Coimisiún na Scrúduithe Stáit
State Examination Commission

Scrúdu
an Teastais Shóisearaigh

JUNIOR CERTIFICATE EXAMINATION

2011

MARKING SCHEME

MATHEMATICS (PROJECT MATHS) HIGHER LEVEL

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Introduction

The Higher Level Mathematics examination for candidates in the 24 initial schools for *Project Maths* shared a common Paper 1 and common material on Paper 2 with the examination for all other candidates. The marking scheme used for the common elements was identical for the two groups.

This document contains the complete marking scheme for both paper for the candidates in the 24 schools.

Readers should note that, as with all marking schemes used in the state examinations, the detail required in any answer is determined by the context and the manner in which the question is asked, and by the number of marks assigned to the question or part. Requirements and mark allocations may vary from year to year.

GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips- numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase **"and stops"** means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The ***same*** **error** in the ***same*** **section** of a question is penalised ***once*** only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5·50 may be written as €5,50.

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MARKING SCHEME

MATHEMATICS (PROJECT MATHS) HIGHER LEVEL PAPER 1

Coimisiún na Scrúduithe Stáit
State Examinations Commission

QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 3,3**
Part (c)	**20 marks**	**Att 2,2,2,2**

Part (a) **10 marks** **Att 3**

Peter and Anne share a lotto prize in the ratio $3\frac{1}{2}$ to $2\frac{1}{2}$.

Peter's share is €35 000.

✍ What is the total prize fund?

(a) **10 marks** **Att 3**

I

$3\frac{1}{2} \ : \ 2\frac{1}{2}$

$3\frac{1}{2} + 2\frac{1}{2}$

$= 6$

€35,000 ÷ $3\frac{1}{2}$

€10,000 × 6 or € 10,000 ×2½ = €25,000
€35,000 + €25,000

The total prize fund = €60,000

II

$3\frac{1}{2} \ : \ 2\frac{1}{2}$

7 : 5

€35,000 ÷ 7 = €5,000

7 + 5 = 12

€5,000 × 12 or €5,000 × 5 = €25,000
€35,000 + €25,000

The total prize fund = €60,000

* Other methods may be used

* €10,000 is 4 marks; €25,000 is 7 marks; €20,416·67 is 4 marks (B3 & B4 with work)

Blunders (-3)

B1 Correct answer no work shown. ✍

B2 Fails to finish - stops at €10,000 ×6, or €5,000 × 12 or €35,000 + €25,000 or equivalent

B3 Incorrect divisor

B4 Incorrect multiplier

B5 Decimal error

Slips (-1)
S1 Numerical errors to a maximum of −3

Misreadings (-1)
M1 Incorrect digit which does not oversimplify the question

Attempts (3 marks)
A1 $3\frac{1}{2} + 2\frac{1}{2}$ and stops
A2 6 or 5,000 or 25,000 or 10,000 or 12 with no work shown
A3 Any relevant step

Worthless (0)
W1 Incorrect answer no work shown, but note A2
W2 Work of no merit

Part (b) **20 (10,10) marks** **Att (3,3)**

(i)	The diameters of Venus and Saturn are $1\cdot21\times10^4$ km and $1\cdot21\times10^5$ km. ✍ What is the difference between the diameters of the two planets? Give your answer in the form of $a\times10^n$ where $n\in\mathbb{Z}$ and $1\le a<10$.
(ii)	✍ Write $\dfrac{\sqrt{3}\times27}{3^2}$ in the form of 3^n where $n\in\mathbb{Q}$.

(b) (i) **10marks** **Att3**

I

$1\cdot21\times10{,}000$

12,100 **3m**

↕ Interchangeable

$1\cdot21\times100{,}000$

121,000 **4m**

121,000 – 12,100

108,900 **7m**

Difference: $1\cdot089\times10^5$ **10m**

II

$(1\cdot21\times10^5)-(1\cdot21\times10^4)$ **3m**

108,900 **7m**

$1\cdot089\times10^5$ **10m**

* One correct calculation 3 marks, two correct calculations 4 marks, subtraction 7 marks, conversion 10 marks

* 108,900 **only** is B1 and B5

Blunders (-3)

B1 Correct answer no work shown. ✍

B2 Reversed subtraction

B3 Index error

B4 Misplaced decimal

B5 Answer not given in required form

Slips (-1)

S1 Numerical errors to a maximum of −3

Misreadings (-1)

M1 Incorrect digit provided it doesn't oversimplify the question

Attempts (3 marks)

A1 10,000 or 100,000 or 12,100 or 121,000

A2 Digits 1089 with incorrectly placed decimal or index without work

A3 Any relevant step

Worthless (0)

W1 Incorrect answer, no work shown

W2 $1\cdot21\times10^1$ or $1\cdot21\times10^9$

W3 Work of no merit

(b) (ii) **10 marks** **Att3**

I

$$\frac{\sqrt{3}\times 27}{3^2}$$

$(3^{½} \times 3^3) \div 3^2$

$3^{7/2} \div 3^2$

$3^{3/2}$

II

$$\frac{1{\cdot}732050808 \times 27}{9}$$

$$\frac{46{\cdot}7653718}{9}$$

5·196152423 **3m**

$5{\cdot}196152423 = 3^{3/2}$ **10m**

* Accept $3^{1\cdot 5}$ or $3^{1½}$ for full marks.

Blunders (-3)

B1 Correct answer no work shown. ✍

B2 Each index error

B3 Incorrect operation

B4 Fails to finish

Slips (-1)

S1 Numerical error to a maximum of −3

Misreadings (-1)

M1 Misreads a digit provided it doesn't oversimplify

Attempts (3 marks)

A1 $27 = 3^3$

A2 $\sqrt{3} = 3^{½}$

A3 1·732

A4 $3^2 = 9$

A5 $\sqrt{}$ = power of ½

A6 5·196152423

A7 $3\sqrt{3}$

A6 Some relevant step

Worthless (0)

W1 Incorrect answer with no work

W2 Work of no merit

Part (c) **20 (5,5,5,5) marks** **Att 2,2,2,2**

(i) ✍ By rounding to the nearest whole number estimate the value of

$$\frac{\sqrt{(7\cdot17)^2+14\cdot59}}{8\cdot29-1\cdot64\times2\cdot23}.$$

Then evaluate $\dfrac{\sqrt{(7\cdot17)^2+14\cdot59}}{8\cdot29-1\cdot64\times2\cdot23}$, correct to one decimal place.

(ii) Úna and Conor were travelling to South Africa.
They bought 5760 rand in the bank.
The bank charged them €630, which included a 5% service charge.

✍ What was the value of the euro in rand (the exchange rate) on that day?

(c) (i) Estimate **5 marks** **Att2**

$$\frac{\sqrt{(7\cdot17)^2+14\cdot59}}{8\cdot29-1\cdot64\times2\cdot23}$$

$$\frac{\sqrt{7^2+15}}{8-2\times2}$$

$$\frac{\sqrt{49+15}}{8-4}$$

$$\frac{\sqrt{64}}{4}$$

$$\frac{8}{4}$$

$= \quad 2$

Blunders (-3)

- B1 Correct answer no work shown. ✍
- B2 Precedent error (*i.e.* incorrect order)
- B3 Mishandles square root
- B4 Incorrect squaring
- B5 Incorrect use of indices
- B6 Decimal error
- B7 Mathematical error
- B8 Calculates first, then rounds (*i.e.* 1·8, rounded to 2)

Slips (-1)

- S1 Numerical errors to a maximum of −3
- S2 Incorrect rounding to a max of −3 if it affects answer

Misreadings (-1)

- M1 Misreads a digit, provided it doesn't oversimplify the question

Attempts (2 marks)
A1 Some correct rounding
A2 Any correct step without rounding

Worthless (0)
W1 1·8 without work
W2 Work of no merit

(c) (i) Evaluate **5 marks** **Att 2**

$$\frac{\sqrt{(7\cdot17)^2+14\cdot59}}{8\cdot29-1\cdot64\times2\cdot23}$$

$$\frac{\sqrt{51\cdot4089\ +14\cdot59}}{8\cdot29-3\cdot6572}$$

$$\frac{\sqrt{65\cdot9989}}{4\cdot6328}$$

$$\frac{8\cdot123970704}{4\cdot6328}$$

1·7535 = 1·8

Blunders (-3)
B1 Apply once in (c) (i). Correct answer with no work shown. ✍
B2 Mishandles square root
B3 Incorrect squaring
B4 Precedent error
B5 Incorrect use of indices
B6 Decimal error

