



**Coimisiún na Scrúduithe Stáit**  
**State Examinations Commission**

**Junior Certificate 2012**

**Marking Scheme**

**Project Maths (Phase 2)**

**Higher Level**



| <b>Contents</b>    | <i>Page</i> |
|--------------------|-------------|
| Introduction ..... | 2           |

## **Paper 1**

|  |    |
|--|----|
| Model Solutions .....                  | 3  |
| Structure of the marking scheme .....  | 23 |
| Marking Scheme (Questions 1 – 13)..... | 25 |
| Marking Scheme (Question 14).....      | 32 |

## **Paper 2**

|                                       |    |
|---------------------------------------|----|
| Model Solutions .....                 | 41 |
| Structure of the marking scheme ..... | 57 |
| Marking Scheme . . . . .              | 59 |

## **Introduction**

The Higher Level Mathematics examination for candidates in the 24 initial schools for *Project Maths* shared a common question on Paper 1 with the examination for all other candidates. The marking scheme used for the common question was identical for the two groups.

This document contains the complete marking scheme for both papers for the candidates in the 24 schools.

Readers should note that, as with all marking schemes used in the state examinations, the detail required in any answer is determined by the context and the manner in which the question is asked, and by the number of marks assigned to the question or part. Requirements and mark allocations may vary from year to year.

### Question 1

- (a) Give two reasons why  $-7.3$  is not a natural number.

Reason 1:

$-7.3$  is not a positive number OR  $-7.3$  is a minus number

Reason 2:

$-7.3$  is not a whole number OR  $-7.3$  is a decimal

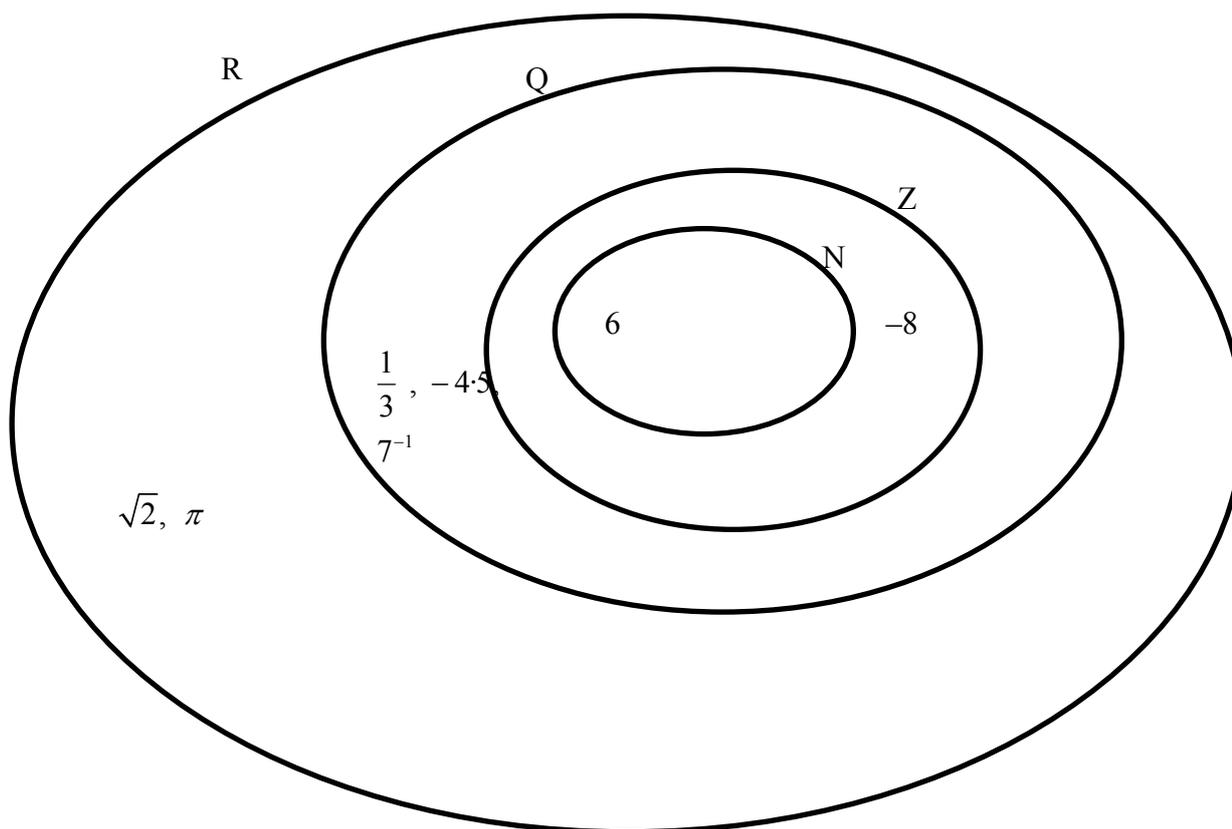
- (b) The diagram represents the sets:

Natural Numbers N

Integers Z

Rational Numbers Q

Real Numbers R

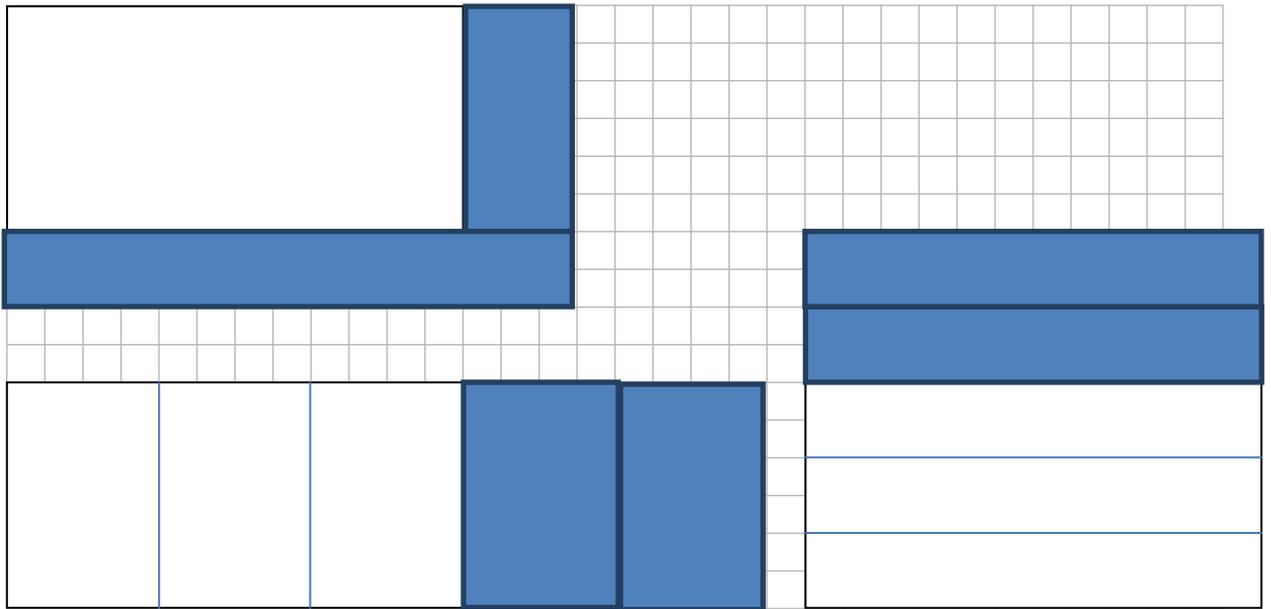


Insert each of the following numbers in the correct place on the diagram:

$-8$ ,  $\pi$ ,  $\frac{1}{3}$ ,  $6$ ,  $\sqrt{2}$ ,  $-4.5$  and  $7^{-1}$ .

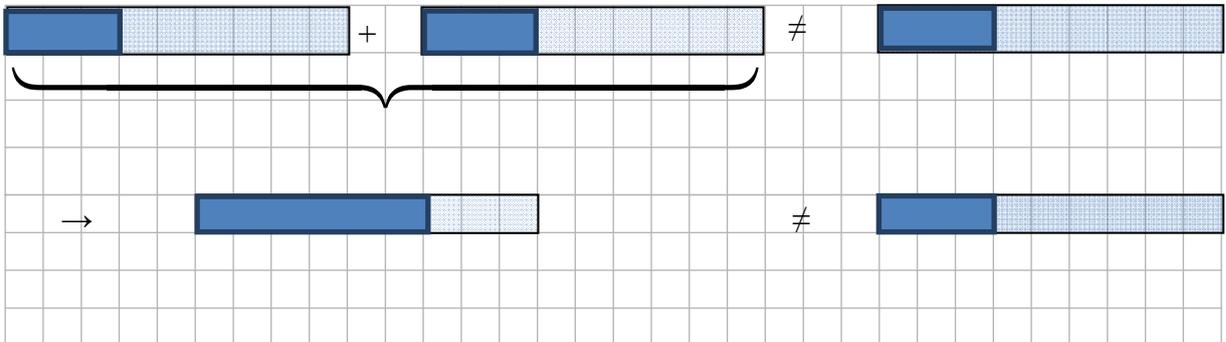
**Question 2**

(a) The diagram below shows three fifths of a rectangle. Complete the rectangle on the grid.



(b) By shading appropriate sections of the strips below, show that

$$\frac{1}{3} + \frac{2}{6} \neq \frac{3}{9}$$



### Question 3

The value of one euro against other currencies on a particular day is shown in the table below.

| Currency         | Rate (€) |
|------------------|----------|
| US Dollar        | 1.4045   |
| Pound Sterling   | 0.87315  |
| Lithuanian Litas | 3.4528   |
| Latvian Lats     | 0.7093   |
| Polish Zloty     | 4.0440   |

- (a) Mary was going to America for a few months. She changed €1200 into US Dollars using the exchange rate in the table.

- (i) How many dollars should she receive at this exchange rate?

$$\text{€}1200 \times 1.405 = \$1685.4$$

- (ii) The bank charged 3% commission on the transaction. How many dollars did she receive?

$$\$1685 \times 0.97 = \$1634.84$$

- (b) On returning to Ireland Mary had \$3060. She changed this amount into euro. The bank again charged her 3% commission on the transaction. She received €2047.

Find the exchange rate on that day, correct to two decimal places.

$$\begin{aligned} \frac{2047}{97} \times 100 &= 2110.3 \\ 2110.3 \times R &= 3060 \\ R &= 1.45 \end{aligned}$$

$$\begin{aligned} 360 \times 0.03 &= 91.8 \\ 3060 - 91.8 &= 2968.2 \\ \frac{2968.2}{2047} &= 1.45 \end{aligned}$$

- (c) David changed a certain amount of sterling into euro at the exchange rate in the table above. A few days later he again changed the same amount of sterling into euro at a different exchange rate. He received fewer euro this time. No commission was charged on these transactions. Write down one possible value for the exchange rate for the second transaction.

|   |                                    |
|---|------------------------------------|
| 1 Euro = £ 0.87315<br>=> £1 = 1.1453 Euro | €1 = anything greater than 0.87315 |
|---|------------------------------------|

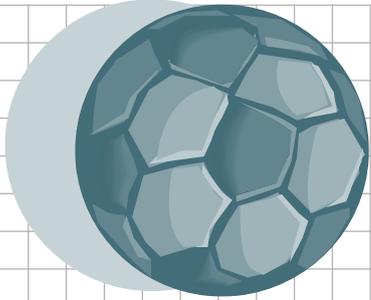
£1 = anything less than €1.1453

#### Question 4

A soccer team has three strikers John, Paul and Michael. The number of minutes each had played by the end of a particular season is shown on the table. The team divided a bonus of €150 000 between its strikers in proportion to the time each had played.

| Name    | Minutes Played |
|---------|----------------|
| John    | 2250           |
| Paul    | 2600           |
| Michael | 150            |

- (a) Calculate the amount each player received.

|   |   |
|---|---|
| $2250 + 2600 + 150 = 5000$ $\frac{150000}{5000} \times 2250 = €67\,500$ $\frac{150000}{5000} \times 2600 = €78\,000$ $\frac{150000}{5000} \times 150 = €4500$ |  |
|---|---|

(b) At the end of the following season a larger total bonus was paid. At that time, John said: “The bonus should be paid according to the number of goals scored by the striker. Paul scored 50% more goals than Michael. I scored as many as both of them together. I would get €140 000 if the team used this method.”

