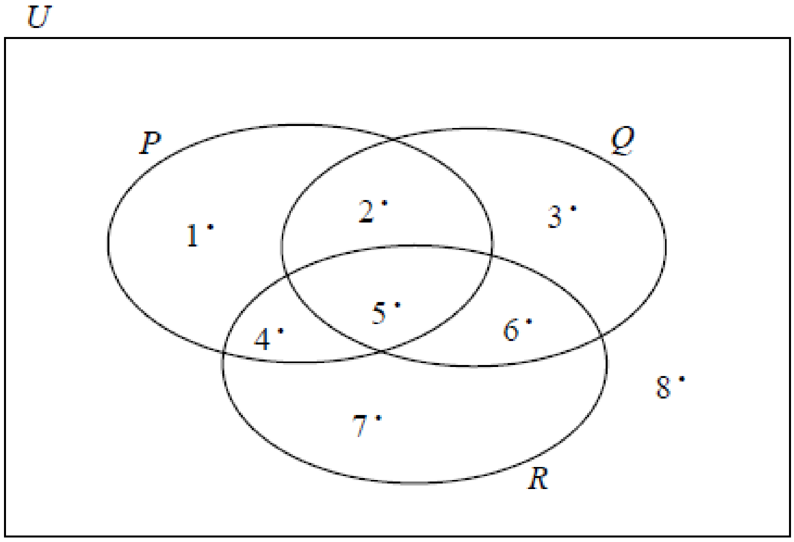
2014 Junior Certificate Higher Level
Official Sample Paper 1

Question 1 **(Suggested maximum time: 5 minutes)**

The sets U, P, Q, and R are shown in the Venn diagram below.



(a) Use the Venn diagram to list the elements of:

(i) $P \cup Q$

$P \cup Q$ is the set containing everything in P and everything in Q, so $\{1,2,3,4,5,6\}$.

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(ii) $Q \cap R$

$Q \cap R$ is the set containing everything that is in both Q and R, so $\{5,6\}$.

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(iii) $P \cup (Q \cap R)$

$P \cup (Q \cap R)$ is the set containing everything in P, and also everything in both Q and R, so $\{1,2,4,5,6\}$.

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(b) Miriam says: "For all sets, union is distributive over intersection." **Name** a set that you would use along with $P \cup (Q \cap R)$ to show that Miriam's claim is true for the sets P, Q, and R in the Venn diagram above.

Miriam's statement means that $P \cup (Q \cap R) = (P \cup Q) \cap (P \cup R)$. To show this, we would need to use the sets $P \cup Q$ and $P \cup R$. So either of these sets $P \cup Q$ or $P \cup R$ is a valid answer to this question.

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Question 2 **(Suggested maximum time: 5 minutes)**

$$U = \{2,3,4,5,\ldots,30\}, A = \{\text{multiples of } 2\}, B = \{\text{multiples of } 3\}, C = \{\text{multiples of } 5\}$$

(a) Find $\#[(A \cup B \cup C)']$ the number of elements in the complement of the set $A \cup B \cup C$.

The elements in the set $(A \cup B \cup C)'$ will be the whole numbers between 2 and 30 inclusive which are **not** multiples of 2, 3 or 5. These numbers are $\{7,11,13,17,19,23,29\}$. This means that the number of elements $\#[(A \cup B \cup C)'] = 7$.

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(b) How many divisors does each of the numbers in $(A \cup B \cup C)'$ have?

Each of these numbers has exactly two divisors: itself and 1.

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(c) What name is given to numbers that have exactly this many divisors?

These numbers are called **prime** numbers.

Question 3 **(Suggested maximum time: 10 minutes)**

A group of 100 students were surveyed to find out whether they drank tea (T), coffee (C), or a soft drink (D) at any time in the previous week. These are the results:

24 had not drunk any of the three
51 drank tea or coffee, but not a soft drink
41 drank tea
8 drank tea and a soft drink, but not coffee
9 drank a soft drink and coffee
20 drank at least two of the three
4 drank all three.

(a) Represent the above information on the Venn diagram.

We suggest you draw out the Venn diagram on paper, and complete it as you go through the following explanation, to make it easier to follow the solution.

We can fill in the following data from the above statements: 24 students are in $(T \cup C \cup D)'$, 8 students are in $(T \cap D) \setminus C$ and 4 students are in $T \cap C \cap D$.

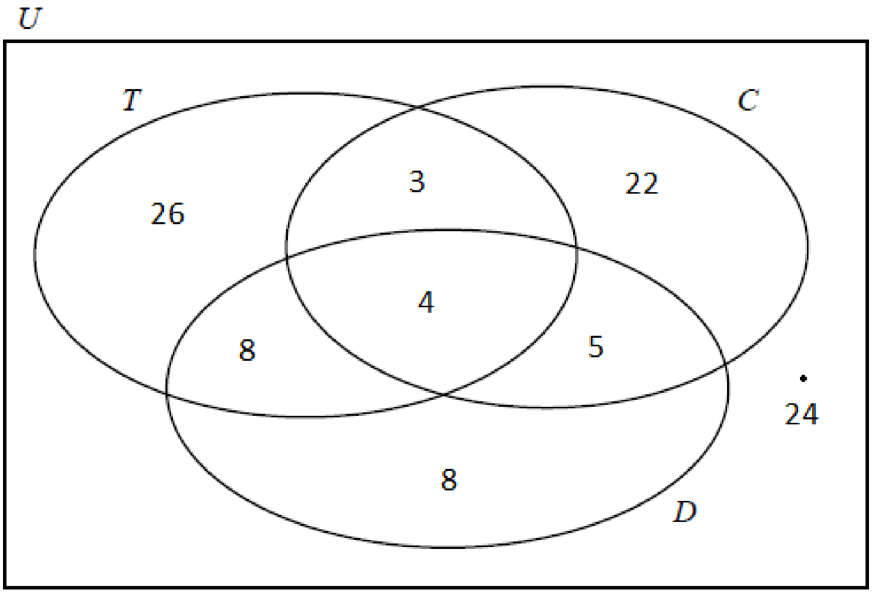
We are told that 9 students are in $D \cap C$. Since there are 4 in $T \cap D \cap C$, this means that there are 5 students in $(D \cap C) \setminus T$.