Slips (-1)
S1 Numerical error to a maximum of −3
S2 Incorrect or no rounding, apply once if it affects final answer

Misreadings (-1)
M1 Misreads a digit, provided it doesn't oversimplify the question

Attempts (2 marks)
A1 Any relevant step *e.g. squaring, multiplying, square root etc.*

Worthless (0)
W1 Incorrect answer no work shown
W2 Work of no merit

(c) (ii) **10 (5,5) marks** **Att 2,2**

I 105% = €630 1% = €630 ÷ 105 1% = €6 100% = €600 **5m** €600 = 5760 rand €1 = 5760 ÷ 600 €1 = 9·6 rand **5m** Value of euro in rand: €1 = 9·6 rand **II** 5760 rand × 105% = 6048 rand **5m** €630 = 6048 rand €1 = 6048 ÷ 630 €1 = 9·6 rand **5m**

* Accept final answer €1 = 9·6 rand with some work for 10 marks
* Two parts in marking this question; dealing with the 5% and the conversion in any order. 5 marks each (but note first *)

Blunders (-3)

B1 Correct answer no work shown. ✍
B2 Incorrect operation
B3 Inverted division
B4 Decimal error

Slips (-1)
S1 Numerical error to a maximum of −3

Misreadings (-1)
M1 Misreads a digit provided it doesn't oversimplify the question

Attempts (2,2 marks)
A1 105% or 100% + 5%
A2 105% = €630 and stops
A3 Any relevant step

Worthless (0)
W1 Incorrect answer no work shown
W2 Work of no merit

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**25 marks**	**Att 3,3,2**
Part (c)	**15 marks**	**Att 2,2,2**

Part (a)	**10 marks**	**Att 3**

A computer salesperson is paid an annual salary of €30 000.
He is also paid a commission of 4% on sales.
Last year the salesperson earned €38 000.

✍ Calculate the value of the sales.

(a)	**10 marks**	**Att 3**

I

€38,000 − €30,000 = €8000
Commission of 4% = €8000
1% = €8,000 ÷ 4
1% = €2,000
100% = Sales = €200,000

II

€30,000 + 4% of Sales = €38,000
€30,000 + ·04 Sales = €38,000
·04 Sales = €38,000 − €30,000
·04 Sales = €8,000
Sales = €8,000 ÷ ·04
Sales = €200,000

Blunders (-3)

B1 Correct answer no work shown. ✍
B2 Decimal error
B3 Percentage error
B4 Incorrect transposition
B5 Mathematical error
B6 Expresses % as an incorrect fraction and continues
B7 In Method **I,** stops at €2,000
B8 Fails to finish

Slips (-1)

S1 Numerical errors to a maximum of −3

Misreadings (-1)

M1 Incorrect figure if it doesn't oversimplify the question *e.g.* uses €36,000

Attempts (3 marks)

A1 Indicates some knowledge of percentages *e.g.* 4% = 4/100 or ·04
A2 Subtraction involving €38,000 and €30,000 or €8,000 mentioned without work
A3 Any relevant step

Worthless (0)

W1 Incorrect answer no work shown
W2 Adds €30,000 and €38,000
W3 Adds or subtracts 4 and €30,000 or 4 and €38,000
W4 Work of no merit

Part (b) **25 (10,10,5) marks** **Att 3,3,2**

Aoife is single and earned €40 000 last year. Aoife's tax credits are listed below.

Single Person Tax Credit	€1830
PAYE Tax Credit	€1830
Rent Allowance Tax Credit	€400
Trade Union Payment Tax Credit	€70

(i) ✍ Calculate Aoife's total tax credits.

The standard rate cut-off point for a single person was €36 400.
The standard rate of income tax was 20% and the higher rate was 41%.

(ii) ✍ Calculate the tax paid by Aoife on her income.

Aoife also had to pay a 2% income levy on her gross income.

(iii) ✍ Calculate Aoife's net income after all deductions had been made.

(b) (i) **10 marks** **Att 3**

€1,830 + €1,830 + €400 + €70 =	**7m**
Total tax credits = €4,130	**10m**

Blunders (-3)

B1 Correct answer no work shown. ✍
B2 Omits one tax credit
B3 Addition indicated but fails to complete

Slips (-1)

S1 Numerical error to a maximum of −3

Misreadings (-1)

M1 Incorrect number written *e.g.* €1,380 *etc.* provided it doesn't oversimplify the question

Attempts (3 marks)

A1 Adds two numbers together from list
A2 Any relevant step

Worthless (0)

W1 Incorrect answer no work shown
W2 Work of no merit

(b) (ii) **10 marks** **Att 3**

€40,000 − €36,400 = €3,600	**3m**	
€36,400 × 20% = €7,280	**3m**	
		↕ Interchangeable
€3,600 × 41% = €1,476	**4m**	
€7,280 + €1,476 = €8,756	**7m**	
€8,756 − €4,130	**7m**	
The tax paid = €4,626	**10m**	

* Accept candidate's tax credit figure from (b)(i)

* If candidate gets 41% of €36,400 (€14,924) and 20% of €3,600 (€720) **and** continues correctly, this is one blunder (Total tax €15,644 minus tax credits €4,130 = €11,514 is worth 7 marks)

Blunders (-3)

B1 Correct answer no work shown. ✍
B2 Decimal error
B3 Percentage error
B4 20% of an incorrect figure but note *2
B5 41% of an incorrect figure but note *2
B6 Mishandles tax credits
B7 Mathematical error
B8 Fails to finish

Slips (-1)
S1 Numerical error to a maximum of −3

Misreadings (-1)
M1 Uses 21%
M2 Uses 40% or 42%

Attempts (3 marks)
A1 Finds 20% or 41% of any number and stops
A2 Writes 20% as 20/100, 1/5 or ·2 without any further work of merit
A3 Writes 41% as 41/100 or 0·41 without any further work of merit
A4 Some knowledge of tax paid *e.g.* writes tax paid = total tax – tax credits
A5 €7,280 **or** €1,476 with or without work
A6 Any relevant step

Worthless (0)
W1 Incorrect answer no work shown, but note A2, A3 and A5
W2 Work of no merit

(b) (iii) **5 marks** **Att 2**

€40,000 × 2% = €800 €4,626 + €800 = € 5,426 €40,000 − € 5,426 Net income = €34,574

* Accept candidate's "tax paid" figure from (b) (ii)

Blunders (-3)

B1 Correct answer no work shown. ✍
B2 Decimal error
B3 Percentage error
B4 Finds 2% of incorrect figure and continues
B5 Mathematical error
B6 Fails to finish
B7 Ignores "tax paid" figure when calculating net income

Slips (-1)
S1 Numerical errors to a maximum of −3

Attempts (2 marks)
A1 Finds 2% of any number and stops
A2 2% = ·02 or 2/100 or 1/50
A3 Demonstrates some knowledge of net income *e.g.* Net income = Gross – Tax
A4 Demonstrates some knowledge of income levy *e.g.* Levy = Gross × %
A5 Any relevant step

Worthless (0)
W1 Incorrect answer no work shown, but note A2
W2 Work of no merit

Part (c) **15 (5,5,5) marks** **Att 2,2,2**

U is the universal set and P and Q are two subsets of U.
$\# U = 30$, $\#P = 16$ and $\# Q = 6$.

(i) ✍ Find with the aid of a Venn diagram the minimum value of $\# (P \cup Q)'$.

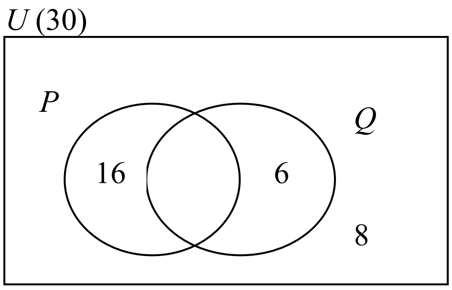
(ii) ✍ Find with the aid of a Venn diagram the maximum value of $\# (P \cup Q)'$.

$\# U = u$, $\#P = p$, $\# Q = q$ and $\# (P \cup Q)' = x$.

(iii) ✍ Show with the aid of a Venn diagram, that if $p > q$ and x is a maximum, then $u = p + x$.