(i) Calculate the total bonus on offer that season.

| Michael | Paul | John |
|---------|------|------|
| 1       | 1.5  | 2.5  |

  
$$\frac{140\,000}{2.5} \times 5 = \text{€}280\,000$$
  
$$50\% = 140\,000$$
$$\Rightarrow 100\% = \text{€}280\,000$$

(ii) How much each would Paul and Michael get under John’s system?

$$\frac{280\,000}{5} = 56\,000 \text{ (one part)}$$
  
$$\text{Michael } 56\,000 \times 1 = \text{€}56\,000$$
$$\text{Paul } 56\,000 \times 1.5 = \text{€}84\,000$$

### Question 5

The USC (Universal Social Charge) is calculated on gross income. The rates of the USC are:

- 2% on the first €10 036 of gross annual income
- 4% on the next €5980
- 7% on the balance.

(a) Niamh earned €45 000 in 2011. Find her USC for that year.

|  |
|--|
| $10\,036 \times 0.02 = 200.72 \text{ (2\% USC charge)}$ $5980 \times 0.04 = 239.2 \text{ (4\% USC charge)}$ $[45\,000 - (10\,036 + 5980)] \times 0.07 = 2028.88 \text{ (7\% USC charge)}$ $2028.88 + 239.2 + 200.72 = \text{€}2468.8 \text{ Total USC Charge}$ |
|--|

The table shows a selection of the tax credits available in Ireland in 2011.

| Individual's Tax Credits           | Tax Credit 2011 |
|------------------------------------|-----------------|
| Single Person                      | €1650           |
| Married or Civil Partner           | €3300           |
| Widowed or Surviving Civil Partner | €2190           |
| Home Carer                         | €810            |
| PAYE                               | €1650           |
| One Parent Family                  | €1650           |

(b) Niamh is a single person who is a PAYE worker. Calculate her total tax credits for 2011.

|                              |
|------------------------------|
| $1650 + 1650 = \text{€}3300$ |
|------------------------------|

- (c) The standard rate of tax is 20% and the higher rate is 41%. The Standard Rate Cut-off Point for a single person is €32 800. Calculate Niamh's tax bill for 2011.

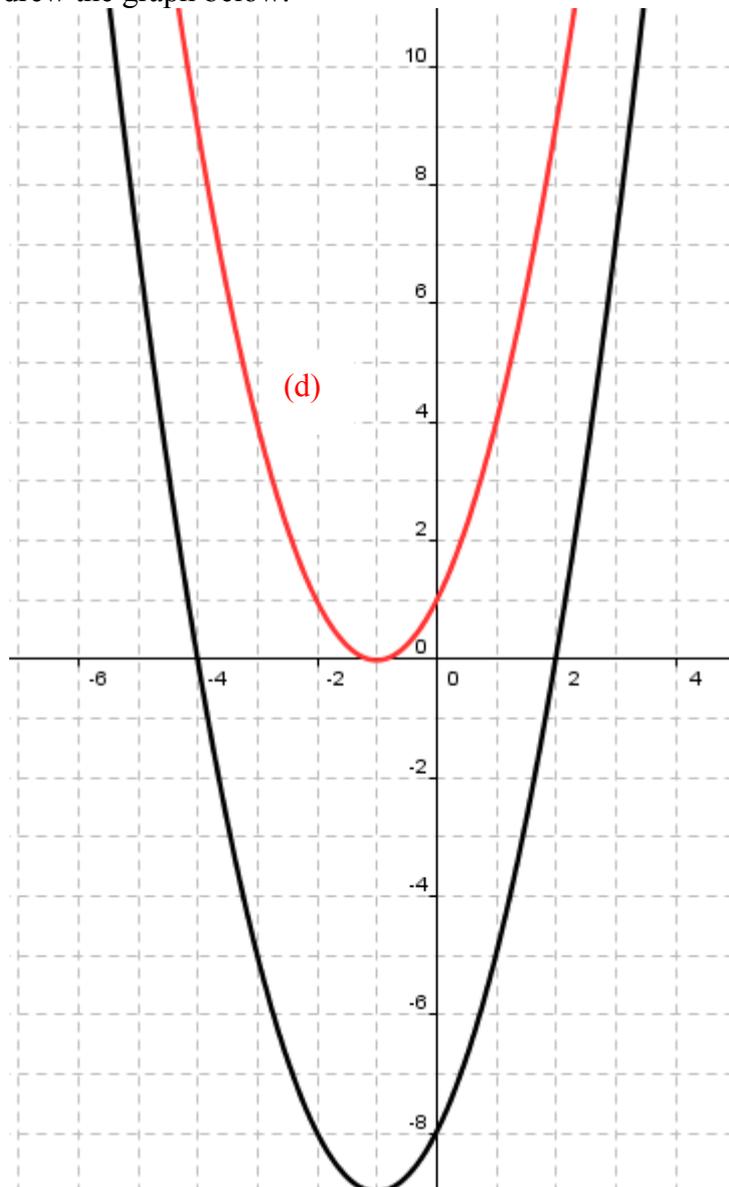
|  |
|--|
| $32\,800 \times 0.2 = 6560$ Tax at lower rate  |
| $45\,000 - 32\,800 = 12\,200$                  |
| $12\,200 \times 0.41 = 5002$ Tax at upper rate |
| $6560 + 5002 = 11\,562$ Total gross Tax        |
| $11\,562 - 3300 = \text{€}8262$ Total Net Tax  |

- (d) Calculate Niamh's net pay for the year, after tax and USC are paid.

|   |  |
|---|--|
| $45\,000 - (2468.8 + 8262) = \text{€}34\,269.2$ | $2468.8 + 8262 = 10\,730.8$<br>$45\,000 - 10\,730.8 = \text{€}34\,269.2$ |
|---|--|

### Question 6

A group of four students is studying graphs of functions of the form  $f : x \mapsto x^2 + 2x + k$ ,  $x \in \mathbb{R}$ . Each takes an integer value of  $k$  and draws the graph of their function in a suitable domain. Maria took  $k = -8$  and drew the graph below.



- (a) Use the graph to write down the roots of the equation  $x^2 + 2x - 8 = 0$ .

|                            |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|----------------------------|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|
| $Roots\ 2\ \text{and}\ -4$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|----------------------------|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|

- (b) Keith's graph passes through the point (3, 2). Find the value of  $k$  that Keith used.

|   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|---|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|
| $f(x) = x^2 + 2x + k$<br>$2 = 3^2 + 2(3) + k$<br>$2 = 9 + 6 + k$<br>$k = -13$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|---|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|

- (c) On Alice's graph, the two roots of the function are the same. Find the value of  $k$  that Alice used.

|   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|---|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|
| $(x+t)^2 = x^2 + 2x + k$<br>$x^2 + 2tx + t^2 = x^2 + 2x + k$<br>$\Rightarrow 2t = 2$<br>$t = 1$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|---|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|

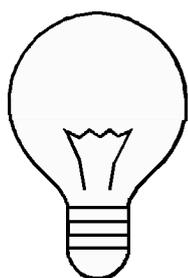
(d) Draw a sketch of Alice’s function on the diagram shown in part (a).

(e) Emma’s graph shows that the roots of her function are  $-5$  and  $3$ . Find the value of  $k$  that she used.

|   |
|---|
| $(x+5)(x-3) = 0$ $x^2 + 2x - 15 = 0$ $\Rightarrow k = -15$                |
| Constant is product of roots<br>$-5 \times 3 = -15$ $\Rightarrow k = -15$ |

### Question 7

Lisa is on a particular payment plan called “Plan A” for her electricity. She pays a standing charge each month even if no electricity is used. She also pays a rate per unit used. The table shows the cost, including the standing charge, of using different amounts of units, in a month.

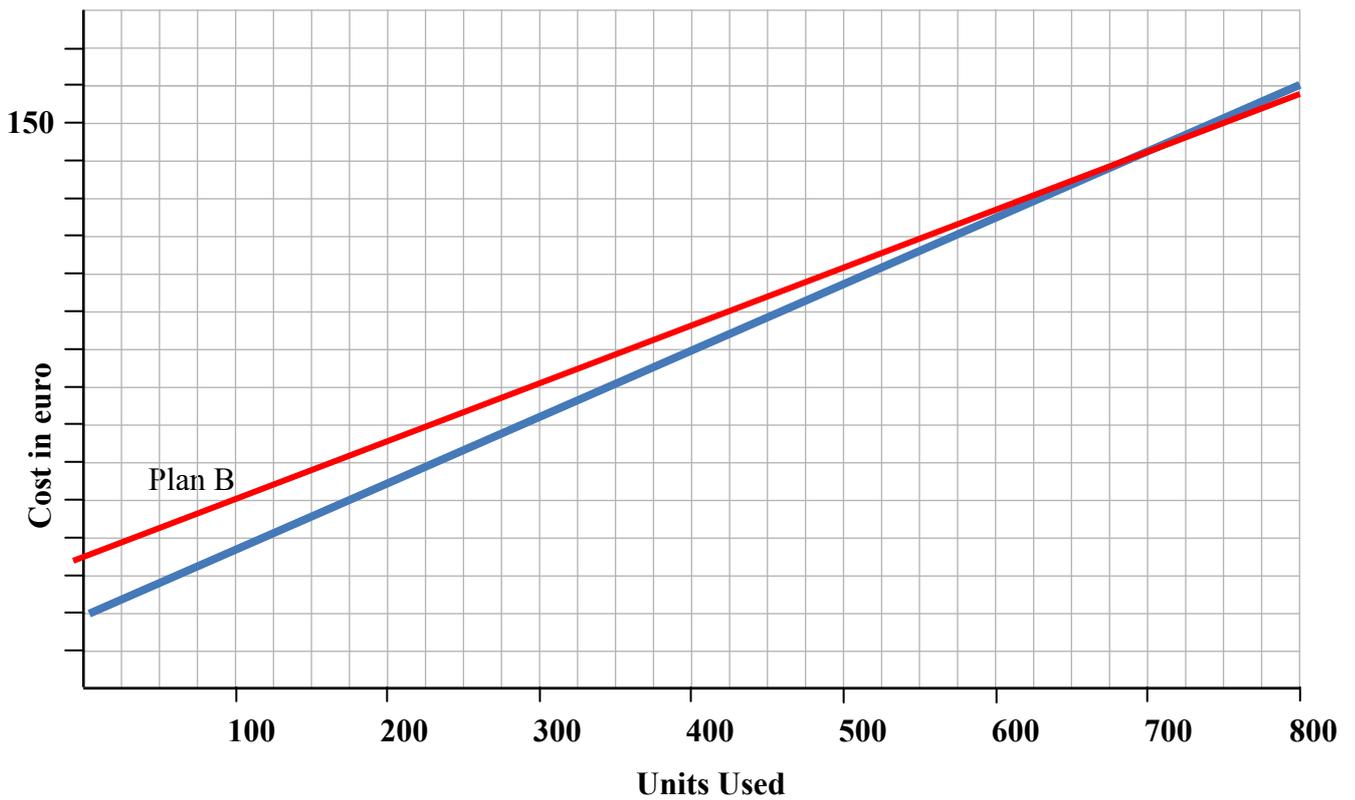


| Units Used | Plan A<br>Cost in euro |
|------------|------------------------|
| 100        | 38                     |
| 200        | 56                     |
| 300        | 74                     |
| 400        | 92                     |
| 500        | 110                    |
| 600        | 128                    |
| 700        | 146                    |
| 800        | 164                    |

(a) Use the data in the table to show that the relationship between the number of units used and the cost is linear.

|  |
|--|
| $56 - 38 = 18, 74 - 56 = 18, 92 - 74 = 18,$ $110 - 92 = 18, 128 - 110 = 18,$ $146 - 128 = 18, 164 - 146 = 18$ <p>Common first difference of 18</p> |
|--|

- (b) Draw a graph to show the relationship between the number of units used and the cost of electricity.



- (c) Use your graph to estimate the standing charge. \_\_\_\_\_ €20 \_\_\_\_\_

- (d) Write down a different method of finding the standing charge.  
Find the standing charge using your method.

Method:

*When the units used go down by 100 then the cost goes down by 18.*  
 $\Rightarrow 38 - 18 = 20$

$$m = \frac{56 - 38}{200 - 100} = 0.18 \left( \text{or } \frac{9}{50} \right)$$

$$y - 38 = 0.18(x - 100)$$

$$0.18x - y + 20 = 0$$

$$\text{sub } x = 0$$

$$\Rightarrow y = 20$$

Standing charge: €20 \_\_\_\_\_



- (g) Lisa is offered a new plan, “Plan B”, where the standing charge is €36 and the rate per unit used is 15.5 cent. Complete the following table for Plan B.

| Units Used | Plan B<br>Cost in euro |
|------------|------------------------|
| 100        | €51.50                 |
| 200        | €67.00                 |
| 300        | €82.50                 |
| 400        | €98.00                 |
| 500        | €113.50                |
| 600        | €129.00                |
| 700        | €144.50                |
| 800        | €160.00                |

- (h) Which plan do you think Lisa should choose? Give a reason for your answer.

