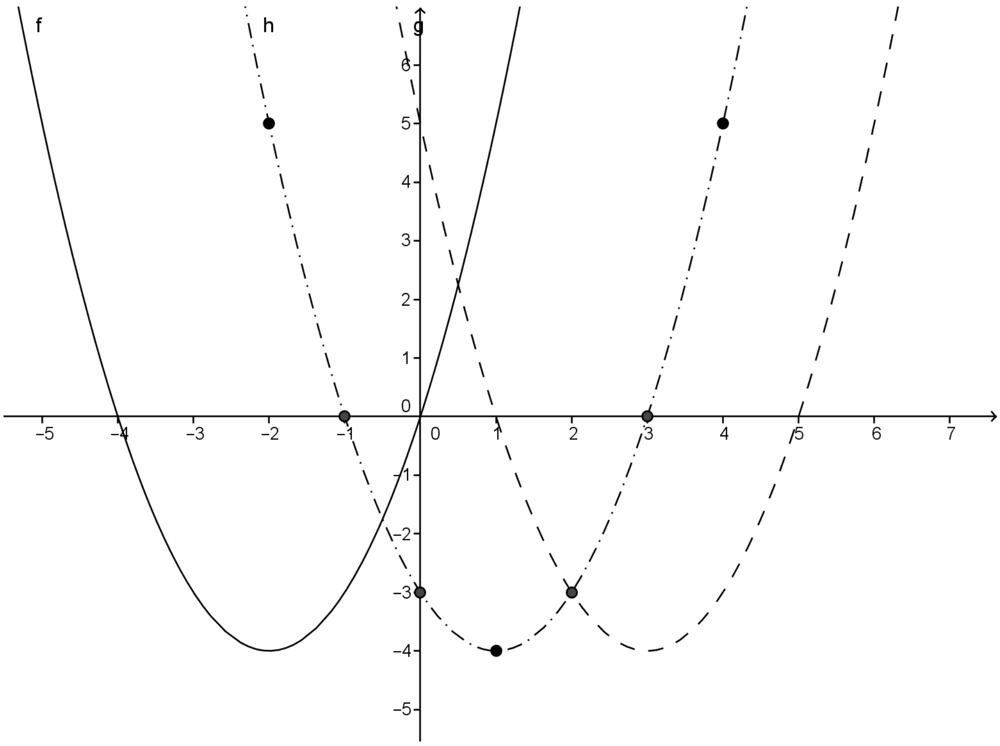


ALTERNATE SOLUTION

We begin by substituting some values for x:

x	$h(x)$	y
-2	$(-2-1)^2-4$	5
-1	$(-1-1)^2-4$	0
0	$(0-1)^2-4$	-3
1	$(1-1)^2-4$	-4
2	$(2-1)^2-4$	-3
3	$(3-1)^2-4$	0
4	$(4-1)^2-4$	5

Plotting these points, we get



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(d) p is a natural number, such that $(x-p)^2-2=x^2-10x+23$. Find the value of p.

Firstly, we'll rearrange our equation to read

$$(x-p)^2 = x^2 - 10x + 25$$

Now, we can factorise the right hand side as $x^2 - 10x + 25 = (x-5)^2$, so we have $(x-p)^2 = (x-5)^2$ and thus $p = 5$.

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(e) Write down the equation of the axis of symmetry of the graph of the function:

$$k(x) = x^2 - 10x + 23$$

We know from part **(d)** that we can write $k(x) = (x-5)^2 - 2$. Now, the functions of f, g and h, are all of the form $(x-c)^2 - 4$, and we can see that the axis of symmetry for f is $x = -2$, for g it is $x = 3$ and for h it is $x = 1$, i.e. in all cases $x = c$ is the axis of symmetry. Now, we know that $k(x) = (x-5)^2 - 2$, so in the same way, the axis of symmetry for $k(x)$ will be $x = 5$.

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Question 12 **(Suggested maximum time: 5 minutes)**

Give a reason why the graph below does not represent a function of x.

For any function, each x value has no more than one value for y. However, we can see from this graph that almost all x points have two y values. So this graph does not represent a function.

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