(c) (i) **5 marks** **Att 2**

$16 + 6 = 22$
$30 - 22 = 8$



Minimum value of $\# (P \cup Q)' = 8$

* Accept correct Venn diagram for full marks
* Ignore notation

Blunders (-3)

B1 Correct answer no work shown. ✍
B2 Incorrect operation
B3 Venn diagram correct but no minimum included or stated

Slips (-1)

S1 Correct value of 8 for minimum with work and no Venn diagram or incorrect Venn diagram

Attempts (2 marks)

A1 Venn diagram
A2 16 + 6 and stops
A3 30 – 6 or 30 – 16 and stops
A4 Any relevant step

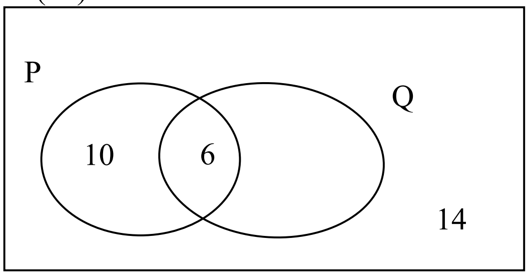
Worthless (0)

W1 Incorrect answer no work shown
W2 Work of no merit

(c) (ii) **5 marks** **Att 2**

16 – 6 = 10
30 – 16 = 14

U (30)



Maximum value of # ($P \cup Q$) ′= 14

* Accept correct Venn diagram for full marks
* Ignore notation

Blunders (-3)

B1 Correct answer no work shown. ✍
B2 Incorrect operation
B3 Venn diagram correct but no maximum included or stated

Slips (-1)

S1 Correct value of 14 for maximum with work and no Venn diagram or incorrect Venn diagram

Attempts (2 marks)

A1 Venn diagram
A2 16 – 6
A3 Any relevant step

Worthless (0)

W1 Incorrect answer no work shown
W2 Work of no merit

(c) (iii) 5 marks Att 2

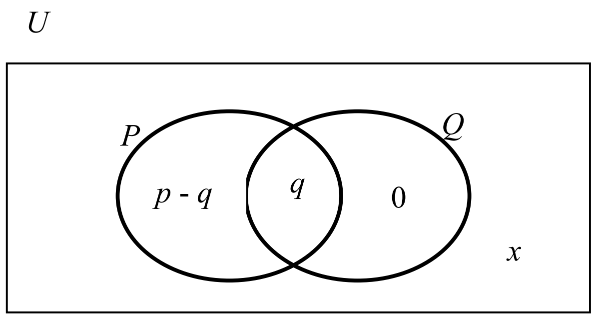
0 in Q only for x to be a maximum

$u = p - q + q + x$

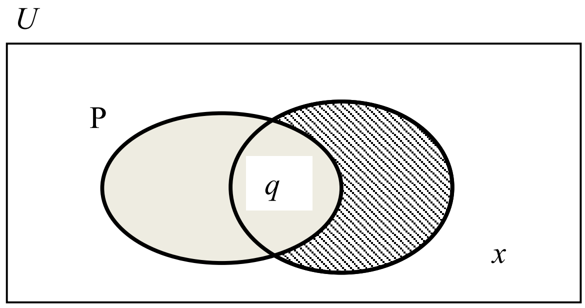
$u = p + x$

Venn diagram to give: $\boldsymbol{u = p + x}$

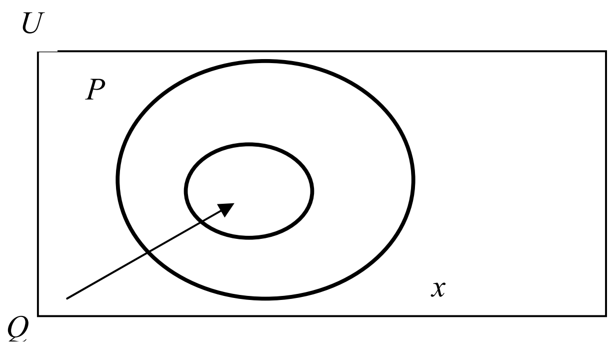
I



II



III



* Accept correct Venn diagram for full marks **with conclusion** $u = p + x$

* Ignore notation

Blunders (-3)

B1 Omits term from equation

B2 Fails to finish

B3 # Q/P ≠ 0

B4 #P/Q ≠ $p - q$

B5 # (P ∩ Q) ≠ q

Slips (-1)

S1 $u = p - q + q + x \rightarrow u = p + x$ only. No Venn diagram.

S2 No conclusion

Attempts (2 marks)

A1 Venn diagram

A2 $p - q$

A3 Any relevant step

Worthless (0)

W1 Incorrect answer no work shown

W2 $u = p + x$ only (Given)

W3 No work of merit

QUESTION 3

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 3,3**
Part (c)	**20 marks**	**Att 2,2,2,2**

Part (a) **10 marks** **Att 3**

> ✍ Given that $t^2 - s = r$, express t in terms of r and s.

(a) **10 marks** **Att 3**

$t^2 - s = r$	**Given**
$t^2 = r + s$	**7m**
$t = \sqrt{r+s}$	**10m**

* Two steps in this question, transposition and square root

* $t - s = r$ and continues correctly to get $t = r + s$ is worth 6 marks - Misread and B2.

* Finds s correctly in terms of t and r is 6 marks - Misread and B2. ($s = t^2 - r$)

Blunders (-3)

B1 Correct answer no work shown. ✍

B2 Mishandles or fails to get square root

B3 Incorrect operation *e.g.* May attempt to square everything

B4 Transposition error

Misreadings (-1)

M1 Note *2 and *3

M2 $t^2 + s = r$ and continues correctly to get $t = \sqrt{r - s}$

Attempts (3 marks)

A1 Effort at square root

A2 $t^2 - s - r$ (with or without = 0)

A3 Any relevant step

Worthless (0)

W1 Incorrect answer no work shown

W2 Work of no merit

Part (b) **20 (10,10) marks** **Att (3,3)**

> **(i)** ✍ Divide $3x^2 + 5x - 28$ by $x + 4$.
>
> **(ii)** ✍ Solve the equation $\dfrac{4x+2}{5} - \dfrac{6-x}{3} = -5$.

(b) (i) **10 marks** **Att3**

I

$3x^2 + 5x - 28 \div x + 4$

$$\frac{(3x-7)(x+4)}{x+4}$$

$= 3x - 7$

II

$3x^2 + 5x - 28 \div x + 4$

$3x^2 + 12x - 7x - 28 \div x + 4$

$3x(x + 4) + 7(x + 4) \div x + 4$

$(3x - 7)(x + 4) \div x + 4$

$= 3x - 7$

III

Division to give answer $3x - 7$

$$\begin{array}{r|l} & \quad 3x - 7 \\ \cline{2-2} x + 4 & 3x^2 + 5x - 28 \\ & \underline{3x^2 + 12x} \\ & \qquad -7x - 28 \\ & \qquad \underline{-7x - 28} \\ & \qquad\qquad 0 \end{array}$$

* $(3x+7)(x-4)$ and continues is one blunder (B4), will also incur B5 **or** B6. All other attempts to factorise apply B2, B3 and/or B4.