**Scenario 1: Concentrates on 650 units**

$[36 + 0.155 \times 650 = €136.75]$

The cost of Plan A and Plan B are very similar therefore it doesn't really matter which plan Lisa chooses

OR

Lisa should choose plan B as it is 25c cheaper

**Scenario 2: Concentrates on low and/or high usage**

If Lisa tends to use a low number of units on average then plan A is better but if she uses a high number of units on average then Plan B is better.

- (i) On your diagram for part (b), draw a graph to show the relationship between the number of units used and the cost of electricity for Plan B. Label this graph “Plan B”.
- (j) Use your diagram to find the number of units for which both plans have the same cost.

640 units

### Question 8

A capacitor is a device which stores electricity. The formula  $W = \frac{1}{2}CV^2$  gives the energy stored in the capacitor, where  $W$  is the energy,  $C$  is the capacitance and  $V$  is the voltage, and standard units are used throughout.

- (a) Find the amount of energy stored in a capacitor when  $C = 2500$  and  $V = 32$ .

|   |
|---|
| $W = \frac{1}{2}CV^2$ $W = \frac{1}{2}(2500)(32)^2$ $W = 1\,280\,000$ |
|---|

- (b) Write  $V$  in terms of  $W$  and  $C$ .

|  |
|--|
| $W = \frac{1}{2}CV^2$ $2W = CV^2$ $\frac{2W}{C} = V^2$ $\sqrt{\frac{2W}{C}} = V$ |
|--|

### Question 9

Consideration is being given to changing the number of points a team gets for a win and also the number of points a team gets for a draw in a soccer league. No points will be awarded for a loss. The table below shows the standing of two teams after six games under the proposed new system.

| Team   | Played | Won | Drawn | Lost | Points (new system) |
|--------|--------|-----|-------|------|---------------------|
| Team A | 6      | 2   | 2     | 2    | 12                  |
| Team B | 6      | 1   | 5     | 0    | 10                  |

- (a) Find the number of points which would be awarded for (i) a win and (ii) a draw under this proposed system.

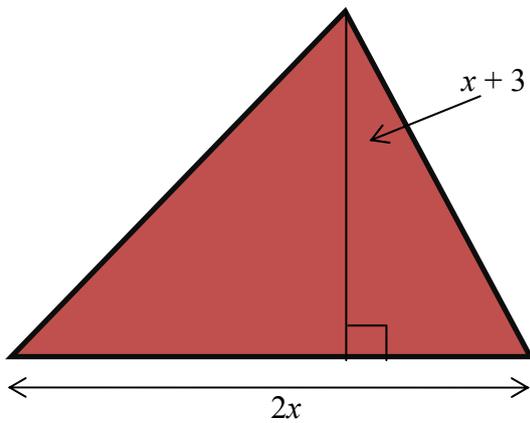
|  |  |   |
|--|--|---|
| $\begin{aligned} 2w+2d &= 12 \\ 1w+5d &= 10 \\ 2w+2d &= 12 \\ -2w-10d &= -20 \\ \hline -8d &= -8 \\ 8d &= 8 \\ \Rightarrow D &= 1 \text{ point} \end{aligned}$ | $\begin{aligned} \text{Substitute } d &= 1 \\ 2w &= 12 - 2 \\ 2w &= 10 \\ W &= 5 \text{ points} \end{aligned}$ | <p>Trial and error with verification of both solutions is awarded full marks.</p> |
|--|--|---|

- (b) The current system awards 3 points for a win and 1 point for a draw. Suggest one reason why it might be preferable to change to the system proposed in part (a).

|   |
|---|
| <p>With new system the ratio of win:draw is higher which rewards a victory more and might encourage a team to go for a win.</p> |
|---|

**Question 10**

A triangle has a base length of  $2x$  cm and a perpendicular height of  $(x + 3)$  cm. The area of the triangle is  $10 \text{ cm}^2$ . Find the distance  $x$ .



$$\frac{1}{2} (2x)(x+3) = 10$$

$$x^2 + 3x - 10 = 0$$

$$(x+5)(x-2) = 0$$

$$\Rightarrow x = -5 \text{ (not possible) and } x = 2 \text{ cm}$$

### Question 11

Factorise fully each of the following expressions:

(i)  $5x^3 - 10x^2$

$$5x^2(x-2)$$

(ii)  $4x^2 - 81y^2$

$$(2x-9y)(2x+9y)$$

(iii)  $a^2 - ab + 3a - 3b$

$$a(a-b) + 3(a-b)$$

$$(a+3)(a-b)$$

### Question 12

(a) Solve each of the following equations:

(i)  $x^2 - 5x - 6 = 0$

$$(x-6)(x+1) = 0$$

$$\Rightarrow x=6 \text{ or } x=-1$$

$$a = 1, b = -5, c = -6$$

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 + 24}}{2}$$

$$x = \frac{5 \pm 7}{2}$$

$$x = 6, x = -1$$

(ii)  $8x^2 - 14x + 3 = 0$

$$(4x-1)(2x-3) = 0$$
$$\Rightarrow x=1/4 \text{ or } x=3/2$$

$$x = \frac{14 \pm \sqrt{(-14)^2 - 4(8)(3)}}{2(8)}$$
$$x = \frac{14 \pm \sqrt{196 - 96}}{2(8)}$$
$$x = \frac{1}{4}, x = \frac{3}{2}$$

(iii)  $\frac{2x+5}{3} - \frac{4x-1}{2} = -\frac{1}{2}$

$$2(2x+5) - 3(4x-1) = 3(-1)$$
$$4x+10 - 12x+3 = -3$$
$$8x = 16$$
$$x = 2$$

- (b) Find the roots of the equation  $2x^2 - 7x - 6 = 0$ .  
Give your answers correct to two decimal places.

$$x = \frac{7 \pm \sqrt{49 - 4(2)(-6)}}{4}$$
$$x = \frac{7 \pm \sqrt{49 + 48}}{4}$$
$$x = \frac{7 \pm \sqrt{97}}{4}$$
$$x = \frac{7 \pm 9.848857}{4}$$
$$x = 4.21 \text{ or } x = -0.71$$

### Question 13

For real numbers  $a$ ,  $b$  and  $c$ , complete the table below. Indicate whether each statement is always true, never true or sometimes true.

| Statement  | Always true | Never true | Sometimes true |
|--|-------------|------------|----------------|
| If $a > b$ and $b > c$ , then $a > c$  | ✓           |            |                |
| If $-a < 4$ and $b < -4$ , then $a < b$  |             | ✓          |                |
| If $a > b$ , then $-a > -b$  |             | ✓          |                |
| If $a > b$ and $b < c$ , then $a < c$  |             |            | ✓              |
| If $3a + 1 > 2$ , then $a > 0$   | ✓           |            |                |
| If $2b - 4 < 3b - 8$ , then $b > 4$  | ✓           |            |                |
| If $a$ and $b$ are both positive and $a < b$ , then<br>$\frac{1}{a} < \frac{1}{b}$ |             | ✓          |                |

### Question 14

Let  $g$  be the function  $g : x \mapsto 2^{x-3}$ .

- (a) Find the value of  $g(3)$ .

$$g(3) = 2^0 = 1$$

- (b) Let  $h$  be the function  $h : x \mapsto x^2 - 3x$ .

- (i) Express  $h(t)$  and  $h(2t + 1)$  in terms of  $t$ .

$$h(t) = t^2 - 3t$$

$$h(2t+1) = (2t+1)^2 - 3(2t+1)$$

$$= 4t^2 + 4t + 1 - 6t - 3$$

- (ii) Hence, find the values of  $t$  for which  $h(t) = h(2t + 1)$ .

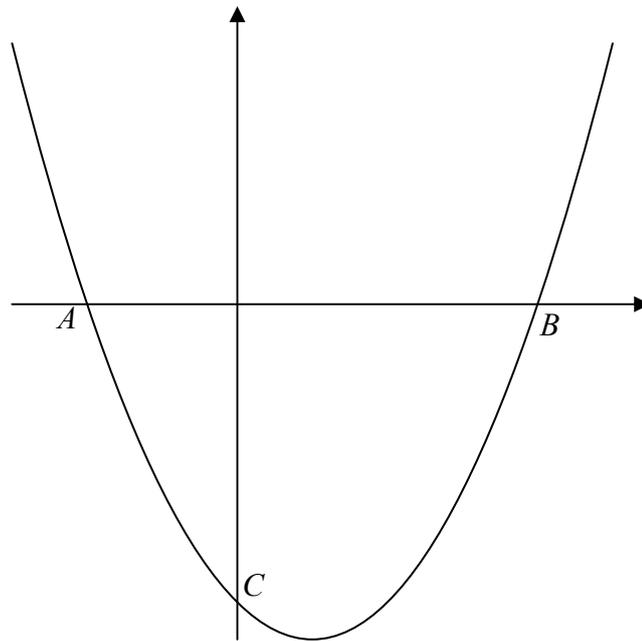
$$4t^2 - 2t - 2 = t^2 - 3t$$

$$3t^2 + 2t - 2 = 0$$

$$(3t - 2)(t + 1) = 0$$

$$t = \frac{2}{3}, t = -1$$

(c) The diagram shows part of the graph of the function  $f : x \mapsto x^2 - 2x - 8, x \in \mathbb{R}$ .



The graph intersects the  $x$ -axis at  $A$  and  $B$ , and the  $y$ -axis at  $C$ .

(i) Find the co-ordinates of  $A$ ,  $B$  and  $C$ .

|   |   |
|---|---|
| <p>C<br/>Sub <math>x = 0</math><br/><math>y = -8</math><br/><math>C(0, -8)</math></p> | <p>A, B<br/><math>f(x) = 0</math><br/><math>(x-4)(x+2) = 0</math><br/><math>\Rightarrow x = 4</math> or <math>x = -2</math></p> |
|---|---|

(ii) Hence, write down the range of values of  $x$  for which  $x^2 - 2x - 8 \leq 0$ .  
 $-2 \leq x \leq 4$

## Structure of the marking scheme (Questions 1 – 13) Paper 1

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

| Scale label      | A     | B         | C             |
|------------------|-------|-----------|---------------|
| No of categories | 2     | 3         | 4             |
| 2 mark scale     | 0, 2  | 0, 1, 2   |               |
| 5 mark scale     | 0, 5  | 0, 3, 5   | 0, 3, 4, 5    |
| 10 mark scale    | 0, 10 | 0, 5, 10  | 0, 5, 8, 10   |
| 15 mark scale    | 0, 15 | 0, 10, 15 | 0, 10, 12, 15 |

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

### Marking scales – level descriptors

#### *A-scales (two categories)*

- incorrect response (no credit)
- correct response (full credit)

#### *B-scales (three categories)*

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

#### *C-scales (four categories)*

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk.

Thus, for example, *scale 10C\** indicates that 9 marks may be awarded.

## Summary of mark allocations and scales to be applied Paper 1

### Question 1 (20)

- (a) 15B
- (b) 5C

### Question 2 (10)

- (a) 5B
- (b) 5C

### Question 3 (30)

- (a) (i)10B\* (ii) 10B\*
- (b) 5C\*
- (c) 5B

### Question 4 (17)

- (a) 10C\*
- (b) (i) 5C\* (ii) 2B\*

### Question 5 (30)

- (a)10C\*
- (b) 10B\*
- (c) 5C\*
- (d) 5B\*

### Question 6 (12)

- (a) 2B
- (b)5C
- (c) and (d) and (e)5C

### Question 7 (55)

- (a)10B
- (b) +(c) 10C\*
- (d) and (e) 5C
- (f) 5C\*
- (g)10C
- (h)5C
- (i) 5B\*
- (j)5A

### Question 8 (17)

- (a)15B
- (b)2B

### Question 9 (15)

- (a)10C
- (b)5B

### Question 10 (5)

5C\*

### Question 11 (15)

- (i)5B
- (ii)5B
- (iii)5B

### Question 12 (19)

- (a)(i)10C
- (ii)5C
- (iii)2B
- (b)2B\*

### Question 13 (5)

5C

### Question 14 (50)

- 10
- 20
- 20

## Detailed Marking Notes (Questions 1 – 13) Paper 1

The \* for units to be applied only if answered fully right. The \* to be applied once only per question. The \* penalty is not applied to currency solutions.