We are told that 20 students are in $(T \cap C) \cup (C \cap D) \cup (D \cap T)$, so there must be $20 - 5 - 4 - 8 = 3$ students in $(T \cap C) \setminus D$.

We are told that 41 students are in T. This means that in $T \setminus (C \cup D)$, there must be $41 - 8 - 4 - 3 = 26$ students.

We are told that 51 students are in $(T \cup C) \setminus D$, so there must be $51 - 26 - 3 = 22$ students in $C \setminus (T \cup D)$.

Finally, the total number of students is 100, so the remaining section $D \setminus (T \cup C)$ must have $100 - 26 - 3 - 22 - 8 - 4 - 5 - 24 = 8$ students. Our completed Venn diagram:



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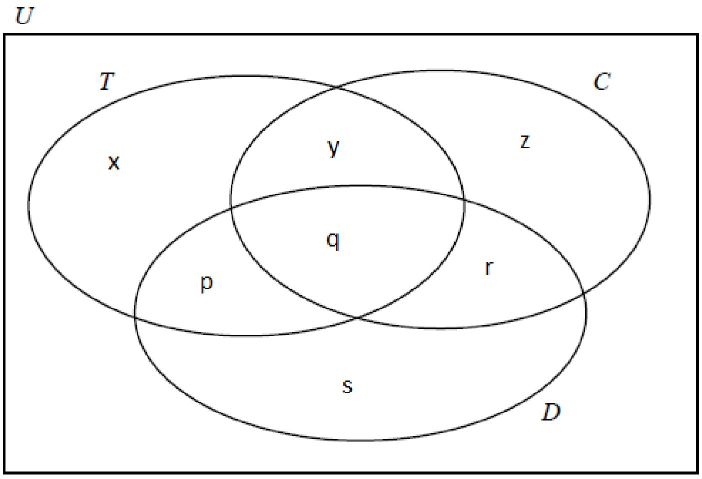
ALTERNATE SOLUTION

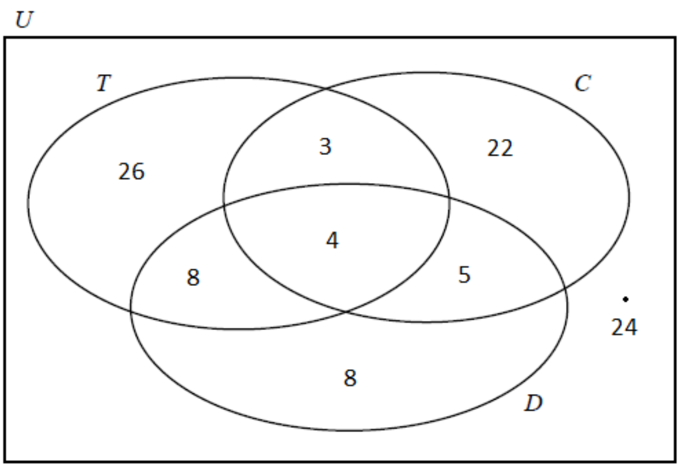
We will label the number of students in the separate sections of our Venn diagram as follows:



We know the following data directly from the above statements: 24 students are not in any of the three sets, $y = 8$ and $q = 4$. We are told that 9 students drank a soft drink and coffee, so $q + r = 9$. This means that $r = 5$. We are told that 20 students drank at least two of the three, so $20 = y + p + q + r$. This means that $p = 20 - 5 - 4 - 8 = 3$.

We are told that 41 drank tea, so $x + y + p + q = 41$. This means that $x = 41 - 8 - 4 - 3 = 26$. We are told that 51 students drank tea or coffee, but no a soft drink, so $x + y + z = 51$. This means that $z = 51 - 26 - 3 = 22$. Finally, the total number of students is 100, so $x + y + z + p + q + r + s + 24 = 100$. This means that $s = 100 - 26 - 3 - 22 - 8 - 4 - 5 - 24 = 8$.

Our completed Venn diagram:



(b) Find the probability that a student chosen at random from the group had drunk tea or coffee.

> The total number of students who had drunk tea or coffee, i.e. the number of students in $T \cup C$, is $26+8+4+3+5+22=68$. Thus, there is a probability of $\frac{68}{100}=0.68$ of choosing such a student.
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(c) Find the probability that a student chosen at random from the group had drunk tea and coffee but **not** a soft drink.

> The total number of students who had drunk tea and coffee but not a soft drink, i.e. the number of students in $(T \cap C) \setminus D$, is 3. Thus, there is a probability of $\frac{3}{100}=0.03$ of choosing such a student.
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Question 4 **(Suggested maximum time: 10 minutes)**

Dermot has €5,000 and would like to invest it for two years. A special savings account is offering a rate of 3% for the first year and a higher rate for the second year, if the money is retained in the account. Tax of 33% will be deducted each year from the interest earned.

(a) How much will the investment be worth at the end of one year, after tax is deducted?