Blunders (-3)

B1 Correct answer no work shown. ✍

B2 Incorrect factors of $3x^2$ in method **I**

B3 Incorrect factors of -28 in method **I**

B4 Incorrect factors leading to an incorrect middle term in method **I**

B5 Fails to finish *i.e.* no cancellation in method **I**

B6 Incorrect cancellation

B7 Mathematical error, once if consistent

Slips (-1)

S1 Numerical errors to a maximum of -3

Attempts (3 marks)

A1 Some effort at factorising
A2 Sets up division
A3 Multiplies instead of dividing, with at least one correct term
A4 Finds guide number ($-$ 84) in method **II** and stops
A5 Quadratic with some correct substitution
A6 Sets up quadratic and identifies *a,b* or *c*
A7 Uses quadratic formula and stops at correct roots ($x = -4$ and $x = 7/3$)
A8 Any relevant step

Worthless (0)

W1 Incorrect answer no work shown
W2 (   ) (   )
W3 Work of no merit

(b) (ii) **10 marks** **Att3**

I

$$\frac{4x+2}{5} - \frac{6-x}{3} = -5$$

$$\frac{3(4x+2) - 5(6-x)}{15} = -5$$

$3(4x + 2) - 5(6 - x) = 15\,(-5)$

$12x + 6 - 30 + 5x = -75$

$17x - 24 = -75$

$17x = -75 + 24$

$17x = -51$

$x = -51 \div 17$

$x = -3$

II

$$\frac{4x+2}{5} - \frac{6-x}{3} = -5$$

$$\frac{(4x+2)(3)(5)}{5} - \frac{(6-x)(3)(5)}{3} = -5(3)(5)$$

$3(4x + 2) - 5(6 - x) = 15\,(-5)$

$12x + 6 - 30 + 5x = -75$

$17x - 24 = -75$

$17x = -75 + 24$

$17x = -51$

$x = -51 \div 17$

$x = -3$

III

$$\frac{4x+2}{5} - \frac{6-x}{3} = -5$$

$3(4x + 2) - 5(6 - x) = (5)(3)\,(-5)$

$12x + 6 - 30 + 5x = -75$

$17x - 24 = -75$

$17x = -75 + 24$

$17x = -51$

$x = -51 \div 17$

$x = -3$

* $x = -3$, by trial and error or similar, **fully verified** merits 10 marks

Blunders (-3)

B1 Correct answer no work shown. ✍

B2 Distribution error, once if consistent

B3 Incorrect common denominator or mishandles denominator

B4 Transposition error, once if consistent

B5 Mathematical error

B6 Mishandles numerator

B7 Combines unlike terms and continues

Slips (-1)
S1 Numerical errors to a maximum of −3

Attempts (3 marks)
A1 Common denominator and stops
A2 Oversimplified but some correct work
A3 Cross multiplies
A4 Any relevant step

Worthless (0)
W1 Incorrect answer no work shown
W2 Adds or subtracts terms incorrectly *e.g.* $5 + 3$, or $4x + 2 \pm 6 - x$ etc.
W3 Work of no merit

Part (c) **20 (5,5,5,5) marks** **Att 2,2,2,2**

A car park can accommodate cars and mini-buses.
On a particular day there were x cars and y mini-buses in the car park, giving a total of 520 vehicles.
The parking area for a car is 7 m^2 and the parking area for a mini-bus is 12 m^2.
On that day a total area of 3840 m^2 was occupied by cars and mini-buses.

(i) Write down two equations to represent the above information.

(ii) ✍ Solve these equations to find the number of cars and the number of mini-buses in the car park on that day.

There is a flat rate charge per day for parking.
The flat rate for mini-buses is 3 times that for cars. On that day €3000 was taken in.

(iii) ✍ What is the flat rate for cars?

(c) (i) **10 (5,5) marks** **Att 2,2**

$$x + y = 520 \quad \textbf{5m}$$
$$7x + 12y = 3840 \quad \textbf{5m}$$

* Two equations to mark in (c) (i)
* Each equation is marked separately
* Each equation is worth 5 marks, attempt 2
* Answer is sufficient for full marks (No ✍ in question)

Blunders (-3)
B1 Incorrect term

Attempts (2,2 marks)
A1 x or y or 7 x or 12 y
A2 Effort at creating an equation equal to 520 or 3,840
A3 Any relevant step

(c) (ii) **5 marks** **Att 2**

I

$$x + y = 520 \quad (-7)$$
$$7x + 12y = 3840$$

$$-7x - 7y = -3640$$
$$\underline{7x + 12y = 3840}$$
$$5y = 200$$
$$y = 200 \div 5$$
$$y = 40$$

$$x + y = 520$$
$$x + 40 = 520$$
$$x = 520 - 40$$
$$x = 480$$
$x = 480$ and $y = 40$.

II

$$\begin{aligned} x + y &= 520 \quad (-12) \\ 7x + 12y &= 3840 \end{aligned}$$

$$\begin{aligned} -12x - 12y &= -6240 \\ \underline{7x + 12y} &= \underline{3840} \\ -5x &= -2400 \\ 5x &= 2400 \\ x &= 2400 \div 5 \\ x &= 480 \end{aligned}$$

$$\begin{aligned} 480 + y &= 520 \\ y &= 520 - 480 \\ y &= 40 \end{aligned}$$

$x = 480$ and $y = 40$.

III

$$\begin{aligned} x &= 520 - y \\ 7(520 - y) + 12y &= 3840 \\ 3640 - 7y + 12y &= 3840 \\ -7y + 12y &= 3840 - 3640 \\ 5y &= 200 \\ y &= 200 \div 5 \\ y &= 40 \end{aligned}$$

$$\begin{aligned} x + y &= 520 \\ x + 40 &= 520 \\ x &= 520 - 40 \\ x &= 480 \end{aligned}$$

$x = 480$ and $y = 40$.

IV

$$\begin{aligned} y &= 520 - x \\ 7x + 12(520 - x) &= 3840 \\ 7x + 6240 - 12x &= 3840 \\ 7x - 12x &= 3840 - 6240 \\ -5x &= -2400 \\ 5x &= 2400 \\ x &= 2400 \div 5 \\ x &= 480 \end{aligned}$$

$$\begin{aligned} 480 + y &= 520 \\ y &= 520 - 480 \\ y &= 40 \end{aligned}$$

$x = 480$ and $y = 40$

*1 Equations may be also solved by substituting $y = \frac{3840 - 7x}{12}$ or $x = \frac{3840 - 12y}{7}$

*2 Accept candidate's equations from (c) (i) provided oversimplification does not occur

*3 Apply only one blunder in establishing the first equation in terms of x only or the first equation in terms of y only.

*4 Finding the second variable is subject to a maximum deduction of 3 marks

*5 Correct values of x and y without algebraic work, **both verified in both equations** merits full marks

*6 Correct values of x and y without algebraic work **not verified or not fully verified in both equations** merits attempt mark only

Blunders (-3)
B1 Finds one variable only
B2 Distribution error
B3 Mathematical error
B4 Incorrect substitution when finding second variable, but note M1
B5 Transposition error in solving first variable
B6 Transposition error in solving second variable
B7 Error(s) in establishing the first equation in terms of x ($-5x = -2400$) only or the first equation in terms of y ($-5y = -200$) only through elimination by cancellation **I** and **II**
B8 Error(s) in establishing the first equation in terms of x ($-5x = -2400$) only or the first equation in terms of y ($5y = 200$) only through elimination by substitution **III** and **IV**

Slips (-1)
S1 Numerical errors to a max of -3

Misreadings (-1)
M1 Misreads digits, providing it doesn't oversimplify

Attempts (2 marks)
A1 Any correct manipulation of either given equation and stops
A2 Some correct partial substitution and stops
A3 Any relevant step

Worthless (0)
W1 Incorrect answer, no work shown
W2 Trial and error, but see *5 and *6 above
W3 Work of no merit

(c) (iii) **5 marks** **Att 2**

$3x : x$

$480x + 3x(40) = 3{,}000$
$480x + 120x = 3{,}000$
$600x = 3{,}000$
$x = 3{,}000 \div 600$
$x = 5$

Flat rate for cars = €5

* Accept candidate's answers from (c) (ii)

Blunders (-3)

B1 Correct answer no work shown. ✍
B2 Mathematical error
B3 Uses ratio Buses: Cars 1:3, but note M1

Slips (-1)
S1 Numerical errors to a maximum of −3

Misreadings (-1)
M1 Misreads values for buses and cars (*i.e.* uses 40 cars and 480 buses – note B3)

Attempts (2 marks)
A1 States ratio 3:1 and stops
A2 Some correct partial substitution and stops
A3 3000 ÷ 4 or 750 or 3 + 1
A4 Some relevant step *e.g.* divides 3,000

Worthless (0)
W1 Incorrect answer no work shown
W2 Trial and error with incorrect value(s)
W3 Work of no merit

QUESTION 4

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 2,3,2**
Part (c)	**20 marks**	**Att 2,2,2,2**

Part (a) **10 marks** **Att 3**

Graph on the number line the solution set of

$$-2x + 1 > -7, \; x \in \mathbb{N}.$$

(a) **10 marks** **Att 3**

I

$-2x + 1 > -7$

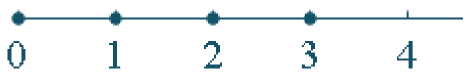
$-2x > -7 - 1$

$-2x > -8$

$2x < 8$

$x < 4$

$x \in \{1,2,3\}$



II

$-2x + 1 > -7$

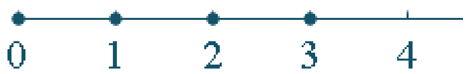
$1 + 7 > 2x$

$8 > 2x$

$4 > x$

$x < 4$

$x \in \{1,2,3\}$



* Accept $x \in \{0, 1, 2, 3\}$ plotted

* Graph must be based on candidate's inequality

Blunders (-3)

B1 Correct answer no work shown.