### Question 1

(a) Scale 15B

*Partial Credit:*

- One correct reason

(b) Scale 5C

*High Partial Credit:*

- Four, five or six elements in correct location

*Low partial Credit:*

- One, two or three elements in correct location

### Question 2

(a) Scale 5B

*Partial Credit:*

- Rectangle divided into three equal sections
- Draws three fifths of existing rectangle
- Draws a separate rectangle of area 120 square units
- Demonstrates an understanding of three fifths

(b) Scale 5C

*High Partial Credit:*

- One Lower section not shaded correctly  
(i.e. top three sections and one lower section shaded correctly)

*Low Partial Credit:*

- One, two or three of the top sections shaded correctly

### Question 3

(a) (i) Scale 10B\*

*Partial Credit:*

- US \$ exchange rate identified from the table

(ii) Scale 10B\*

*Partial Credit:*

- 0.03 or 0.97 or 97% or 97/100 written
- $1685.4/100 \times 3$  (or candidate's answer part (i) used here)
- $1200/100 \times 3$  or  $1200 \times 0.03$

(b) Scale 5C\*

*High Partial Credit:*

- Calculates €2110.31

*Low partial Credit:*

- Identifies 97%
- Some correct work on calculating 3% of 3060

(c) Scale 5B

*Partial Credit:*

£1 = 1.453 or identifies 0.87315

#### **Question 4**

(a) Scale 10C\*

*High Partial Credit:*

- One or two of John, Paul, Michael calculated correctly

*Low partial Credit:*

- Writes 5000
- Any three sums of money which sum to €150 000

(b) (i) Scale 5C\*

*High Partial Credit:*

- All three correct ratios correctly identified
- 140000/ 2.5
- Identifies €140 000 of the fund as being 50% of the fund

*Low Partial Credit:*

- Any two correct ratios identified e.g. 1, 2, 3

(ii) Scale 2B\*

*Partial Credit:*

- The one part found (56 000)
- Any two sums of money which sum to half of the fund
- Michael's or Paul's found

#### **Question 5**

(a) Scale 10C\*

*High Partial Credit:*

- Fully correct setup for calculating the 7%, 4% and 2% USC charges

*Low partial Credit:*

- Some correct work in calculating the 2% USC charge and/or the 4% USC charge
- 45 000 – 10 036 and/or 45 000 – 5980

(b) Scale 10B\*

*Partial Credit:*

- One of the tax credits written down

(c) Scale 5C\*

*High Partial Credit:*

- Fully correct setup for calculating the 41% and 20% tax amounts

*Low partial Credit:*

- Some correct work in calculating the 20% and /or the 41% tax amounts

(d) Scale 5B\*

*Partial Credit:*

- Some correct work towards adding the USC (part a) and the net tax (part c)
- Either tax (net or gross) or USC alone is taken away from the 45 000

## Question 6

(a) Scale 2B

*Partial Credit:*

- One of the roots identified (The 2 or the  $-4$ )
- $(x+4)(x-2)$  written

(b) Scale 5C

*High Partial Credit:*

- $2 = 9 + 6 + k$

*Low partial Credit:*

- Substitutes in either the 3 for the  $x$  or the 2 for the  $y$

(c) , (d) And (e) Scale 5C

*High Partial Credit:*

- Either part (c) or part (d) or part (e) fully correct
- Part (d) fully correct means a quadratic function drawn which is consistent with candidate's answer to part (c)

*Low Partial Credit:*

- Partial credit in (c) or (d) or both
- Squared out a perfect square (c)
- Says 'cuts  $x$ -axis once' (c)
- Any work with two equal roots (c)
- The  $-1$  identified on the graph (d)
- A quadratic function drawn which is not consistent with candidate's answer to part (d)
- Use  $(x-5)(x+3)$  consistently and correctly (**Note:** this will lead to  $k = -15$  also) in part (e)
- $(x+5)(x-3) = 0$  or one or two correct factors written in part (e)
- Any correct substitution of  $-5$  and/or  $3$

## Question 7

(a) Scale 10B

*Partial Credit:*

- One up to six of the first differences
- Draws a line using the data in table at **part (a)**

(b) And (c) Scale 10C\*

*High Partial Credit:*

- Correct line or histograms drawn but no standing charge identified

*Low Partial Credit:*

- Marks any one point or histogram correctly from table

**Note:** No line drawn but 20 written as standing charge gets 0 marks

(d) And (e) Scale 5C

*High Partial Credit:*

- Either part (d) or part (e) fully correct

*Low Partial Credit:*

- Partial credit in (d) or (e) or both
- Identifies a valid method but doesn't go as far as calculating the standing charge (d)
- Identifies the 20 (e)
- Uses rise/run correctly or incorrectly (e)
- $C = 20 + 18x$  (e)
- Writes  $0.18x$

(f) Scale 5C\*

*High Partial Credit:*

- $155.5 - 137$

*Low Partial Credit:*

- The €137 calculated
- Uses  $20 + 18x$  correctly or incorrectly
- Some correct use of  $C = 20 + 0.18x$
- Reads the cost of 650 units from graph (Tolerance 117 to 157)

(g) Scale 10C

*High Partial Credit:*

- Four, five, six or seven of the plan B costs calculated

*Low Partial Credit:*

- One or two or three of the plan B costs calculated
- 

(h) Scale 5C

*High Partial Credit:*

- Reason not consistent with previous calculations

*Low Partial Credit:*

- Calculates cost of 650 units using plan B (€136.75)

**Note:** Looking for a specific reason on the 650 units or a general reason on the use of high or low units for full marks

(i) Scale 5B\* (\* is for label on plan B graph)

*Partial Credit:*

- One or more points plotted correctly but line not joined

(j) Scale 5A (Tolerance  $\pm 40$  units)

### Question 8

(a) Scale 15B

*Partial Credit:*

- Some correct substitution into  $\frac{1}{2} CV^2$

(b) Scale 2B

*Partial Credit:*

- One or more correct manipulation
- Leaves as  $v$  or  $v^2 = \frac{2W}{C}$

### Question 9

(a) Scale 10C

*High Partial Credit:*

- Calculates one coordinate only i.e. points for a win or a draw
- Gives draw = 1 point AND win = 5 points without supporting work

*Low partial Credit:*

- Set up one or two simultaneous equations correctly
- Gives draw = 1 point OR win = 5 points without supporting work

(b) Scale 5B

*Partial Credit:*

- Reason given but not fully correct
- Says that in the new system more points given for a win

### Question 10

Scale 5C\*

*High Partial Credit:*

- Correct set-up  $\frac{1}{2} (2x)(x+3) = 10$  or  $(2x)(x+3) = 20$

*Low Partial Credit:*

- Correct substitution of base and/or  $\perp h$
- $2x(x+3) = 10$

**Question 11**

i. Scale 5B

*Partial Credit:*

- Any common factor identified

ii. Scale 5B

*Partial Credit:*

- $2x$  or  $9y$  identified
- $(4x+81y)(4x-81y)$

iii. Scale 5B

*Partial Credit:*

- Any two terms correctly factorised

**Question 12**

(a)

i. Scale 10C

*High Partial Credit:*

- One of 6 and/or  $-1$  without work
- $(x-6)(x+1) = 0$
- $(x+6)(x-1) = 0$  and continues to get  $x = -6$  and  $x = 1$
- $x = \frac{5 \pm \sqrt{25 + 24}}{2}$

*Low Partial Credit:*

- One or both factors correctly identified
- Some correct substitution into  $-b$  formula

ii. Scale 5C

*High Partial Credit*

- One of  $1.5$  or  $0.25$
- $(4x-1)(2x-3) = 0$
- $\frac{14 \pm \sqrt{196 - 96}}{2(8)}$

*Low Partial Credit:*

- One or both factors correctly identified
- Some correct substitution into  $-b$  formula

iii. Scale 2B

*Partial Credit:*

- Some correct use of any correct CD

(b) Scale 2B\*

*Partial Credit:*

- Some correct substitution into  $-b$  formula

**Question 13**

Scale 5C

*High Partial Credit:*

- Four, five or six of the 'If's correctly ticked (or written 'yes')

*Low partial Credit:*

- One, two or three of the 'If's' correctly ticked (or written 'yes')



## QUESTION 14

|                 |                 |                      |
|-----------------|-----------------|----------------------|
| <b>Part (a)</b> | <b>10 marks</b> | <b>Att 3</b>         |
| <b>Part (b)</b> | <b>20 marks</b> | <b>Att (2,2,2,2)</b> |
| <b>Part (c)</b> | <b>20 marks</b> | <b>Att (3,2,2)</b>   |

|                 |                 |              |
|-----------------|-----------------|--------------|
| <b>Part (a)</b> | <b>10 marks</b> | <b>Att 3</b> |
|-----------------|-----------------|--------------|

Let  $g$  be the function  $g : x \rightarrow 2^{x-3}$ .

Find the value of  $g(3)$ .

|            |                 |              |
|------------|-----------------|--------------|
| <b>(a)</b> | <b>10 marks</b> | <b>Att 3</b> |
|------------|-----------------|--------------|

|          |             |            |  |
|----------|-------------|------------|--|
| <b>I</b> |             |            |  |
| $g(3)$   | $= 2^{3-3}$ | <b>4m</b>  |  |
|          | $= 2^0$     | <b>7m</b>  |  |
|          | $= 1$       | <b>10m</b> |  |

|                             |                     |            |  |
|-----------------------------|---------------------|------------|--|
| <b>II</b>                   |                     |            |  |
| $g : x \rightarrow 2^{x-3}$ |                     |            |  |
|                             | $= \frac{2^x}{2^3}$ | <b>3m</b>  |  |
|                             | $= \frac{2^3}{2^3}$ | <b>4m</b>  |  |
|                             | $= \frac{8}{8}$     | <b>7m</b>  |  |
|                             | $= 1$               | <b>10m</b> |  |

*Blunders (-3)*

- B1 Correct answer no work shown
- B2 Mishandles  $2^0$
- B3 Mishandles indices, once if consistent
- B4  $2^{3-3} = 2^3 - 2^3$  and continues correctly
- B5 Fails to finish e.g. stops at  $2^0$

*Attempts (3 marks)*

- A1  $x = 3$  and stops
- A2 8

*Worthless (0)*

- W1 Incorrect answer no work shown, note A2
- W2  $2 \times 3 = 6$
- W3 No work of merit

**Part (b)**

**20 (5,5,5,5) marks**

**Att (2,2,2,2)**

Let  $f$  be the function  $f: x \rightarrow x^2 - 3x$ .

(i) ✍ Express  $f(t)$  and  $f(2t+1)$  in terms of  $t$ .

(ii) ✍ Hence, find the values of  $t$  for which  $f(t) = f(2t+1)$ .

**(b)(i)**

**10(5,5) marks**

**Att (2,2)**

$f: x \rightarrow x^2 - 3x$  Given

$$f(t) = t^2 - 3t \quad \mathbf{5m}$$

$$f(2t+1) = (2t+1)^2 - 3(2t+1) \quad \mathbf{5m}$$

\* 2 parts to mark  $f(t)$  and  $f(2t+1)$  **5m** each

\* Accept  $t^2 - 3t$  for 5 marks

\* Accept  $(2t+1)^2 - 3(2t+1)$  for 5 marks

\* Ignore notation if consistent

*Blunders (-3)*

B1 Substitution error

B2  $(2t+1)^2 + 3(2t+1)$

*Misreadings (-1)*

M1 Misreads a digit providing it does not oversimplify the question

*Attempts (2,2 marks)*

A1 Some correct substitution

A2 States  $x = t$

A3 States  $x = 2t + 1$

A4 Any relevant step

*Worthless (0)*

W1 No work of merit

(b) (ii)

(5,5) marks

Att (2,2)

$$t^2 - 3t = 4t^2 + 4t + 1 - 6t - 3 \quad \mathbf{2m}$$

$$t^2 - 3t = 4t^2 - 2t - 2$$

$$4t^2 - 2t - 2 - (t^2 - 3t) = 0$$

$$4t^2 - 2t - 2 - t^2 + 3t = 0$$

$$3t^2 + t - 2 = 0 \quad \mathbf{5m}$$

and

I

$$(t+1)(3t-2) = 0$$

$$\rightarrow t = -1, t = \frac{2}{3} \quad \mathbf{5m}$$

or

II

$$3t^2 + t - 2 = 0$$

$$3t^2 + 3t - 2t - 2 = 0$$

$$3t(t+1) - 2(t+1) = 0$$

$$(t+1)(3t-2) = 0$$

$$\rightarrow t = -1, t = \frac{2}{3} \quad \mathbf{5m}$$

or

III

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad a = 3, b = 1, c = -2$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(3)(-2)}}{2(3)}$$

$$x = \frac{-1 \pm \sqrt{1^2 - (-24)}}{6}$$

$$x = \frac{-1 \pm \sqrt{25}}{6}$$

$$x = \frac{-1 \pm 5}{6}$$

$$x = \frac{-6}{6}$$

$$x = \frac{4}{6}$$

$$\rightarrow t = -1, t = \frac{2}{3} \quad \mathbf{5m}$$

\* Mark in 2 parts, **5m** for establishing an equation and **5m** for solving

\* Accept candidate's answers from (b)(i), but note A1

*Blunders (-3)*

- B1 Correct answer no work shown ✍
- B2 Sign error
- B3 Transposition error
- B4 Squaring error
- B5 Mathematical error
- B6 Substitution error
- B7 Error in quadratic formula
- B8 Distribution error
- B9  $(2t^2 + 1) + 3(2t + 1)$  and continues correctly, if not already penalised in **(b)(i)**
- B10 Error in solving
- B11 Finds one solution only
- B12 Fails to finish