> At the end of the first year, the gross interest earned will be 3% of the sum invested, so $5000 \times \frac{3}{100} = €150$. We must deduct 33% tax from the gross interest, so $150 - \left(150 \times \frac{33}{100}\right) = 150 - 49.50 = €100.50$ is the net interest earned. Thus at the end of the first year, the investment will contain the original amount plus the net interest earned: $5000 + 100.50 = €5,100.50$
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(b) Dermot calculates that, after tax has been deducted, his investment will be worth €5,268 at the end of the second year. Calculate the rate of interest for the second year.

Firstly, note that instead of calculating 33% of the interest and then subtracting the tax, to find the net interest we can just find 67% of the gross interest.

The value of the investment at the start of the second year is the same as the value at the end of the first year. Let our new rate of interest be denoted by $i\%$. The gross interest earned at the end of the second year will be $5100.50 \times \frac{i}{100}$. The net interest earned at the end of the second year will be $5100.50 \times \frac{i}{100} \times \frac{67}{100}$. Thus the value of the investment at the end of the second year will be the net interest plus the value at the start of the second year:

$$
\begin{aligned}
& & 5100.50 + \left(5100.50 \times \frac{i}{100} \times \frac{67}{100}\right) &= 5268 \\
&\Leftrightarrow & 5100.50\left(1 + \frac{i}{100} \times \frac{67}{100}\right) &= 5268 \\
&\Leftrightarrow & 1 + \frac{i}{100} \times \frac{67}{100} &= \frac{5268}{5100.50} \\
&\Leftrightarrow & \frac{i}{100} \times \frac{67}{100} &= \frac{5268}{5100.50} - 1 \\
&\Leftrightarrow & i &= \frac{10000}{67}\left(\frac{5268}{5100.50} - 1\right)
\end{aligned}
$$

Thus, the rate of interest is $i = 4.9\%$ correct to one decimal place.

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Question 5 (Suggested maximum time: 10 minutes)

A meal in a restaurant cost Jerry €136.20. The price included VAT at 13.5%. Jerry wished to know the price of the meal before the VAT was included. He calculated 13.5% of €136.20 and subtracted it from the cost of the meal.

(a) Explain why Jerry will not get the correct answer using this method.

€136.20 represents the cost plus 13.5% VAT, so it is equal to 113.5% of the price before VAT.

13.5% of €136.20 will therefore be $113.5 \times \frac{13.5}{100} = 15.3225\%$ of the price before VAT. When this value is subtracted from the original €136.20, Jerry will be left with $113.5 - 15.3225 = 98.1775\%$ of the cost of the meal before VAT.

(b) From July 1, 2011, the VAT rate on food, in restaurants, was reduced to 9%. How much would Jerry have paid for the meal after this date if the VAT reduction was correctly applied?

We divide €136.20 by 113.5%, giving $136.20 \div \frac{113.5}{100} =$ €120 which is the cost of the meal before VAT. We wish to add 9% to this figure, so we want 109% of €120 which is $120 \times \frac{109}{100} =$ €130.80

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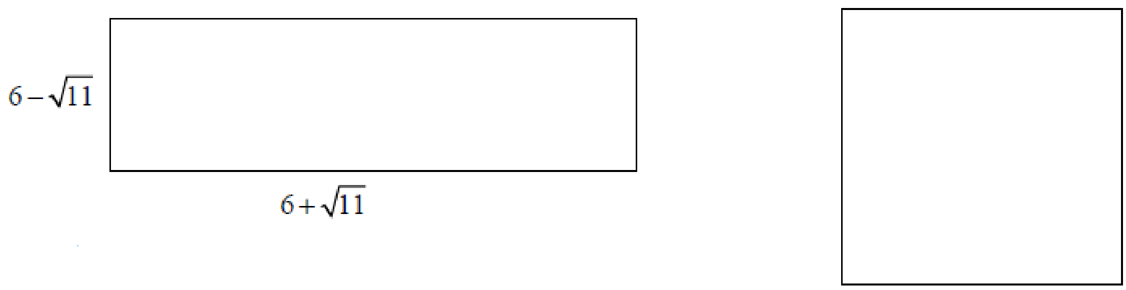
Question 6 **(Suggested maximum time: 5 minutes)**

The rectangle and square below have the same area. The dimensions of both are in cm. The diagrams are not drawn to scale.



(a) Find the area of the rectangle.

The area of the rectangle is the width multiplied by the length, so

$$\begin{aligned}
\text{Area} &= (6-\sqrt{11})\times(6+\sqrt{11}) \\
&= (6\times 6)+(6\times\sqrt{11})-(6\times\sqrt{11})-(\sqrt{11}\times\sqrt{11}) \\
&= 36-11 \\
&= 25\ \text{cm}^2
\end{aligned}$$

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(b) Find the length of one side of the square.

If the area of the two shapes are the same, then the area of the square is 25 cm^2. If the length of one side of the square is denoted l, then the area is $l^2 = 25$. Thus, $l = \sqrt{25} = 5$ cm since the length must be positive.

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Question 7 **(Suggested maximum time: 15 minutes)**

Given any two positive integers m and n ($n > m$), it is possible to form three numbers a, b and c where:

$$a = n^2 - m^2 \qquad b = 2nm \qquad c = n^2 + m^2$$

These three numbers a, b and c are then known as a "Pythagorean triple".

(a) For $m = 3$ and $n = 5$ calculate a, b and c.

$$\begin{aligned}
a &= (5)^2-(3)^2 = 25-9 = 16 \\
b &= 2(5)(3) = 30 \\
c &= (5)^2+(3)^2 = 25+9 = 34
\end{aligned}$$

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(b) If the values of a, b, and c from part **(i)** are the lengths of the sides of a triangle, show that the triangle is right-angled.