B2 Mishandles inequality

B3 Fails to graph

B4 Transposition error

B5 Mathematical error

B6 $x \in R$ indicated

B7 $2x + 1 > -7$ solved to get $x > -4$ with correct graph

Slips (-1)

S1 Numerical errors to a max of −3
S2 Includes 4 on graph
S3 Each incorrect or missing number to a maximum of 3

Misreadings (-1)

M1 Includes equals in inequality

Attempts (3 marks)

A1 Tests any value in inequality and stops
A2 Draws any number line
A3 Any relevant step

Worthless (0)

W1 List given with no correct value
W2 Work of no merit

Part (b) 20 (5,10,5) marks Att 2,3,2

(i)	Factorise $x^2 - 1$.
(ii)	✍ Factorise fully $ax - 3 - a + 3x$.
(iii)	Factorise $6x^2 + x - 35$

(b) (i) **5 marks** **Att 2**

$x^2 - 1 =$	**Given**
$x^2 - 1^2 =$	**2m**
$(x - 1)(x + 1)$	**5m**

* Answer is sufficient for full marks (No ✍ in question)
* Accept also (with or without brackets) for full marks any of the following:
  $(x - 1)$ and $(x + 1)$ [The word 'and' is written down]
  $(x - 1)$ or $(x + 1)$ [The word 'or' is written down]
  $(x - 1), (x + 1)$ [a comma is used]
* Quadratic equation method is subject to slips and blunders

Blunders (-3)
B1 Incorrect factors of x^2
B2 Incorrect factors of -1
B3 $(1 - x)(1 + x)$
B4 Answer left as roots $x = \pm 1$

Slips (-1)
S1 $(x - 1) + (x + 1)$
S2 $(x - 1) - (x + 1)$

Attempts (2 marks)
A1 $x^2 - 1^2$
A2 Correct factors of x^2 only
A3 Correct factors of 1 or -1 only
A4 $\pm x$ or ± 1
A5 $x^2 - 1 = x \times x - 1 \times 1$
A6 Difference of two squares mentioned
A7 $\sqrt{1}$
A8 $\sqrt{x^2}$
A9 Correct quadratic formula with some correct substitution

Worthless (0)
W1 Combines terms incorrectly and stops *e.g.* $-1\,x^2$
W2 Work of no merit

(b) (ii) **10 marks** **Att 3**

I

$a x - 3 - a + 3 x$ **Given**
$a x - a + 3 x - 3$ **3m**
$a (x - 1) + 3 (x - 1)$ **7m**
$(a + 3) (x - 1)$ **10m**

II

$a x - 3 - a + 3 x$ **Given**
$ax + 3x - a - 3$ **3m**
$x (a + 3) - 1 (a + 3)$ **7m**
$(a + 3) (x - 1)$ **10m**

* Accept also (with or without brackets) for full marks any of the following with work
$(a + 3)$ and $(x - 1)$ [The word 'and' is written down]
$(a + 3)$ or $(x - 1)$ [The word 'or' is written down]
$(a + 3), (x - 1)$ [A comma is used]

Blunders (-3)

B1 Correct answer no work shown. ✍
B2 Failure to complete last step *e.g.* stops at $a(x - 1) + 3(x - 1)$
B3 Error in factorising any pair of terms, apply once if consistent
B4 Incorrect last step *e.g.* $3a (x - 1)$ or $(a + 3) (- 1x)$
B5 Incorrect common factor and continues *e.g.* $x (a + 3) + 1 (-a -3)$. B4 will also apply.

Slips (-1)
S1 $(a + 3) + (x - 1)$
S2 $(a + 3) - (x - 1)$

Attempts (3 marks)
A1 Pairing off matching terms, or indication of common factors and stops
A2 Correctly factorises any pair and stops
A3 Any relevant step

Worthless (0)
W1 ( )( )
W2 Pairing of terms with nothing in common *e.g.* $a x - 3$ and no further work of merit
W3 Work of no merit

(b) (iii) **5 marks** **Att 2**

I
$6x^2 + x - 35$
$(2x + 5)\ (3x - 7)$

II
$6x^2 + x - 35$
$6x^2 - 14\,x + 15\,x - 35$
$2\,x\,(3\,x - 7) + 5\,(3\,x - 7)$
$(2x + 5)\ (3x - 7)$

III
$6\,x^2 + x - 35$
$6x^2 + 15\,x - 14\,x - 35$
$3x\,(2\,x + 5) - 7\,x\,(2\,x + 7)$
$(2x + 5)\ (3x - 7)$

* Answer is sufficient for full marks (No ✍ in question)

* Quadratic may be used to solve $6x^2 + x - 35 = 0 \;\rightarrow\; x = -\frac{5}{2}, x = \frac{7}{3}$ and continues

* Accept also (with or without bracket) for full marks any of the following
$(2x + 5)$ and $(3x - 7)$ [The word 'and' is written down]
$(2x + 5)$ or $(3x - 7)$ [The word 'or' is written down]
$(2x + 5), (3x - 7)$ [A comma is used]

Blunders (-3)
B1 Incorrect factors of $6x^2$
B2 Incorrect factors of -35
B3 Factors leading to an incorrect middle term
B4 Substitution error in quadratic
B5 Uses quadratic to get roots and stops

Slips (-1)
S1 $(2x + 5) + (3x - 7)$
S2 $(2x + 5) - (3x - 7)$

Attempts (2 marks)
A1 Some correct factors
A2 Identifies *a,b* or *c* for quadratic
A3 Quadratic with some correct substitution
A4 Any correct relevant step

Worthless (0)
W1 Incorrect answer no work shown but note attempts
W2 ( )( )
W3 Quadratic formula only
W4 Work of no merit

Part (c) **20 (5,5,5,5) marks** **Att 2,2,2,2**

The new Lansdowne Road stadium has seating capacity for 200 journalists.
It was decided initially that this seating would be in x rows of equal value.

(i) Write, in terms of x, the number of seats per row required to accommodate the 200 journalists.

During the construction it was decided to have 3 fewer rows to accommodate the 200 journalists.

(ii) Write, in terms of x, the number of seats per row now required.
It was found that 15 extra seats per row were required compared to the initial plan.

(iii) ✍ Write an equation using the above information and solve for x.

(c) (i) **5 marks** **Att 2**

$$\frac{200}{x}$$

* Answer is sufficient for full marks (No ✍ in question)

Misreadings (-1)
M1 Uses letter other than x

Blunders (-3)
B1 Inversion

Attempts (2 marks)
A1 Effort at forming expression using 200 and x

Worthless (0)
W1 x only or 200 or similar

(c) (ii) **5 marks** **Att 2**

$$\frac{200}{x-3}$$

* Answer is sufficient for full marks (No ✍ in question)

Misreadings (-1)
M1 Uses letter other than x if not penalised already

Blunders (-3)
B1 Inversion, but do not penalise if already blundered in (c) (i)

B2 $\frac{200}{x+3}$

B3 $\frac{200}{x} \pm 3$ (Linear in (c)(iii) and subject to further penalty there)

Attempts (2 marks)
A1 Forms an incorrect expression with at least two of the following x, 3, 200

(c) (iii) **10 (5,5) marks** **Att 2,2**

Establish equation

$\dfrac{200}{x-3} - \dfrac{200}{x} \quad = \quad 15$	**2m**
$\dfrac{200\,x - 200\,(x-3) = 15\,x\,(x-3)}{x\,(x-3)}$	**2m**
$200\,x - 200\,x + 600 = 15x^2 - 45\,x$	**2m**
$600 = 15x^2 - 45\,x$	**5m**
$15x^2 - 45\,x - 600 = 0$	
$x^2 - 3\,x - 40 = 0$	

Solve

$(x-8)(x+5) = 0$	**2m**
$\rightarrow$ 8 and -5	**4m**
Solution: $x = 8$	**5m**

* Mark in two parts: 5 marks for **equation** and 5 marks for **solving**

* Accept candidate's expressions from (c) (i) and (ii). Linear merits Att 2,2 at most

Blunders (-3)

B1 Correct answer no work shown. ✍

B2 Distribution error - apply each time but once if consistent

B3 Transposition error - apply each time but once if consistent

B4 Mathematical error in forming equation *e.g.* line 1

B5 Incorrect factors

B6 Correct factors and stops, will also incur S2

B7 Error in quadratic formula

B8 Each error in grouping terms or fails to group

B9 Error in establishing equation *e.g.* line 1

Slips (-1)

S1 Numerical errors to a maximum of 3

S2 Stops at $x = 8$, $x = -5$ or concludes $x = -5$

Attempts (2,2 marks)

A1 Linear equation merits attempt marks at most

A2 Trial and error merits attempt at most

A3 Any correct relevant step

Worthless (0)

W1 Incorrect answer no work shown

W2 (   )(   )

W3 Substitution of any number other than 8 or −5

QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 3,3**
Part (c)	**20 marks**	**Att 2,3,2**

Part (a) **10 marks** **Att 3**

✍ Given that $f(x) = 3x - 4$ and that $f(k) = 11$, find the value of k.