*Slips (-1)*

- S1 Numerical error to a max of 3

*Misreadings (-1)*

- M1 Misreads a digit providing it does not oversimplify the question

*Attempts (2,2 marks)*

- A1 Linear equation merits *Att 2*, *Att 2* at most
- A2 Attempt to form equation
- A3 Attempt to solve
- A4 Some use of answer(s) from **(b)(i)**
- A5 Any relevant step

*Worthless (0)*

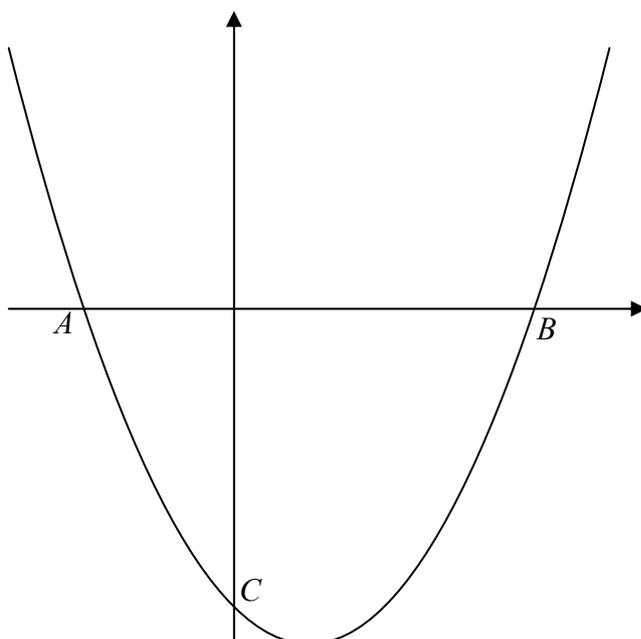
- W1 Incorrect answer(s), no work shown
- W2  $t = 2t + 1$  and stops
- W3 No work of merit

Part (c)

20 (10,5,5) marks

Att (3,2,2)

The diagram below shows part of the graphs of the function  $f : x \rightarrow x^2 - 2x - 8, x \in \mathbb{R}$ .



(i) The graph intersects the  $x$  axis at  $A$  and  $B$  and the  $y$  axis at  $C$ .

~~✎~~ Find the co-ordinates of  $A$ ,  $B$  and  $C$ .

(ii) Hence, write down the range of values of  $x$  for which  $x^2 - 2x - 8 \leq 0$ .

(c) (i)

15(10,5) marks

Att (3,2)

Intersects  $x$  axis:

|                      |           |
|----------------------|-----------|
| <b>I</b>             |           |
| $x^2 - 2x - 8 = 0$   | <b>3m</b> |
| $(x - 4)(x + 2) = 0$ | <b>7m</b> |
| $x = 4, x = -2$      | <b>9m</b> |

→  $A(-2, 0) \quad B(4, 0)$  **10m**

|                           |           |
|---------------------------|-----------|
| <b>II</b>                 |           |
| $x^2 - 2x - 8 = 0$        | <b>3m</b> |
| $x^2 - 4x + 2x - 8 = 0$   | <b>3m</b> |
| $x(x - 4) + 2(x - 4) = 0$ | <b>4m</b> |
| $(x - 4)(x + 2) = 0$      | <b>7m</b> |
| $x = 4, x = -2$           | <b>9m</b> |

→  $A(-2, 0) \quad B(4, 0)$  **10m**

**III**

$$x^2 - 2x - 8 = 0 \quad \mathbf{3m}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{+2 \pm \sqrt{4 - 4(1)(-8)}}{2} \quad \mathbf{3m}$$

$$x = \frac{+2 \pm \sqrt{4 + 32}}{2} \quad \mathbf{4m}$$

$$x = \frac{+2 \pm 6}{2} \quad \mathbf{7m}$$

$$x = 4, x = -2 \quad \mathbf{9m}$$

$$\rightarrow A(-2, 0) \quad B(4, 0) \quad \mathbf{10m}$$

**Intersects y axis:**

$$f(0) = (0)^2 - 2(0) - 8 \quad \mathbf{2m}$$

$$f(0) = -8 \quad \mathbf{4m}$$

$$\rightarrow C(0, -8) \quad \mathbf{5m}$$

- \* Mark in two separate parts. **10m** for finding where graph intersects x axis, **5m** for where graph intersects y axis
- \* Correct answer fully verified is full marks

*Blunders (-3)*

- B1 Correct answer no work shown ✗
- B2 Sign error
- B3 Incorrect operation
- B4 Incorrect factors
- B5 Mathematical error
- B6 Error in quadratic
- B7 Error in substitution
- B8 Transposition error
- B9 Square root error
- B10 Finds one solution only ( *i.e.*  $x = 4$  **or**  $x = -2$  )

*Slips (-1)*

- S1 Does not write co-ordinate
- S2 Does not label A or B, or labels them incorrectly

*Misreadings (-1)*

- M1 Misreads a digit providing it does not oversimplify the question

*Attempts (3,2 marks)*

- A1 Effort to solve equation ( $= 0$ ) or any indication of  $y = 0$
- A2 Effort to substitute ( $x = 0$ )
- A3 Quadratic with some correct substitution
- A4  $x = 4$  and/or  $x = -2$  only with no work
- A5 Some relevant step

*Worthless (0)*

- W1 Incorrect answer(s), no work shown
- W2 No work of merit

**(c) (ii)**

**5 marks**

**Att 2**

|       |                    |
|-------|--------------------|
| Range | $-2 \leq x \leq 4$ |
|-------|--------------------|

- \* Accept candidate's  $x$  values from **(c)(i)**
- \* Accept "from  $-2$  to  $4$  inclusive" or similar for full marks

*Blunders (-3)*

- B1 Reversed inequalities
- B2  $2 \leq x \leq 4$ , minus sign omitted

*Slips (-1)*

- S1 Does not include equals in inequalities

*Attempts (2 marks)*

- A1 Some identification on graph
- A2  $f(x) \leq 0$
- A3 Some relevant step

*Worthless (0)*

- W1 Incorrect answer(s) with no work of merit
- W2 No work of merit

*Coimisiún na Scrúduithe Stáit  
State Examination Commission*

*Scrúdu  
an Teastais Shóisearaigh*



**JUNIOR CERTIFICATE  
EXAMINATION  
MARKING SCHEME  
MATHEMATICS  
(PROJECT MATHS – PHASE 2)  
PAPER 2  
HIGHER LEVEL**



**Coimisiún na Scrúduithe Stáit**  
*State Examinations Commission*

### Question 1

A tissue measures 300 mm × 260 mm. There are 100 tissues in a box.  
Find the total area of tissue in the box in m<sup>2</sup>.

$$0.3 \times 0.26 = 0.078 \text{ m}^2$$

$$0.078 \times 100 = 7.8 \text{ m}^2$$

$$300 \times 260 = 78\,000$$

$$78\,000 \times 100 = 7\,800\,000 \\ = 7.8 \text{ m}^3$$

### Question 2

- (a) A container in the shape of a cylinder has a capacity of 50 litres. The height of the cylinder is 0.7 m.  
Find the length of the diameter of the cylinder.  
Give your answer correct to the nearest whole number.

$$\pi r^2 h = V$$

$$\pi \times r^2 \times 70 = 50\,000$$

$$r^2 = \frac{50\,000}{\pi \times 70}$$

$$r^2 = 227.36$$

$$r = 15$$

$$\text{Diameter} = 30 \text{ cm}$$

- (b) A rectangular tank has a length of 0.6 m, a width of 0.35 m and its height measures 15 cm.  
Find the capacity of the rectangular tank.

$$60 \times 35 \times 15 = 31\,500 \text{ cm}^3$$

$$\text{or } 31.5 \text{ L}$$

- (c) The rectangular tank is full of water. This water is then poured into the cylindrical container in (a) above. Find the depth of water in the cylinder.  
Give your answer correct to one decimal place.

$$\pi \times 15 \times 15 \times h = 31500$$

$$h = \frac{31500}{\pi \times 15 \times 15}$$

$$h = 44.6 \text{ cm}$$

$$\frac{31.5}{50} \times 100 = 63\%$$

63% of the height of 70 cm is 44.1 cm

### Question 3

An ornament is carved from a rectangular block of wood which has a square base and a height of 24 cm. The ornament consists of two identical spheres and two identical cubes as illustrated in the diagram. The diameter of each sphere is equal to the length of the side of each cube. The ornament has the same width as the original block.

- (a) Find the length of a side of one of the cubes.

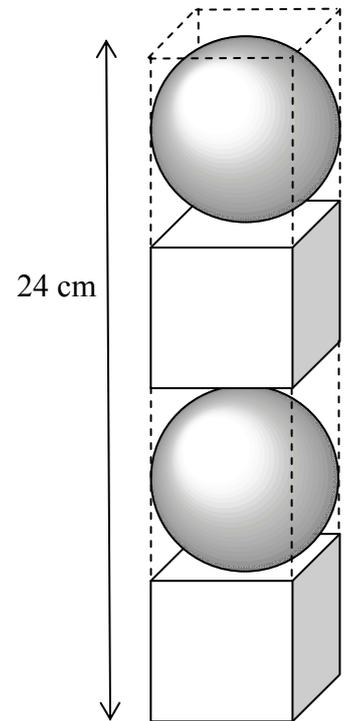
$$\frac{24}{4} = 6 \text{ cm}$$

- (b) Find the volume of the ornament.

$$2(6 \times 6 \times 6) + 2\left(\frac{4}{3}\pi \times 3^3\right)$$

$$= 432 + 226$$

$$= 658 \text{ cm}^3$$



- (c) In making the ornament, what percentage of the original block of wood is carved away?

$$24 \times 6 \times 6 - 658.2 = 864 - 658.2 = 205.8$$

$$\frac{205.8}{864} \times \frac{100}{1} = 23.82\%$$

$$6 \times 6 \times 6 = 216$$

$$\frac{4}{3} \times \pi \times 3^3 = 113.097$$

$$216 - 113.097 = 102.9$$

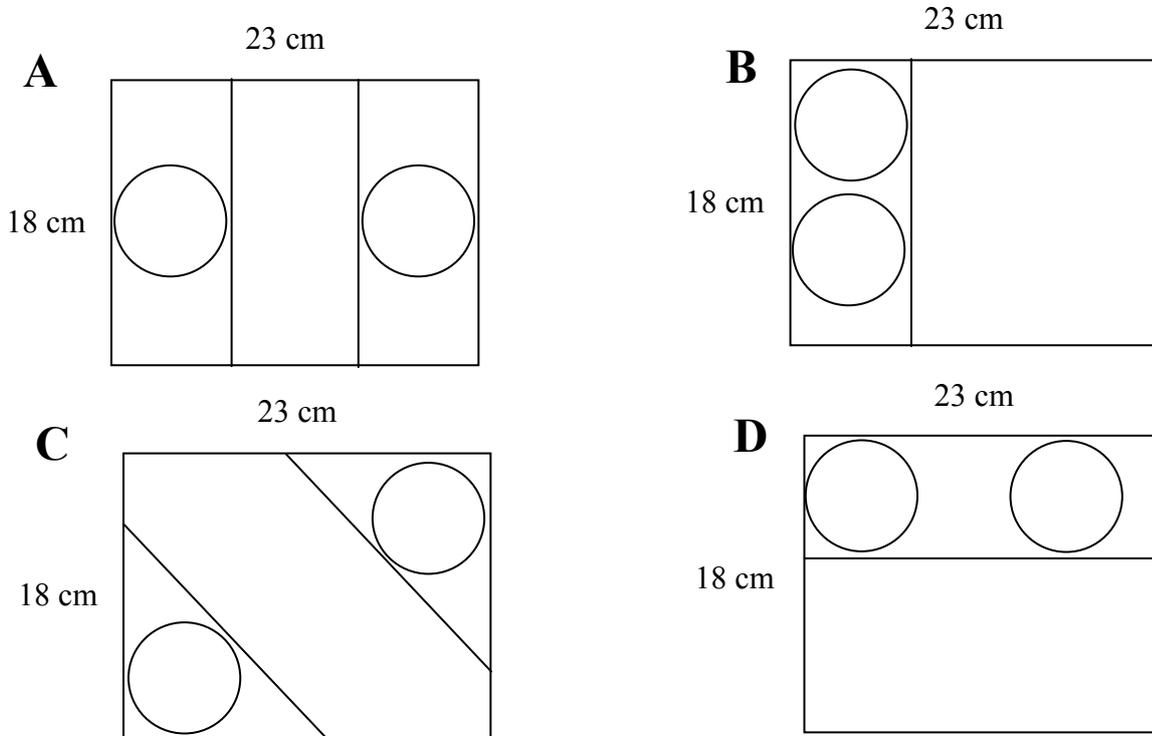
$$102.9 \times 2 = 205.8$$

$$\frac{205.8}{864} \times 100 = 23.82\%$$

**Question 4**

A soup tin in the form of a cylinder has a diameter of 7 cm and a height of 10 cm. The cylinder is constructed from pieces of metal cut from a thin sheet measuring 23 cm by 18 cm.