The largest of these three numbers is c, and so it will represent the hypotenuse of the triangle. Thus, we wish to conclude that $c^2 = a^2 + b^2$. Now, $c^2 = (34)^2 = 1156$ and $a^2 + b^2 = (16)^2 + (30)^2 = 256 + 900 = 1156$. Thus, $c^2 = a^2 + b^2$ and so by Pythagoras' Theorem, the triangle is right-angled.

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(c) If $n^2 - m^2$, $2nm$, and $n^2 + m^2$ are the lengths of the sides of a triangle, show that the triangle is right-angled.

We wish to conclude that $(n^2 + m^2)^2 = (n^2 - m^2)^2 + (2nm)^2$. Note that

$$(n^2 + m^2)^2 = (n^2)^2 + 2(n^2)(m^2) + (m^2)^2 = n^4 + 2n^2m^2 + m^4$$

and

$$\begin{aligned}(n^2 - m^2)^2 + (2nm)^2 &= \left[(n^2)^2 - 2(n^2)(m^2) + (m^2)^2\right] + 4n^2m^2 \\ &= \left[n^4 - 2n^2m^2 + m^4\right] + 4n^2m^2 \\ &= n^4 + 2n^2m^2 + m^4\end{aligned}$$

So, we have shown that $(n^2 + m^2)^2 = (n^2 - m^2)^2 + (2nm)^2$. Thus, by Pythagoras' Theorem, the triangle represented by these lengths is right-angled.

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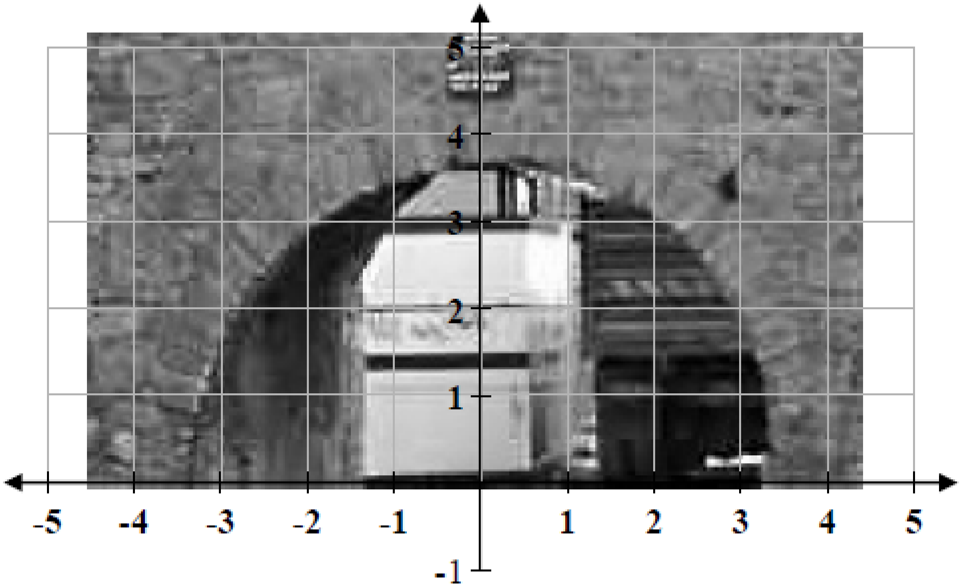
Question 8 **(Suggested maximum time: 5 minutes)**

The picture below shows the top section of the Spanish Arch in Galway city. George wants to see if the arch can be described by a function. He puts a co-ordinate grid over the arch as shown.



(a) Complete the table below to show the value of y for each of the given values of x

We derive the y values below from estimation from the graph.

x	y
-3	1.6
-2	2.9
-1	3.4
0	3.7
1	3.4
2	3.1
3	1.9

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(b) Is it possible to represent this section of the Spanish Arch by a quadratic function? Give a reason for your answer.

No, it is not possible to represent this arch by a quadratic function. The maximum point of the arch is on the y-axis so if had the form of a quadratic function, the arch would be symmetric about the y-axis. We can see from the table above that this is not the case.

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ALTERNATE SOLUTION

The graph seems to cut the x-axis at the points $x_1 = -3.3$ and $x_2 = 3.2$. The equation for a quadratic function with these roots is

$$0 = -(x-(-3.3))(x-3.2) = -(x^2+0.1x-10.56) = -x^2-0.1x+10.56$$

Note that we included the minus sign in front of the equation since the graph has a $\cap$-like shape. If we enter a value into this function, it should correspond to what we determined from the graph. But when $x = 0$, we get 10.56 from the quadratic function and 3.7 from the graph. Hence, this graph cannot be represented by a quadratic function.