(a) **10 marks** **Att 3**

$3k - 4 = 11$		**3m**
$3k$	$= 11 + 4$ *or* 15	**4m**
k	$= 15 \div 3$	**7m**
k	$= 5$	**10m**

* Ignore notation (*e.g.* $x = 5$ or answer $= 5$ for full marks)
* $k = 5$ fully verified = 10 marks

Blunders (-3)
B1 Correct answer no work shown. ✍
B2 Transposition error
B3 $3k - 4 = 0$ and continues correctly
B4 Incorrect operation

Slips (-1)
S1 Numerical error to a maximum of 3

Attempts (3 marks)
A1 Fills in k and stops
A2 Attempt to divide by 3
A3 Tests values *e.g.* $k = 11$ to get answer of 29
A4 Any relevant step

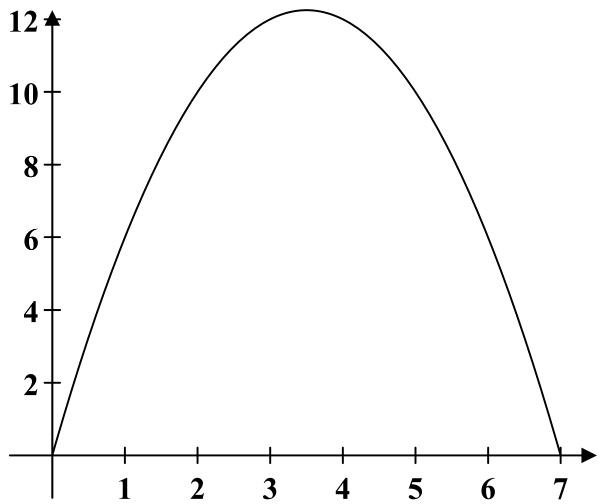
Worthless (0)
W1 Incorrect answer with no work shown

Part (b) **20 (10,10) marks** **Att 3,3**

Let f be the function $f : x \to 7x - x^2$.

✍ Draw the graph of f for $0 \le x \le 7$, $x \in \mathbb{R}$.

(b) Function *f* **20 (10,10) marks** **Att 3,3**



x	0	1	2	3	4	5	6	7
$7x$	0	7	14	21	28	35	42	49
$-x^2$	0	−1	−4	−9	−16	−25	−36	−49
$f(x)$	0	6	10	12	12	10	6	0

or

x	0	1	2	3	4	5	6	7
$f(x)$	0	6	10	12	12	10	6	0

or

$f : x \to 7x - x^2.$
$f : x \to 7(0) - 0^2 = 0 - 0 = 0$
$f : x \to 7(1) - 1^2 = 7 - 1 = 6$
$f : x \to 7(2) - 2^2 = 14 - 4 = 10$
$f : x \to 7(3) - 3^2 = 21 - 9 = 12$
$f : x \to 7(4) - 4^2 = 28 - 16 = 12$
$f : x \to 7(5) - 5^2 = 35 - 25 = 10$
$f : x \to 7(6) - 6^2 = 42 - 36 = 6$
$f : x \to 7(7) - 7^2 = 49 - 49 = 0$

(0.0) (1,6) (2, 10) (3, 12) (4,12) (5, 10) (6, 6) (7,10)

* Table is worth 10 marks, graph is worth 10 marks
* Middle lines of table do not have to be shown
* Candidates may choose not to use a table
* Points might not be listed, mark on position on graph

* Graph constitutes work in this question ✍

Blunders (-3)
B1 Error in calculating $7x$, once if consistent
B2 Error in calculating $-x^2$, once if consistent, but note A1
B3 Error in calculating last line of table, once if consistent
B4 Each incorrect point without work
B5 Point plotted incorrectly, once if consistent
B6 Each missing point
B7 Axes scaled incorrectly, once only
B8 Reversed axes
B9 No curve between (3,12) and (4,12) on graph
B10 Points not joined, most probably incurs B9 also

Slips (-1)
S1 Numerical error to a maximum of -3

Attempts (3,3 marks)
A1 Error leading to a linear graph
A2 Some correct substitution
A3 Draws axes, with some indication of scaling

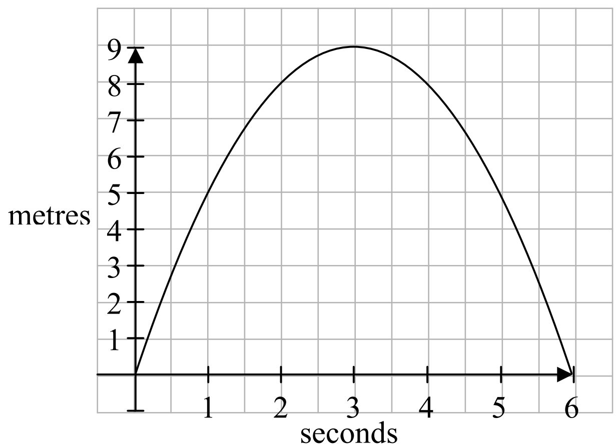
Part (c) **20 (5,10,5) marks** **Att2,3,2**

The formula for the height, y metres, of a golf ball above ground level x seconds after it is hit, is given by $7x - x^2$.
Use your graph from part **(b)**:

(i) ✍ to find the maximum height reached by the golf ball

(ii) ✍ to estimate the number of seconds the golf ball was more than 2 metres above the ground.

The graph below represents the flight of another golf ball.
The flight of the golf ball is given by the formula $ax - x^2$, $x \in \mathbb{R}$.



(iii) ✍ Find the value of a.

(c) (i) **5 marks** **Att 2**

Maximum height → 12·25 m

* Accept answer consistent with candidate's graph, tolerance ± ·2

Blunders (-3)

B1 Correct answer no indication on graph. ✍
B2 Maximum indicated on graph but no value given
B3 Outside of tolerance
B4 States x co-ordinate of maximum point
B5 Fails to use graph

Slips (-1)
S1 Accept maximum as point if y value is correct *i.e.* (3·5, 12·25)

Attempts (2 marks)
A1 Reads maximum from table
A2 Some relevant substitution in effort to find maximum
A3 Uses graph of (c) (iii) to find maximum of 9
A4 x = 3· 5 indicated on graph

Worthless (0)
W1 Incorrect answer no work, but note attempts

(c) (ii) **10 marks** **Att 3**

More than 2 metres above the ground→ 6·7 – 0·3= 6·4 secs

* Accept values from candidate's graph with a tolerance of ± ·2

Blunders (-3)
B1 No subtraction
B2 Value(s) not consistent with candidate's graph
B3 No indication on graph
B4 Indication on graph but no value given each time. B1 also applies
B5 Outside of tolerance, each time
B6 ·3 − 6·7 **= − 6·4** or candidate's equivalent

Slips (-1)
S1 Numerical error

Attempts (3 marks)
A1 Correctly solves $f(x)$ = 2 by formula; graph not used
A2 f (2) found (answer = 10, or candidate's equivalent)
A3 Uses graph of (c)(iii) to find answer (5·7 − ·3 = 5·4)

Worthless (0)
W1 Incorrect answer, no work shown

(c) (i) **5 marks** **Att 2**

Fills in any of the following points (1,5) (2,8) (3,9) (4, 8) (5, 5) (6,0)
or any other correct points to solve equation $ax - x^2 = y$