(a) Which one of the four diagrams A, B, C or D could represent the sheet of metal from which the cylinder has been cut?



Answer = Diagram  D . Give a reason for your choice.

$2\pi r = 2 \times \pi \times 3.5 = 22 \text{ cm}$   
 Need a piece  $10 \text{ cm} \times 22$  to make this cylinder. Only D has this.

(b) Find the area of metal which remains after the pieces have been cut out.

Area of sheet:  $18 \times 23 = 414$

Surface area of cylinder:  $10 \times 22 + 2(\pi \times 3.5^2) = 220 + 77 = 297$

Metal remaining:  $414 - 297 = 117 \text{ cm}^2$

(c) Find the capacity of the soup tin.

$V = \pi \times 3.5^2 \times 10 = 384.85 \text{ cm}^3$

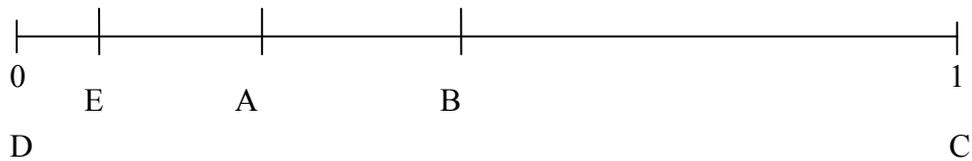
**Question 5****(Suggested maximum time: 5 minutes)**

A, B, C, D and E represent the probabilities of certain events occurring.

(a) Write the probability of each of the events listed into the table below.

| Event  |   | Probability                     |
|--|---|---------------------------------|
| A club is selected in a random draw from a pack of playing cards | A | $\frac{1}{4}$                   |
| A tossed fair coin shows a tail on landing                       | B | $\frac{1}{2}$ OR evens OR 50/50 |
| The sun will rise in the east tomorrow                           | C | 1 OR certain                    |
| May will follow directly after June                              | D | 0 OR impossible                 |
| A randomly selected person was born on a Thursday                | E | $\frac{1}{7}$                   |

(b) Place each of the letters A, B, C, D and E at its correct position on the probability scale below.





(ii) The median age of the male winners and the median age of the female winners.

|  |  |
|--|--|
| <p style="text-align: center;">Male</p> $(45+45)/2$ <p style="text-align: center;">Median = 45</p> | <p style="text-align: center;">Female</p> $(33+33)/2$ <p style="text-align: center;">Median = 33</p> |
| <p>Median age of women is lower so statement is true for median age</p>                            |  |

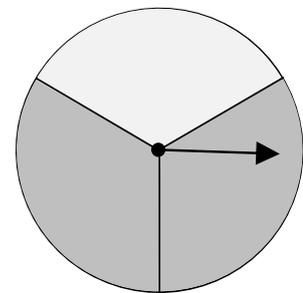
(d) Find the interquartile ranges of the ages of the male winners and of the female winners.

|  |  |
|--|--|
| <p style="text-align: center;">Male</p> $50 - 37.5$ $12.5$ | <p style="text-align: center;">Female</p> $40.5 - 29$ $11.5$ |
|--|--|

**Question 7**

**(25 marks)**

A fair circular spinner consists of three equal sectors. Two are coloured blue and one is coloured red. The spinner is spun and a fair coin is tossed.



(a) What is the probability of the spinner landing on a blue sector?

|   |
|---|
| $\frac{2}{3} \text{ or } \frac{240}{360}$ |
|---|

(b) Find the probability of getting a head and a red.

|  |
|--|
| $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ |
|--|

(c) Find the probability of getting a tail and a blue.

|  |
|--|
| $\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$ |
|--|

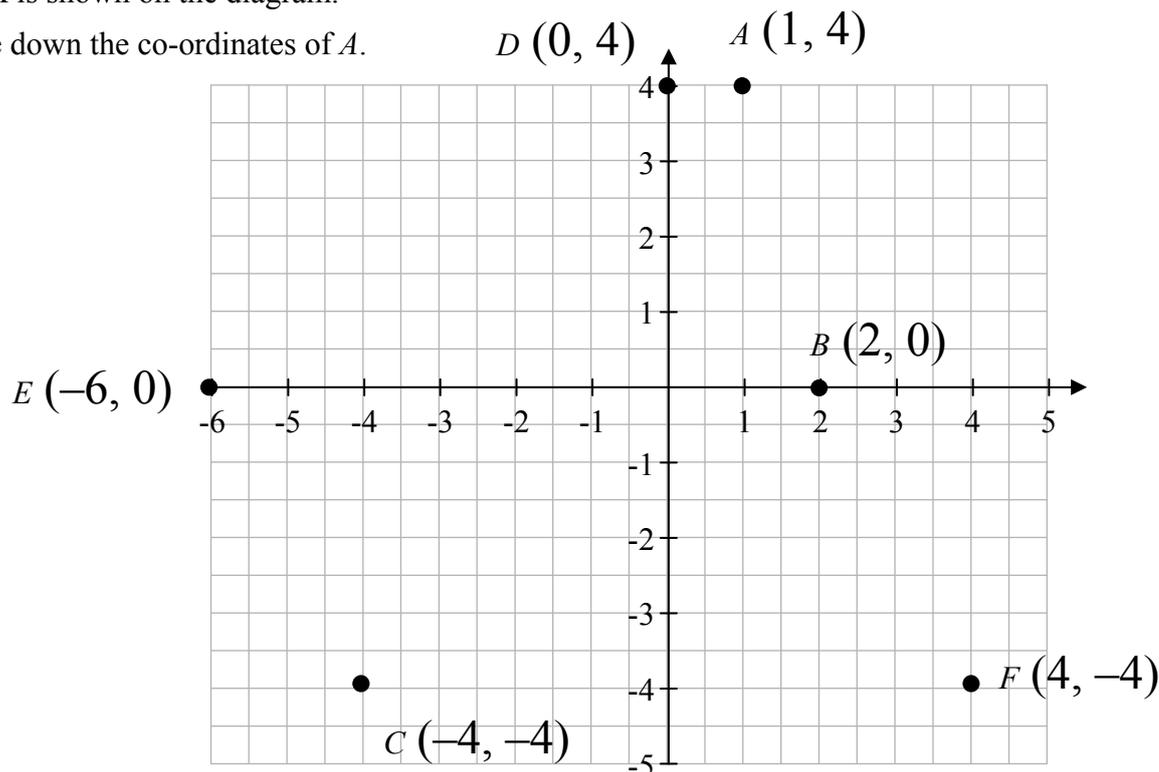


**Question 9**

**(20 marks)**

The point A is shown on the diagram.

(a) Write down the co-ordinates of A.



(b) Plot the following points on the diagram above.

|          |          |          |          |          |
|----------|----------|----------|----------|----------|
| <b>B</b> | <b>C</b> | <b>D</b> | <b>E</b> | <b>F</b> |
| (2, 0)   | (-4, -4) | (0, 4)   | (-6, 0)  | (4, -4)  |

(c) Calculate the midpoint of  $[DF]$ .

$$\left( \frac{0+4}{2}, \frac{4-4}{2} \right) = (2, 0)$$

(d) Find the slope of  $BF$ .

$$\frac{-4-0}{4-2} = \frac{-4}{2} = -2$$

(e) Write down the equation of the line  $BF$  in the form  $y = mx + c$ .

$$y = -2x + 4$$

$$y - 0 = -2(x - 2)$$

$$y = -2x + 4$$

(f) Find the slope of the line  $CE$ .

$$\frac{0 - (-4)}{-6 - (-4)} = \frac{4}{-2} = -2$$

(g) Write the equation of the line  $CE$  in the form of  $ax + by + c = 0$ .

$$\begin{aligned}y - 0 &= -2(x + 6) \\y &= -2x - 12 \\2x + y + 12 &= 0\end{aligned}$$

(h) What is the ratio of the area of the triangle  $BCE$  to the area of the triangle  $BCF$ ?

Area of  $\triangle BCE$

$$\begin{aligned}\frac{1}{2}(8)(4) \\16\end{aligned}$$

Area of  $\triangle BCF$

$$\begin{aligned}\frac{1}{2}(8)(4) \\16\end{aligned}$$

Ratio:

1:1 or 1/1

(i) State whether the two triangles in part (h) above are congruent. Give a reason for your answer.

Answer:

yes

Reason:

$CFBE$  is a parallelogram and  $CB$  is a diagonal which divides the parallelogram into two congruent triangles.

Or

SSS or SAS or ASA argument

**Question 10**

**(50 marks)**

The table below gives the equations of six lines.

Line 1  $y = 3x - 6$

Line 2  $y = 3x + 12$

Line 3  $y = 5x + 20$

Line 4  $y = x - 7$

Line 5  $y = -2x + 4$

Line 6  $y = 4x - 16$

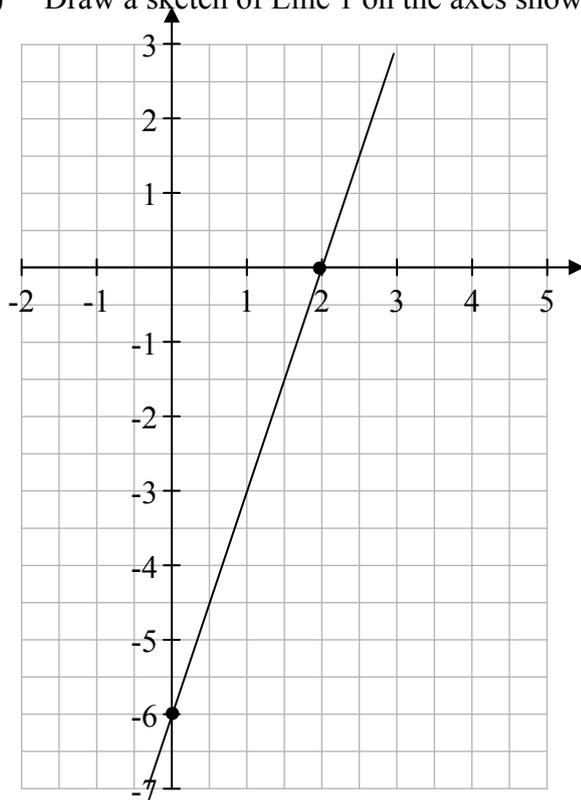
- (a) Which line has the greatest slope? Give a reason for your answer.

Line 3 OR  $y = 5x + 20$   
5 is the biggest number in front of  $x$  for any of the lines

- (b) Which lines are parallel? Give a reason for your answer.

Line 1 and Line 2  
 $y = 3x - 6$  and  $y = 3x + 12$   
They have the same slope (3)

(c) Draw a sketch of Line 1 on the axes shown.