Question 9 **(Suggested maximum time: 10 minutes)**

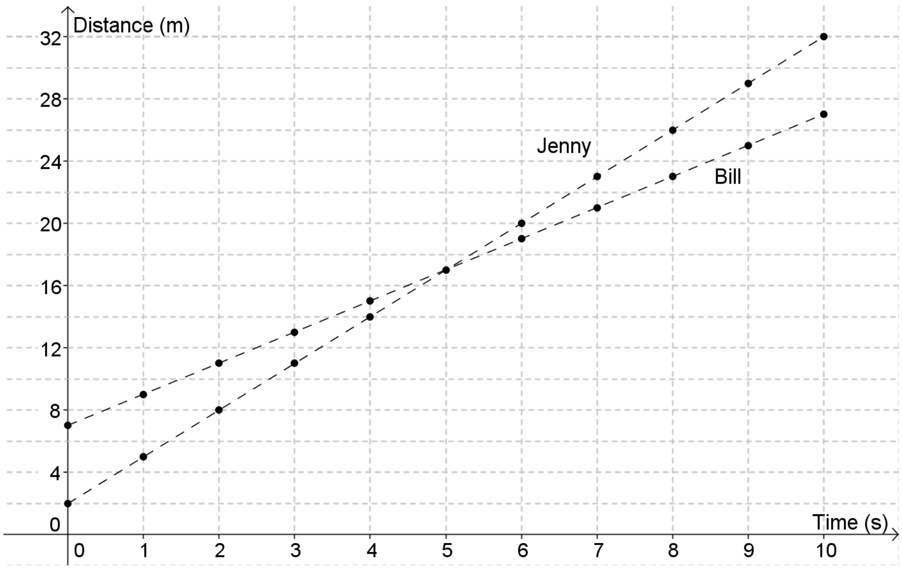
Bill and Jenny are two athletes running in the same direction at steady speeds on a race-track. Tina is standing beside the track. At a particular time, Bill has gone 7 m beyond Tina and his speed is 2 m/s. At the same instant Jenny has gone 2 m beyond Tina and her speed is 3 m/s.

(a) Complete the table below to show the distance between the two runners and Tina over the next 10 seconds.

Bill begins at a distance of 7 m from Tina and this distance increases at a rate of 2 m per second. Similarly, Jenny begins at a distance of 2 m from Tina and this distance increases at a rate of 3 m per second. Thus, our table becomes:

Time	Bill's Distance (m)	Jenny's Distance (m)
0	7	2
1	9	5
2	11	8
3	13	11
4	15	14
5	17	17
6	19	20
7	21	23
8	23	26
9	25	29
10	27	32

(b) On the grid below draw graphs for the distance between Bill and Tina and the distance between Jenny and Tina over the 10 seconds.



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(c) After how many seconds will both runners be the same distance from Tina?

From the graph, both the runners will be the same distance from Tina after 5 seconds.

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(d) After 9 seconds, which runner is furthest from Tina and what is the distance between the runners?

After 9 seconds, Bill is 25m from Tina and Jenny is 29m from Tina, so Jenny is furthest.

There is a distance of $29 - 25 = 4$m between the runners after 9 seconds.

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(e) Write down a formula to represent the distance between Bill and Tina for any given time. State clearly the meaning of any letters used in your formula.

The graph of Bill's distance from Tina is a straight line, so we will use the equation of a line formula. Two points on this line are $(0,7)$ and $(1,9)$. The slope of this line is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-7}{1-0} = 2$$

and so the equation will be

$$y - y_1 = m(x - x_1) \quad \Leftrightarrow \quad y - 7 = 2(x-0) \quad \Leftrightarrow \quad y = 2x+7$$

Thus, if x represents the number of seconds and f represents the distance between Bill and Tina, then $f(x) = 2x+7$

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(f) Write down a formula to represent the distance between Jenny and Tina for any given time.

As with Bill's distance, so we will use the equation of a line formula. Two points on this line are $(0,2)$ and $(1,5)$. The slope of this line is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-2}{1-0} = 3$$

and so the equation will be

$$y - y_1 = m(x - x_1) \quad \Leftrightarrow \quad y - 2 = 3(x-0) \quad \Leftrightarrow \quad y = 3x+2$$

Thus, if x represents the number of seconds and g represents the distance between Jenny and Tina, then $g(x) = 3x+2$

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(g) Use your formulas from (v) and (vi) to verify the answer that you gave to part (iii) above.

We wish to determine the value of x for which the distances are the same, i.e. when $f(x) = g(x)$. So:

$$f(x) = g(x) \quad \Leftrightarrow \quad 7x+2 = 3x+2 \quad \Leftrightarrow \quad x = 5$$

which corresponds to what we determined before.

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(h) After 1 minute, Jenny stops suddenly. From the time she stops, how long will it be until Bill is again level with her?

1 minute corresponds to 60 seconds, so we consider the runners' distances when $x = 60$:

$$f(60) = 2(60) + 7 = 127 \text{ m} \qquad g(60) = 3(60) + 2 = 182 \text{ m}$$

Thus, Bill has to run a further $182 - 127 = 55$ m to catch up with Jenny. Since Bill runs at a rate of 2 m per second, it will take him $\frac{55}{2} = 27.5$ s until he is level with Jenny.

We can verify this by noting that Bill's total run time in seconds will be $60 + 27.5 = 87.5$, so the distance he will have run in this time is $f(87.5) = 2(87.5) + 7 = 182$ which is indeed Jenny's distance after 1 minute.

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(i) If Jenny had not stopped, how long in total would it be until the runners are 100 m apart?

Since Jenny's distance will be bigger than Bill's after 5 seconds, the distance between the two runners will be given by $g(x) - f(x)$, when $x \geqslant 5$. We wish to find the value of x for which the distance is 100 m, so

$$g(x) - f(x) = 100 \quad \Leftrightarrow \quad (3x+2) - (2x+7) = 100 \quad \Leftrightarrow \quad x = 105$$

and so after 105 seconds, the runners will be 100 m apart.