I

e.g. (1, 5)
$a(1) - (1)^2 = 5$
$a - 1 = 5$
$a = 5 + 1$
Value of $a = 6$

II

$ax - x^2 = y$
$x(a - x) = y$
Fill in *e.g.* (1, 5)
$1(a - 1) = 5$
$a - 1 = 5$
$a = 5 + 1$
Value of $a = 6$

Blunders (-3)

B1 Correct answer no work shown. ✍
B2 Co-ordinates reversed when substituting
B3 Incorrect squaring
B4 Transposition error
B5 Substitutes for a instead of x
B6 Fails to finish
B7 Incorrect factors

Slips (-1)

S1 Numerical error to a maximum of −3

Attempts (2 marks)

A1 Some relevant substitution
A2 $ax - x^2 =$ ***f(x)*** or $ax - x^2 =$ **y**
A3 Writes down any point on the curve
A4 Attempt to factorise

Worthless (0)

W1 Incorrect answer no work shown
W2 No work of merit

QUESTION 6

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 3,2,2**
Part (c)	**20 marks**	**Att 3,2,2**

Part (a) **10 marks** **Att 3**

✍ When $a = \frac{1}{4}$, find the value of $\frac{a+5}{3} - \frac{a+4}{2}$.

(a) **10 marks** **Att 3**

I

$$\frac{a+5}{3} - \frac{a+4}{2}$$

$$\frac{\frac{1}{4}+5}{3} - \frac{\frac{1}{4}+4}{2}$$

$$\frac{5\frac{1}{4}}{3} - \frac{4\frac{1}{4}}{2}$$

$$1\tfrac{3}{4} - 2\tfrac{1}{8}$$

$$= \frac{-3}{8}$$

II

$$\frac{a+5}{3} - \frac{a+4}{2}$$

$$\frac{2(a+5) - 3(a+4)}{6}$$

$$\frac{2a + 10 - 3a - 12}{6}$$

$$\frac{-a-2}{6}$$

$$a = \tfrac{1}{4}$$

$$\frac{-\frac{1}{4} - 2}{6}$$

$$\frac{-2\frac{1}{4}}{6}$$

$$-\frac{3}{8}$$

* Accept answer in decimal format (− ·375) or equivalent fraction

Blunders (-3)

B1 Correct answer no work shown. ✍

B2 Incorrect denominator

B3 Mishandles denominator

B4 Mishandles numerator

B5 Mathematical error

B6 Distribution error, once if consistent

B7 Fails to combine like terms (if it affects final answer)

B8 Combines unlike terms

B9 Incomplete step (*e.g.* − 2¼/6 or similar)

Slips (-1)

S1 Numerical errors to a maximum of −3

Attempts (3 marks)

A1 Some correct substitution

A2 No denominator used

A3 Any correct relevant step

Worthless (0)

W1 Incorrect answer no work shown

W2 No work of merit

Part (b)	**20 (10,5,5) marks**	**Att 3,2,2**

(i) ✍ Express in its simplest form:

$$\frac{4}{x-1} - \frac{5}{x+2}.$$

(ii) ✍ Hence, or otherwise, solve the equation:

$$\frac{4}{x-1} - \frac{5}{x+2} = \frac{3}{2},$$

giving your answers correct to one decimal place.

(b)(i)	**10 marks**	**Att 3**

$$\frac{4(x+2) - 5(x-1)}{(x-1)(x+2)}$$

$$\frac{4x + 8 - 5x + 5}{(x-1)(x+2)}$$

$$\frac{-x + 13}{(x-1)(x+2)}$$

* Accept common denominator as $(x - 1)(x+2)$. Penalise incorrect multiplication in (b) (ii)

Blunders (-3)

B1 Correct answer no work shown. ✍
B2 Incorrect denominator
B3 Mishandles denominator
B4 Mishandles numerator
B5 Mathematical error
B6 Distribution error, once if consistent
B7 Fails to combine like terms
B8 Combines unlike terms
B9 Reads as $\frac{4}{x-1} + \frac{5}{x+2}$. Continue to apply slips and blunders

Slips (-1)
S1 Numerical errors to a maximum of −3

Attempts (3 marks)
A1 Identifies common denominator and stops
A2 No denominator used
A3 Oversimplification
A4 Any relevant step

Worthless (0)
W1 Incorrect answer no work shown

W2 Adds or subtracts numerators and denominators *e.g.* $\frac{9}{2x+1}$ or $-\frac{1}{1}$ or $-\frac{1}{2x+1}$ *etc*

(b)(ii) **10 (5,5) marks** **Att 2,2**

$$\frac{-x+13}{(x-1)(x+2)} = \frac{3}{2}$$

$2(-x+13) = 3(x-1)(x+2)$

$-2x + 26 = (3x-3)(x+2)$

$-2x + 26 = 3x^2 + 3x - 6$

$3x^2 + 5x - 32 = 0$ **Equation 5m**

$\dfrac{-5 \pm \sqrt{25-4(3)(-32)}}{2(3)}$

$\dfrac{-5 \pm \sqrt{25+384}}{6}$

$\dfrac{-5 \pm \sqrt{409}}{6}$

$\dfrac{-5 \pm 20{\cdot}22374842}{6}$

$-\dfrac{25{\cdot}22374842}{6}$ and $\dfrac{15{\cdot}22374842}{6}$

$x = -4{\cdot}2$ and $x = 2{\cdot}5$ **Solve 5m**

* Mark in two parts: 5 marks for **equation** and 5 marks for **solving**

* Accept candidate's expressions from (b) (i). Linear merits Att 2,2 at most

Blunders (-3)

B1 Correct answer no work shown. ✍

B2 Distribution error - apply each time but once if consistent

B3 Transposition error - apply each time but once if consistent

B4 Mathematical error in forming equation

B5 Incorrect denominator

B6 Mishandles denominator

B7 Mishandles numerator

B8 Fails to combine like terms

B9 Combines unlike terms

B10 Error in quadratic formula

B11 Error in application of quadratic formula

B12 Finds only one solution

Slips (-1)

S1 Numerical errors to a maximum of −3

S2 Fails to round or rounds incorrectly

Attempts (2,2 marks)

A1 Linear equation merits attempt marks at most

A2 Quadratic formula with some correct substitution

A3 Trial and error of correct solution(s) merits attempt at most

A4 Any relevant step

Worthless (0)

W1 Incorrect answer no work shown

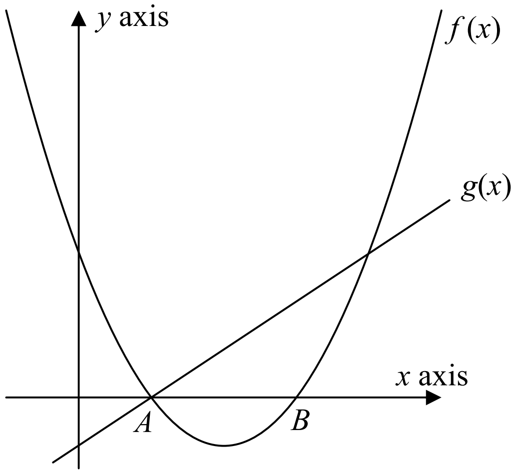
W2 (   ) (   )

W3 Substitution of any number other than 2·5 and − 4·2 or equivalent

W4 No work of merit

Part (c)	20 (10,5,5) marks	Att 3,2,2

The diagram below shows part of the graphs of the functions
$f(x) = x^2 - 4x + 3$ and $g(x) = x + k$.



The graph of $f(x)$ cuts the x axis at A and B.
The graphs of $f(x)$ and $g(x)$ intersect at A.

(i) ✍ Find the coordinates of A and the coordinates of B.

(ii) ✍ Find the value of k.

(iii) ✍ Verify that $f(x)$ and $g(x)$ intersect also at the point (4, 3).

(c) (i)	10 marks	Att3

	Solve $x^2 - 4x + 3 = 0$	**3m**
	$(x - 1)(x - 3) = 0$	**4m**
	$x = 1, x = 3$	**7m**
	(1, 0) *and* (3, 0)	**9m**
→	A (1, 0) B (3, 0)	**10m**

* If trial and error is used, must be fully verified for 10 marks

Blunders (-3)
B1 Correct answer no work shown. ✍
B2 Incorrect factors of x2
B3 Incorrect factors of +3
B4 Factors leading to an incorrect middle term
B5 Fails to find roots
B6 Fails to list coordinates with y = 0 included
B7 Uses quadratic and stops at roots
B8 (0,1) and (0,3) Reversed co-ordinates.