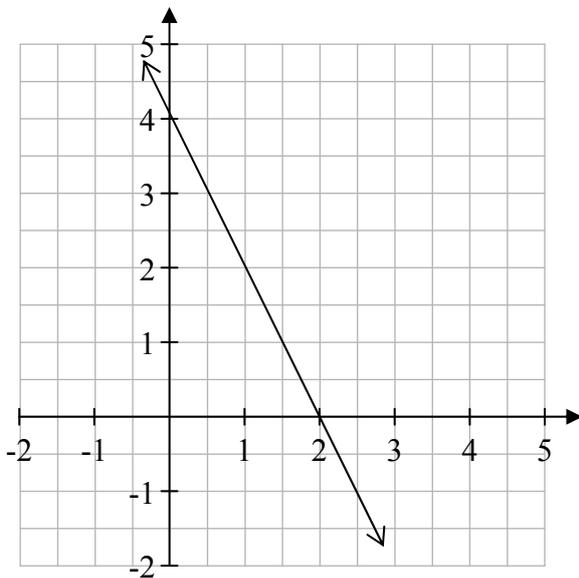
$$y = 3x - 6$$

$$x = 0, y = -6$$

$$y = 0, x = 2$$

$$(2, 0), (0, -6)$$

(d) The diagram below represents one of the given lines. Which line does it represent?



$$\text{slope} = \frac{0 - 4}{2 - 0} = -2$$

$$y\text{-intercept} = 4$$

$$\text{equation} \quad y = -2x + 4$$

Answer = Line \_\_\_\_\_ 5 ( $y = -2x + 4$ )

- (e) The table shows some values of  $x$  and  $y$  for the equation of one of the lines. Which equation do they satisfy?

| $x$ | $y$ |
|-----|-----|
| 7   | 12  |
| 9   | 20  |
| 10  | 24  |

|   |   |
|---|---|
| $y = 4x - 16$ $y = 4(7) - 16 = 12$ $y = 4(9) - 16 = 20$ $y = 4(10) - 16 = 24$ | $m = \frac{20 - 12}{9 - 7} = 4$ $y - 12 = 4(x - 7)$ $y = 4x - 16$ |
|---|---|

Answer = Line \_\_\_\_\_ 6 \_\_\_\_\_

- (f) There is one value of  $x$  which will give the same value of  $y$  for Line 4 as it will for Line 6. Find, using algebra, this value of  $x$  and the corresponding value of  $y$ .

|  |
|--|
| $y = x - 7$ $y = 4x - 16$ $0 = 3x - 9$ $x = 3$ $y = 3 - 7 = -4$<br>$(3, -4)$<br>Ans: $x = 3$ |
|--|

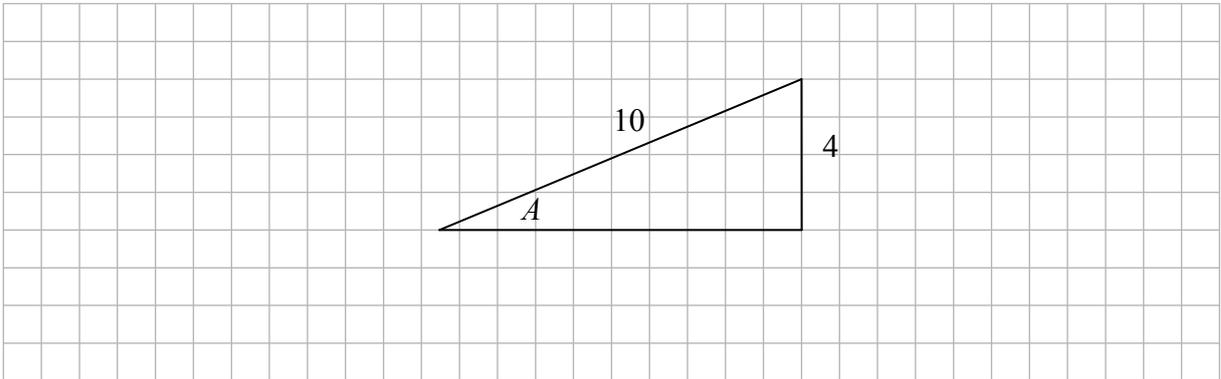
- (g) Verify your answer to (f) above.

|  |
|--|
| Line 4<br>$y = (3) - 7 = -4$<br>Line 6<br>$y = 4(3) - 16 = -4$ |
|--|

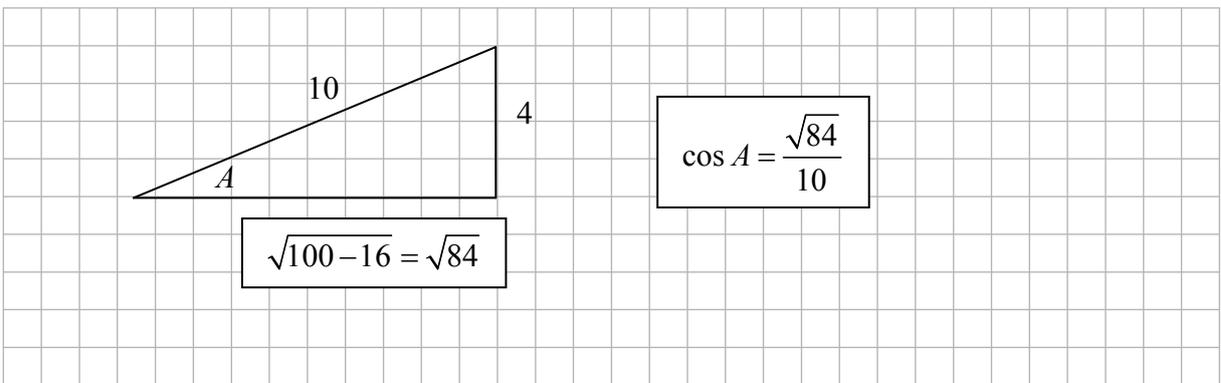
**Question 11**

**(9 marks)**

(a) Construct a right-angled triangle containing an angle  $A$  such that  $\sin A = 0.4$ .



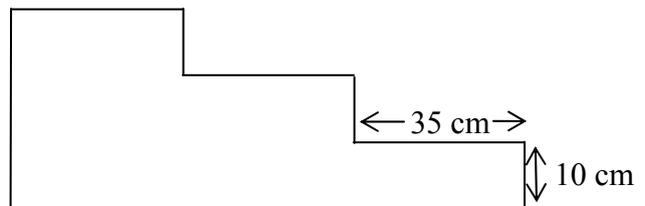
(b) Find, from your triangle,  $\cos A$  in surd form.



**Question 12**

**(30 marks)**

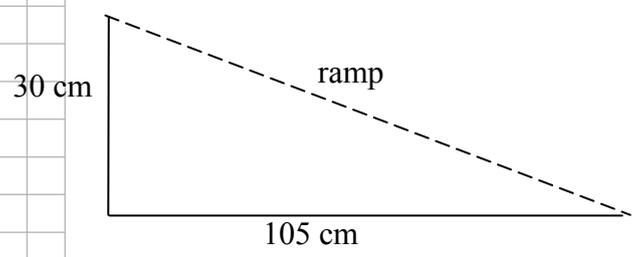
A homeowner wishes to replace the three identical steps leading to her front door with a ramp. Each step is 10 cm high and 35 cm long. Find the length of the ramp. Give your answer correct to one decimal place.



$$\begin{aligned} \text{Ramp} &= \sqrt{30^2 + 105^2} \\ &= 109.2 \text{ cm} \end{aligned}$$

OR

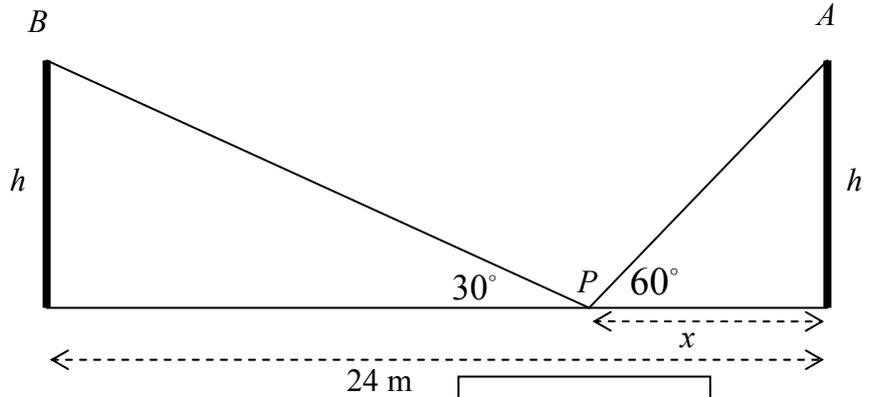
$$\begin{aligned} H^2 &= 35^2 + 10^2 \\ H &= \sqrt{1225 + 100} \\ H &= \sqrt{1325} \\ \text{Ramp} &= 3 \times H \\ \text{Ramp} &= 3 \times \sqrt{1325} \\ \text{Ramp} &= 109.2 \text{ cm} \end{aligned}$$



**Question 13**

**(20 marks)**

Two vertical poles  $A$  and  $B$ , each of height  $h$ , are standing on opposite sides of a level road. They are 24 m apart. The point  $P$ , on the road directly between the two poles, is a distance  $x$  from pole  $A$ . The angle of elevation from  $P$  to the top of pole  $A$  is  $60^\circ$ .



(a) Write  $h$  in terms of  $x$ .

$$\tan 60^\circ = \frac{h}{x}$$

$$\sqrt{3} = \frac{h}{x}$$

$$h = \sqrt{3} x$$

$$\tan 60^\circ = \frac{h}{x}$$

$$1.732 = \frac{h}{x}$$

$$h = 1.732 x$$

$$\tan 30^\circ = \frac{h}{24 - x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{24 - x}$$

$$h = \frac{24 - x}{\sqrt{3}}$$

(b) From  $P$  the angle of elevation to the top of pole  $B$  is  $30^\circ$ . Find  $h$ , the height of the two poles.

$$\tan 30^\circ = \frac{h}{24 - x}$$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{24 - x}$$

$$3x = 24 - x$$

$$4x = 24$$

$$x = 6$$

$$h = 6\sqrt{3} \text{ m}$$

$$\tan 30^\circ = \frac{h}{24 - x}$$

$$\frac{1}{1.732} = \frac{1.732x}{24 - x}$$

$$3x = 24 - x$$

$$4x = 24$$

$$x = 6$$

$$h = 6\sqrt{3} \text{ m or } 6(1.732)\text{m} = 10.39 \text{ m}$$

Question 14

(16 marks)

Prove that the angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.

**Given:** A circle with centre O, with points A, B and C on the circle

**To Prove:**  $|\angle BOC| = 2|\angle BAC|$

**Construction:** Join A to O and extend to R

**Proof:** In the triangle AOB

$$|AO| = |OB| \quad \text{Radii}$$

$$\Rightarrow |\angle OBA| = |\angle OAB| \quad \text{Theorem 2 (isosceles } \Delta)$$

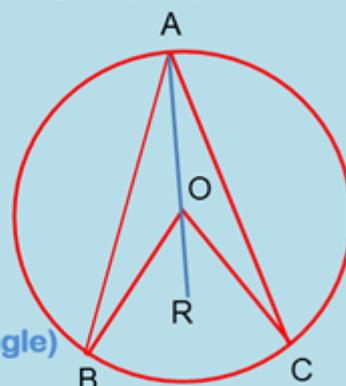
$$|\angle BOR| = |\angle OBA| + |\angle OAB| \quad \text{Theorem 6 (exterior angle)}$$

$$\therefore |\angle BOR| = |\angle OAB| + |\angle OAB|$$

$$\therefore |\angle BOR| = 2|\angle OAB|$$

$$\text{Similarly } |\angle ROC| = 2|\angle OAC|$$

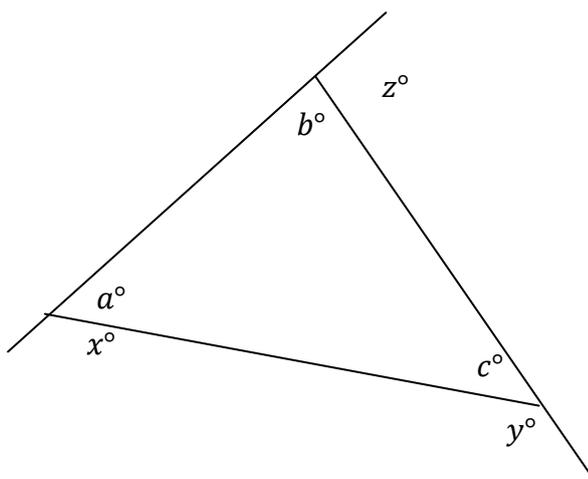
$$\therefore |\angle BOC| = 2|\angle BAC|$$



**Question 15**

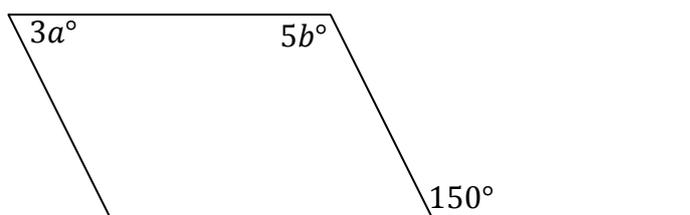
**(16 marks)**

(a) Prove that  $x + y + z = 360$ .



|                              |                              |
|------------------------------|------------------------------|
| $x = b + c$ (external angle) | $a + b + c = 180$ (Triangle) |
| $y = a + b$ (external angle) | $x + y + z = 360$            |
| $z = a + c$ (external angle) |                              |
| $x + y + z = 2(a + b + c)$   |                              |

(b) The diagram below shows a parallelogram and one exterior angle. Find the value of  $a$  and the value of  $b$ .



|                 |
|-----------------|
| $5b = 150(alt)$ |
| $b = 30$        |
| $3a + 5b = 180$ |
| $3a = 30$       |
| $a = 10$        |

## Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

| Scale label      | A     | B         | C             | D                 |
|------------------|-------|-----------|---------------|-------------------|
| No of categories | 2     | 3         | 4             | 5                 |
| 2 mark scale     | 0, 2  | 0, 1, 2   |               |                   |
| 5 mark scale     | 0, 5  | 0, 3, 5   | 0, 3, 4, 5    |                   |
| 10 mark scale    | 0, 10 | 0, 5, 10  | 0, 5, 8, 10   | 0, 5, 8, 9, 10    |
| 15 mark scale    | 0,15  | 0, 10, 15 | 0,10 , 12, 15 | 0, 10, 12, 14, 15 |

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

### Marking scales – level descriptors

#### *A-scales (two categories)*

- incorrect response (no credit)
- correct response (full credit)

#### *B-scales (three categories)*

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

#### *C-scales (four categories)*

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk.