We can verify this by noting that

$$f(105) = 2(105) + 7 = 217 \text{ m} \qquad g(105) = 3(105) + 2 = 317 \text{ m}$$

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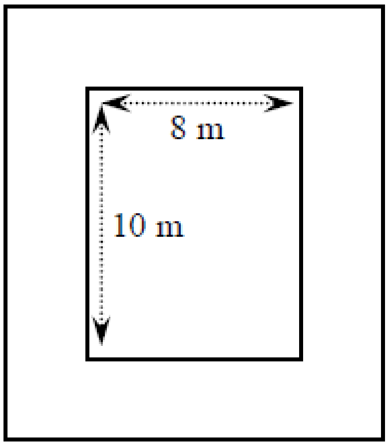
Question 10 **(Suggested maximum time: 20 minutes)**

A plot consists of a rectangular garden measuring 8 m by 10 m, surrounded by a path of constant width, as shown in the diagram. The total area of the plot (garden and path) is 143 m^2.



Three students, Kevin, Elaine, and Tony, have been given the problem of trying to find the width of the path. Each of them is using a different method, but all of them are using x to represent the width of the path.

Kevin divides the path into eight pieces. He writes down the area of each piece in terms of x. He then forms an equation by setting the area of the path plus the area of the garden equal to the total area of the plot.

(a) Write, in terms of x, the area of each section into Kevin's diagram below.

We have four different types of sections in Kevin's diagram: the four corner squares, the rectangles at the top and bottom, the rectangles at the sides and the rectangle in the middle. the areas are given as follows:

Corners: The area of these squares is $x \times x = x^2$ m^2

Top and Bottom: The area of these rectangles is $x \times 8 = 8x$ m^2

Sides: The area of these rectangles is $x \times 10 = 10x$ m^2

Centre: The area of this rectangle is $8 \times 10 = 80$ m^2

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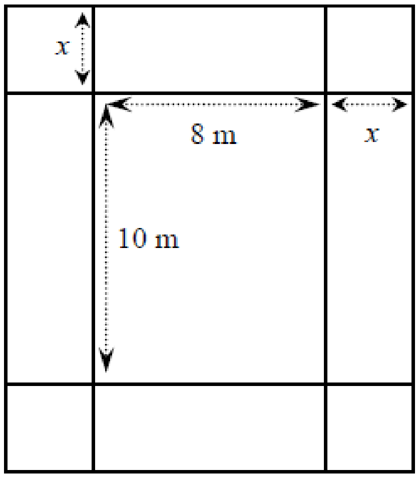


Kevin's Diagram

(b) Write down and simplify the equation that Kevin should get. Give your answer in the form $ax^2 + bx + c = 0$.

If we add the areas from all of Kevin's sections, we will get the total area, which is 143 m^2.

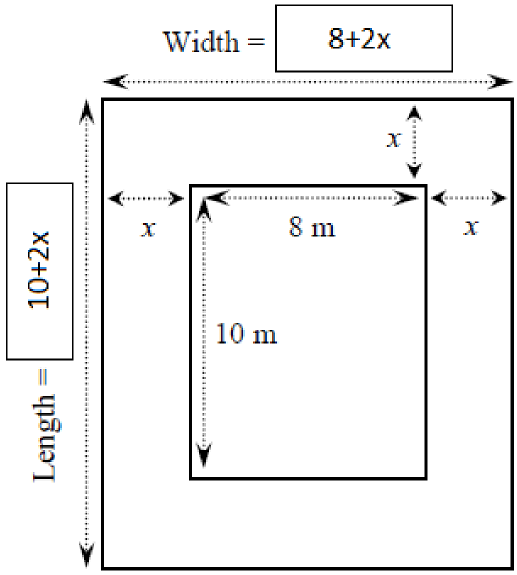
$$4(x^2)+2(8x)+2(10x)+1(80)=143 \quad\Leftrightarrow\quad 4x^2+16x+20x+80=143$$
$$\Leftrightarrow\quad 4x^2+36x-63=0$$

Elaine writes down the length and width of the plot in terms of x. She multiplies these and sets the answer equal to the total area of the plot.

(c) Write, in terms of x, the length and the width of the plot in the spaces on Elaine's diagram.

The overall width of the plot consists of the width of the garden, plus the width of the path on both sides, so $8+2x$.

Similarly, the overall length of the plot consists of the length of the garden, plus the width of the path on both sides, so $10+2x$.



Elaine's Diagram

(d) Write down and simplify the equation that Elaine should get. Give your answer in the form $ax^2+bx+c=0$.

The overall width times the overall length will give the total area of the plot. So:

$$\begin{aligned}(8+2x)(10+2x) = 143 \quad &\Leftrightarrow \quad 80+16x+20x+4x^2 = 143 \\ &\Leftrightarrow \quad 4x^2+36x-63 = 0\end{aligned}$$

As we might have expected, this agrees with Kevin's calculations.

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(e) Solve an equation to find the width of the path.

We can factorise our equation as follows:

$$\begin{aligned}4x^2+36x-63 = 0 \quad &\Leftrightarrow \quad (2x-3)(2x+21) = 0 \\ &\Leftrightarrow \quad 2x-3 = 0 \quad \text{or} \quad 2x+21 = 0\end{aligned}$$

Thus, either $x = \frac{3}{2}$ or $x = -\frac{21}{2}$. Since our path has to have a positive width, we take $x = 1.5$ m as the width of the path.

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(f) Tony does not answer the problem by solving an equation. Instead, he does it by trying out different values for x. Show some calculations that Tony might have used to solve the problem.

We will test some values for x. The simplest approach is to take $x = 1, 2, 3, \ldots$ until we find two points close the correct area.

$$\begin{array}{lcl} x = 1 & \Rightarrow & \text{Area } = (8+2)(10+2) = 120 \text{ m}^2 \\ x = 2 & \Rightarrow & \text{Area } = (8+4)(10+4) = 168 \text{ m}^2 \\ x = 3 & \Rightarrow & \text{Area } = (8+6)(10+6) = 224 \text{ m}^2 \end{array}$$

Since the total area is actually 143, the correct value of x should be between 1 and 2 because $120 < 143 < 168$. We can try $x = 1.5$ as a guess (and in fact we already know this is the correct answer).

$$x = 1.5 \quad \Rightarrow \quad \text{Area } = (8+3)(10+3) = 143 \text{ m}^2$$

(g) Which of the three methods do you think is best? Give a reason for your answer.