Thus, for example, *scale 10C\** indicates that 9 marks may be awarded.

## Summary of mark allocations and scales to be applied Paper 2

### Question 1 (10)

10 C\*

### Question 2 (17)

(a) 10 C\*

(b) 5 B\*

(c) 2B\*

### Question 3 (30)

(a) 5A\*

(b) 10C\*

(c) 15C\*

### Question 4 (12)

(a) 2B

(b) 5C\*

(c) 5C\*

### Question 5 (25)

(a) 15C

(b) 10C

### Question 6 (27)

(a) 5C\*

(b) 10B

(c) 5B 5B

(d) 2B

### Question 7 (12)

(a) 5A

(b) 2B

(c) 5B

### Question 8 (22)

(a) 5A

(b) 5C

(c) 10B\*

(d) 2B

### Question 9 (32)

(a) 2A

(b) 5C

(c) 5B

(d) &(e) 5C

(f) &(g) 10C

(h) &(i) 5C

### Question 10 (52)

(a) 10C

(b) 5B

(c) 15C

(d) 10B

(e) 10C

(f) &(g) 2B

### Question 11 (7)

(a) 5C

(b) 2B\*

### Question 12 (10)

(a) 10C\*

### Question 13 (7)

(a) 5C

(b) 2B\*

### Question 14 (25)

Diagram 5B

To prove and given 5A

Proof 15D

### Question 15 (12)

(a) 2B

(b) 10C

## Detailed Marking Notes Paper 2

The \* for units or rounding off to be applied only if answered fully right. The \* to be applied once only per question.

### Question 1

Scale 10C\*

*High Partial Credit:*

- 0.078
- 7 800 000

*Low Partial Credit:*

- 0.3 or 0.26 or both written
- $300 \times 260$
- $100 \times 300$  or  $100 \times 200$

### Question 2

(a) Scale 10C \*

*High Partial Credit:*

- $r^2 = \frac{50\,000}{\pi 70}$  or equivalent
- Uses 50 as volume and gets  $r = 4.77$  and/or  $d = 10$

*Low partial Credit:*

- Some correct substitution into Volume formula of cylinder

(b) Scale 5B\*

*Partial Credit:*

- Some correct substitution into  $V = L \times B \times H$

(c) Scale 2B\*

*Partial Credit:*

- Some correct substitution into volume of cylinder formula

### Question 3

(a) Scale 5A\*

(b) Scale 10C\*

*High Partial Credit:*

- Full substitution into volume of spheres and cubes formulae

*Low Partial Credit:*

- Some correct substitution into volume of cube or sphere formulae e.g  $24 \times 6 \times 6$

(c) Scale 15C\*

*High Partial Credit:*

- $864 - 658.2$  or  $205.8$

*Low Partial Credit:*

- Some correct substitution into volume of large cuboid formula
- Shows subtraction of candidate's volume from part (b) answer
- $6 \times 6 \times 6$

#### **Question 4**

(a) Scale 2B

*Partial Credit:*

- Valid reason or correct answer
- Shows some understanding of the need to have enough length for the circumference

(b) Scale 5C\*

*High Partial Credit:*

- $18 \times 23 - [10 \times 22 + 2\pi 3.5^2]$

*Low partial Credit:*

- Some correct substitution into candidate's chosen rectangle and circles from part (a)

(c) Scale 5C\*

*High Partial Credit:*

- Full correct substitution into volume of cylinder formula

*Low partial Credit:*

- Some correct substitution into volume of cylinder formula

#### **Question 5**

(a) Scale 15C

*High Partial Credit:*

- Three or four correct probabilities

*Low partial Credit:*

- One or two correct probabilities

(b) Scale 10C

*High Partial Credit:*

- Three or four correct positions

*Low partial Credit:*

- One or two correct positions

### Question 6

(a) Scale 5C\*

*High Partial Credit:*

- 11 to 16 ages placed correctly on **both** male and female

*Low partial Credit:*

- 1 to 10 or ages placed correctly on either male or female

**Note 1:** It is not necessary to order the stem and leaf

**Note 2:** The \* to be applied if the key is missing

(b) Scale 10B

*Partial Credit:*

- One correct similarity or one correct difference

(c)

(i) Scale 5B

*Partial Credit:*

- Some correct substitution into calculation the mean

(ii) Scale 5B

*Partial Credit:*

- Writes 45 and/or 33
- Some correct substitution into calculation of the median e.g.  $\frac{45 + 46}{2}$

(d) Scale 2B

*Partial Credit:*

- Any correct identification of any lower or upper quartile

### Question 7

(a) Scale 5A

(b) Scale 2B

*Partial Credit:*

- Correct probability for either a red or a head
- Correct sample space made out

(c) Scale 5B

*Partial Credit:*

- Correct probability for either a blue or a tail

### Question 8

(a) Scale 5A

(b) Scale 5C

*High Partial Credit:*

- $500 - (70 + 82 + 90 + 91 + 81)$

*Low Partial Credit:*

- 500 written
- Some correct substitution into sum of the frequencies

**Note:** Correct answer written at part (b) full marks

(c) Scale 10B\*

*Partial Credit:*

- Some correct substitution into calculation of any one relative frequency

**Note:** Correct decimals written in part (c) full marks

(d) Scale 2B

*Partial Credit:*

- Reason not fully correct
- $1/6$  and  $0.14$  written

### Question 9

(a) Scale 2A

(b) Scale 5C

*High Partial Credit:*

- Four points plotted correctly

*Low Partial Credit:*

- One to three points plotted correctly

(c) Scale 5B

*Partial Credit:*

- Some correct substitution into midpoint formula

**Note:**  $(2,0)$  written without work full marks

(d) And (e) Scale 5C

*High Partial Credit:*

- Either part (d) or part (e) fully correct

*Low Partial Credit:*

- Some correct substitution into slope formula (d)
- Writes rise/run
- Some correct substitution into equation of line formula
- 4 written and associated with the y intercept in (e)

(f) And (g) Scale 10C

*High Partial Credit:*

- Either part (f) or part (g) fully correct

*Low Partial Credit:*

- Some correct substitution into the slope formula in (f)
- Writes rise/run in (f)
- Some correct substitution into the equation of line formula (g)
- Carries down the slope from part (f) to part (g) and identifies it as slope

(h) And (i) Scale 5C

*High Partial Credit:*

- Either part (h) or part (i) fully correct

*Low Partial Credit:*

- Any correct substitution into area of triangle formula part (h)
- Ratio written consistent with incorrect areas part (i)
- Writes 'yes' in part (i) but no reason or incorrect reason
- Reason given shows some understanding of concept of congruent triangles

## **Question 10**

(a) Scale 10C

*High Partial Credit:*

- Correct line given but no reason or incorrect reason

*Low Partial Credit:*

- Mentions that steeper the line the greater the slope
- Shows recognition that the  $x$  coefficient is the slope of a line in form  $y = mx + c$

(b) Scale 5B

*Partial Credit:*

- Line 1 and/or line 2 but no reason or incorrect reason
- Shows some understanding of the properties of parallel lines

(c) Scale 15C

*High Partial Credit:*

- (2, 0) and (0, -6) or any two correct points from the line calculated but no line drawn

*Low Partial Credit:*

- 0 substituted (once or twice) into  $y = 3x - 6$
- One of (2, 0) or (0, -6) or any one point from line calculated
- Line drawn consistent with two incorrect points calculated

**Note:** Line drawn correctly without algebra shown for full marks

(d) Scale 10B

*Partial Credit:*

- Some correct substitution into slope formula [using (0, 4) and (2, 0)]
- Some correct substitution into rise/run
- States  $y$ -intercept is 4 and/or slope is  $-2$

**Note:** writes  $y = -2x + 4$  or line 5 without work for full marks

(e) Scale 10C

*High Partial Credit:*

- Slope calculated correctly
- Two points correctly tested on  $y = 4x - 16$  but no conclusion
- One point tested on  $y = 4x - 16$  **with** a conclusion

*Low Partial Credit:*

- Correct line identified without supporting work
- Any one point tested on any one line
- Some correct substitution into slope formula
- Identifies another point or points in the pattern

**Note:** Two points correctly tested with a conclusion gets full marks

**Note:** One point tested correctly on ALL six lines with correct conclusion gets full marks

(f) And (g) Scale 2B

*Partial Credit:*

- Either part (f) or part (g) fully correct
- Writes  $y = x - 7$  and  $y = 4x - 16$  in (f)
- Some correct substitution of candidate's answer from (f) into line 4 or line 6
- Point found in part (f) is verified in (g) using the line used to get the second coordinate

## Question 11

(a) Scale 5C

*High Partial Credit:*

- Sketch drawn with 4 and 10 and A shown but angle not  $23.57^\circ$

*Low Partial Credit:*

- Some indication of 0.4 as being opposite and/or 10 as hypotenuse

**Note 1:** A right-angled triangle drawn with  $A = 23.57^\circ$  is full marks

**Note 2:** 4/10 or 0.4/1 written without any indication of opposite or hypotenuse is 0 marks

(b) Scale 2B\*

*Partial Credit:*

- Some correct use of Pythagoras Theorem
- $\cos A = 9.16/10$
- Some correct use of Cos

**Note:** Allow  $\sqrt{84}$  or  $2\sqrt{21}$  written as a decimal over 10 for \* deduction if surd  $\sqrt{84}$  is shown at some stage

### Question 12

Scale 10C\*

*High Partial Credit:*

- $\sqrt{30^2 + 105^2}$
- $R = 3 \times \sqrt{10^2 + 35^2}$

*Low Partial Credit:*

- Some of the additional 35s or 10s identified
- Some correct use of Pythagoras' theorem
- $3 \times 10$  or  $3 \times 35$
- $R = 10^2 + 35^2$

### Question 13

(a) Scale 5C

*High Partial Credit:*

- $\sqrt{3} = \frac{h}{x}$  OR  $1.732 = \frac{h}{x}$

*Low Partial Credit:*

- Calculates  $\tan 60$  ( $\sqrt{3}$  or 1.732) or  $\tan 30$  ( $\sqrt{3}/3$  or  $1/\sqrt{3}$  or 0.577)
- Some correct substitution into  $\tan 60 = \text{opp/adj}$  or  $\tan 30 = \text{opp/adj}$

(b) Scale 2B\*

*Partial Credit:*

- Some correct substitution into  $\tan 30 = \text{opp/adj}$  or  $\tan 60 = \text{opp/adj}$
- Calculates  $\tan 60$  ( $\sqrt{3}$  or 1.732) or  $\tan 30$  ( $\sqrt{3}/3$  or  $1/\sqrt{3}$  or 0.577)
- $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{24-x}$  OR  $\frac{1}{1.732} = \frac{1.732x}{24-x}$  OR  $0.577 = \frac{1.732x}{24-x}$

**Question 14**

**“Diagram”**

Scale 5B

*Partial Credit:*

- Circle drawn

**“To Prove” and “Given”**

Scale 5A

**“Proof”**

Scale 15D

*High Partial Credit:*

- Transferring to the other triangle
- Reasons needed for steps

*Partial Credit:*

- Identifying the exterior angle as twice the angle in the isosceles triangle

*Low Partial Credit:*

- Some correct work identifying the isosceles triangles and/or the exterior angle

**Question 15**

(a) Scale 2B

*Partial Credit:*

- $x + y + z = 2(a + b + c)$  or  $x + y + z = a + b + c + a + b + c$
- Any one or two or three of the external angle equations identified
- Some correct work on the angle in a straight line adding to  $180^\circ$
- Some correct work on the angles in a triangle adding to  $180^\circ$

(b) Scale 10C

*High Partial Credit:*

- Either a or b correct

*Low Partial Credit:*

- $5b = 150$
- $3a = 30$
- $3a + 5b = 180$
- $3a$  or  $5b$  or  $150$  or  $30$  put in correct place in the parallelogram