Answer: Elaine's method is best.

Reason: Although all three methods give the correct answer, Elaine's method is the quickest and simplest. Tony's method involves estimating the correct answer, which could give rise to some errors. Kevin's method requires several calculations before we arrive at the equation to solve.

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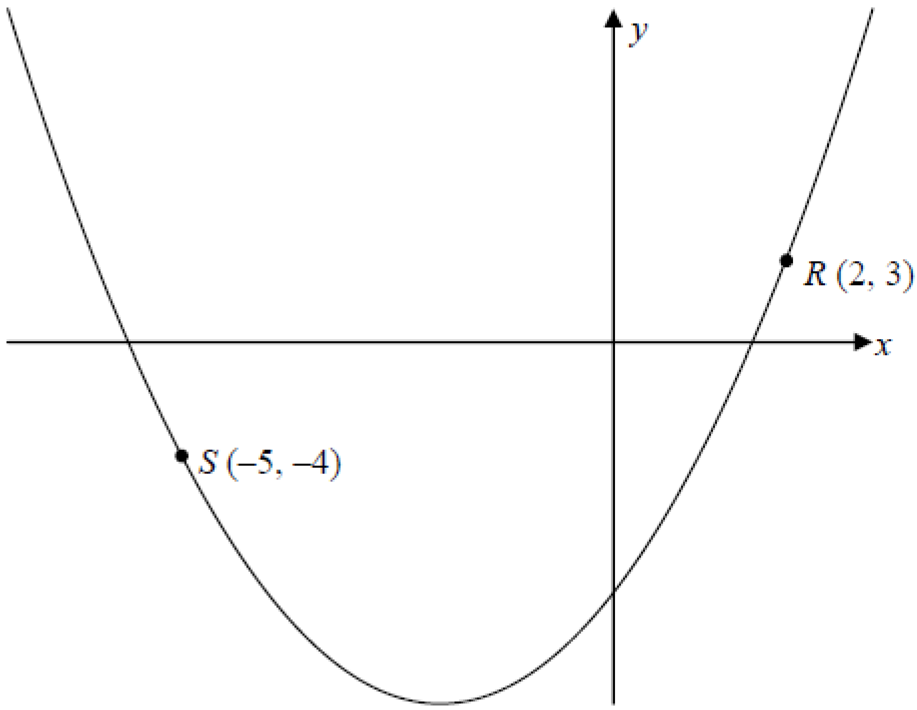
Question 11 **(Suggested maximum time: 20 minutes)**

Part of the graph of the function $y = x^2 + ax + b$, where $a, b \in \mathbb{Z}$, is shown below



The points $R(2,3)$ and $S(-5,-4)$ are on the curve.

(a) Use the given points to form two equations in a and b.

We know that if a point (x_1, y_1) is on a line, then x_1 and y_1 must satisfy the equation of that line. Thus:

$$3 = (2)^2 + a(2) + b \quad \Leftrightarrow \quad 3 = 4 + 2a + b \quad \Leftrightarrow \quad 2a + b = -1$$

Similarly,

$$-4 = (-5)^2 + a(-5) + b \quad \Leftrightarrow \quad -4 = 25 - 5a + b \quad \Leftrightarrow \quad 5a - b = 29$$

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(b) Solve your equations to find the value of a and the value of b.

We can solve the two above equations simultaneously. Adding both equations together we get:

$$\begin{array}{rcl} 2a+b & = & -1 \\ 5a-b & = & 29 \\ \hline 7a & = & 28 \end{array}$$

This means that $a = \frac{28}{7} = 4$. We can now go to the first equation: $2(4)+b=-1$, or $b=-1-8=-9$. This means that the function is of the form $y=x^2+4x-9$.

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(c) Write down the co-ordinates of the point where the curve crosses the y-axis.

The curve will cross the y-axis when $x=0$, so when $y=(0)^2+4(x)-9=-9$

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(d) By solving an equation, find the points where the curve crosses the x-axis. Give each answer correct to one place of decimal.

The curve will cross the x-axis when $y=0$, so when $x^2+4x-9=0$. We will use the quadratic formula to solve for an equation of the form $ax^2+bx+c=0$

$$\begin{aligned}
x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad &\Leftrightarrow \quad x=\frac{-(4)\pm\sqrt{(4)^2-4(1)(-9)}}{2(1)}\\
&\Leftrightarrow \quad x=\frac{-4\pm\sqrt{16+36}}{2}\\
&\Leftrightarrow \quad x=\frac{-4\pm\sqrt{52}}{2}\\
&\Leftrightarrow \quad x=\frac{-4\pm\sqrt{4}\sqrt{13}}{2}\\
&\Leftrightarrow \quad x=\frac{-4\pm2\sqrt{13}}{2}\\
&\Leftrightarrow \quad x=-2\pm\sqrt{13}
\end{aligned}$$

Thus, the curve crosses the x-axis at the points $x=1.6$ and $x=-5.6$ correct to one decimal place.

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Question 12 **(Suggested maximum time: 5 minutes)**

(a) **(i)** Solve the inequality $-2<5x+3\leqslant 18, x\in\mathbb{R}$

$$-2<5x+3\leqslant 18 \quad\Leftrightarrow\quad -5<5x\leqslant 15 \quad\Leftrightarrow\quad -1<x\leqslant 3$$

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(ii) Graph your solution on the number line below.

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(b) Niamh is in a clothes shop and has a voucher which she **must** use. The voucher gives a €10 reduction when buying goods to the value of at least €35. She also has €50 cash.

(i) Write down an inequality in x to show the range of cash that she could spend in the shop.

The voucher requires a minimum of €35 to be used. Since Niamh must use her voucher, she must spend a minimum of €35. If she spends this amount, she gets a €10 discount, meaning the minimum she can spend is €25. The maximum amount of cash she can spend is €50, so the range will be $25 \leqslant x \leqslant 50$.

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(ii) Write an inequality in y to show the price range of articles she could buy.

As before, the minimum value Niamh can spend is €35. The maximum amount of cash she can spend is €50, and including the discount, the item of clothing could cost up to €60. Thus the range will be $35 \leqslant y \leqslant 60$.

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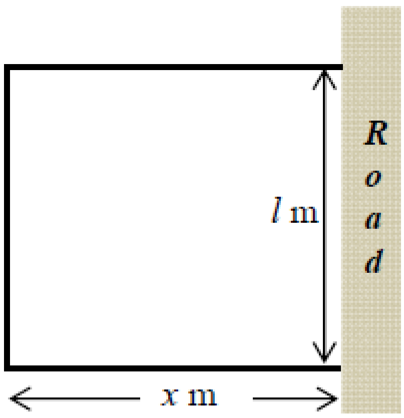
Question 13 **(Suggested maximum time: 20 minutes)**

A rectangular site, with one side facing a road, is to be fenced off.

The side facing the road, which does not require fencing, is l m in length.

The sides perpendicular to the road are x m in length.

The length of fencing that will be used to enclose the rest of the site is 140 m.



(a) Write an expression, in terms of x, for the length (l) of the side facing the road.

The length of the fence is given by $2x+l$. Given that the fence is fixed at 140 m, this means that $2x+l=140$, or $l=-2x+140$.

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(b) Show that the area of the site, in m^2, is $-2x^2+140x$.

The area of the rectangular site is given by $A=l\times x$. Substituting the value of l that we worked out in part **(a)**, we get $A=(-2x+140)\times x=-2x^2+140x$ in m^2 as required.

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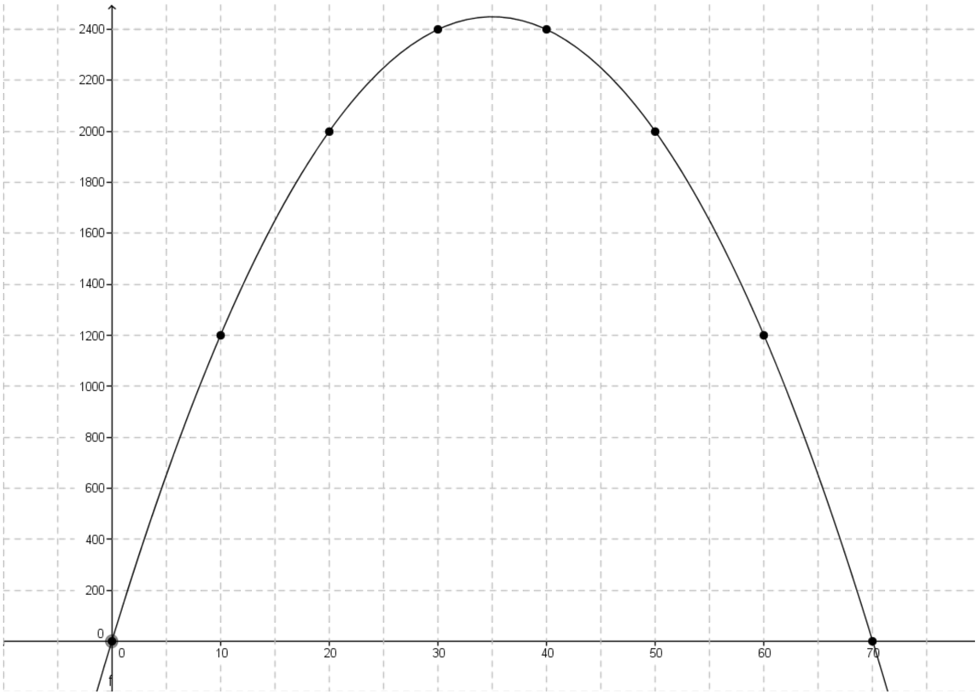
(c) Let f be the function $f:x\mapsto -2x^2+140x$. Evaluate $f(x)$ when $x=0,10,20,30,40,50,60,70$. Hence, draw the graph of f for $0\leqslant x\leqslant 70, x\in\mathbb{R}$.

x	$-2x^2+140x$	y
0	$-2(0)^2+140(0)$	0
10	$-2(10)^2+140(10)$	1200
20	$-2(20)^2+140(20)$	2000
30	$-2(30)^2+140(30)$	2400
40	$-2(40)^2+140(40)$	2400
50	$-2(50)^2+140(50)$	2000
60	$-2(60)^2+140(60)$	1200
70	$-2(70)^2+140(70)$	0

Graphing these points, we get the following:



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Use your graph from part **(c)** to estimate:

(d) the maximum possible area of the site

From the graph, the area reaches its maximum when $x = 35$, which corresponds to an area of 2440 m^2.

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(e) the area of the site when the road frontage (l) is 30 m long.

We know that $l = -2x + 140$, so when $l = 30$, we have $30 = -2x + 140$, or $2x = 110$, giving $x = 55$ m. From the graph, this corresponds to an area of $1,650$ m^2.