Slips (-1)
S1 Fails to specify A **or** B or incorrectly names A and B, apply once.

S2 After solving $x = 1$, $x = 3$ only states one point *i.e* (1,0) or (3,0). May also incur S1.

Attempts (3 marks)

A1 Some correct factors
A2 Identifies *a, b* or *c* for quadratic
A3 Quadratic with some correct substitution
A4 Finds where graph cuts y axis (0,3)
A5 $f(x) = 0$ or $g(x) = 0$ or $(x , \mathbf{0})$
A6 Substitution of 0,1 or 3
A7 Any correct relevant step
A8 Uses graph to read answer

Worthless (0)

W1 Incorrect answer no work shown

(c) (ii) **5 marks** **Att 2**

$g(1) = 1 + k = 0$	**2m**
→ $k = -1$	**5m**

* Accept (4,3) or candidate's A co-ordinate from (c) (i)

Blunders (-3)

B1 Correct answer no work shown. ✍
B2 Fails to let $g(x) = 0$
B3 Fails to let $x = 1$ (or candidate's equivalent value)
B4 Transposition error

Attempts (2 marks)

A1 Substitutes $x = 1$ (or candidate's equivalent) and stops
A2 $g(x) = 0$
A3 Any relevant step

Worthless (0)

W1 Incorrect answer no work shown

(c) (iii)	5 marks	Att 2

	I	
Solve	$x^2 - 4x + 3 = x - 1$	**2m**
	$x^2 - 4x + 3 - x + 1 = 0$	
	$x^2 - 5x + 4 = 0$	
	$(x - 4)(x - 1) = 0$	
	$x = 1$ and $x = 4$	
	$x^2 - 4x + 3 = y$ *or* $x - 1 = y$	
	$(4)^2 - 4(4) + 3 = y$ $\quad 4 - 1 = y$	
	$16 - 16 + 3 = 3 = y$ $\quad 3 = y$	
	$\rightarrow$ point (4, 3)	**5m**
	II	
	Substitute $x = 4$ into $f(x) = x^2 - 4x + 3$ **and** $g(x) = x - 1$	
	$f(x) = x^2 - 4x + 3$	
	$f(x) = (4)^2 - 4(4) + 3$	**2m**
	$f(x) = 16 - 16 + 3$	
	$f(x) = 3$	
	(4,3)	
	$g(x) = x - 1$	
	$g(x) = 4 - 1$	
	$g(x) = 3$	
	(4,3)	**4m**
	(4,3) on both lines → point of intersection	**5m**

* Accept $g(x) = x + k$, based on candidate's k value from (c) (ii)

Blunders (-3)
B1 Fails to equate $f(x)$ and $g(x)$ in method **I**
B2 Incorrect squaring
B3 Transposition error
B4 Does not substitute into second function
B5 Fills in $x = 1$
B6 Fails to finish

Slips (-1)
S1 Does not conclude in method **II**
S2 Numerical errors to a maximum of 3

Attempts (2 marks)
A1 Some relevant substitution
A2 Linear equation attempt at most in Method I
Worthless (0)
W1 Incorrect answer no work shown
W2 $x - 1 = 0$
W3 $x^2 - 4x + 3 = 0$
W4 Work of no merit

JUNIOR CERTIFICATE EXAMINATION

2011

MARKING SCHEME

MATHEMATICS (PROJECT MATHS) HIGHER LEVEL PAPER 2

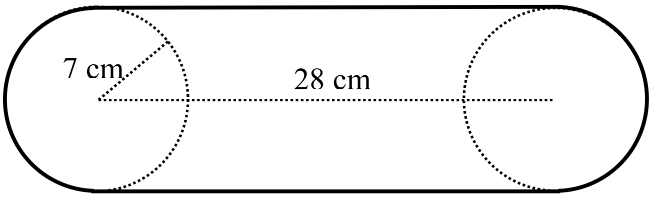
QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10,5,5)marks**	**Att (3,2,2)**
Part (c)	**20 (5,5,10) marks**	**Att (2,2,3)**

Part (a) **10 marks** **Att 3**

The diagram shows two pulley wheels of equal size, connected by a drive belt.
The radius of each wheel is 7 cm and the distance between the centres is 28 cm.

✍ Calculate the length of the belt.
Give your answer correct to the nearest whole number.



(a) **10 marks** **Att 3**

Semicircular lengths	=	$2\pi r$	=	14π	Step 1
Straight lengths	=	2(28)	or	56	Step 2
Total length	=	$14\pi + 56$			
	=	99·98			
	=	100 cm			Step 3

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect relevant formula
B3 Incorrect substitution into correct formula
B4 Incorrect r
B5 Failure to add
B6 Value of π which affects the accuracy of the answer

Slips (-1)
S1 Arithmetic slips to a maximum of *(-3)*
S2 Answer not rounded or incorrectly rounded

Attempts (3 marks)
A1 Correct perimeter formula - $2\pi r + 2l$
A2 Correct formula with some correct substitution
A3 Any use of 28

Worthless (0)
W1 Area of rectangle and/or disc

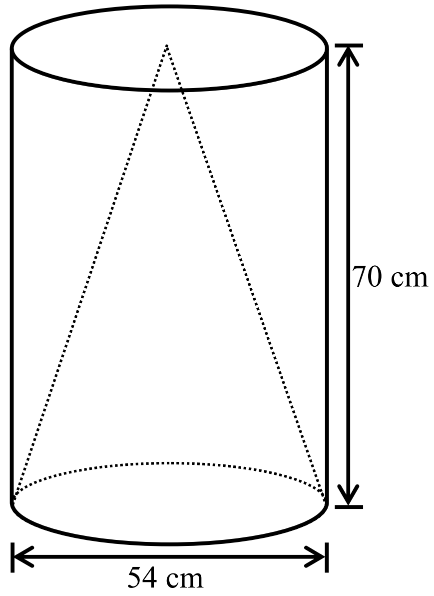
Part (b) **20 (10,5,5) marks** **Att (3,2,2)**

The diagram shows a solid cylinder of diameter 54 cm and of height 70 cm.
A cone, of the same diameter and height as the cylinder,is cut from inside the cylinder.

(i) ✍ Calculate the volume of the cylinder.
Give your answer in terms of π.

(ii) ✍ Calculate the volume of the cone.
Give your answer in terms of π.

(iii) ✍ What fraction of the cylinder remains after the cone is removed?



(b) (i) **10 marks** **Att 3**

Volume of cylinder	=	$\pi r^2 h$
	=	$\pi(27)^2(70)$
	=	51030π cm^3

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect relevant formula
B3 Incorrect substitution into correct formula
B4 Incorrect r
B5 Incorrect h
B6 Incorrect squaring

Slips (-1)
S1 Arithmetic slips to a maximum of *(-3)*
S2 Answer not in terms of π

Attempts (3 marks)
A1 $r = 27$
A2 Indication that radius length is half diameter length
A3 Correct formula with some correct substitution

Worthless (0)
W1 Surface area formula for cylinder

(b) (ii) **5 marks** **Att 2**

Volume of cone	=	$\frac{1}{3}\pi r^2 h$
	=	$\frac{1}{3}\pi(27)^2(70)$
	=	17010π cm^3
		or
Volume of cone	=	$\frac{1}{3}(51030\pi)$
	=	17010π cm^3

* Accept candidate's answer from (b) (i)

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect relevant formula
B3 Incorrect substitution into correct formula
B4 Incorrect *r*
B5 Incorrect *h*
B6 Incorrect squaring

Slips (-1)
S1 Arithmetic slips to a maximum of *(-3)*
S2 Answer not in terms of π

Attempts (2 marks)
A1 $r = 27$
A2 Indication that radius length is half diameter length
A3 Indication that volume of the cone is $\frac{1}{3}$(volume of the cylinder)
A4 Correct formula with some correct substitution

Worthless (0)
W1 Surface area formula for cone

(b) (iii) **5 marks** **Att 2**

Remainder	=	$51030\pi - 17010\pi$
	=	34020π
Fraction	=	$\dfrac{34020\pi}{51030\pi}$
	=	$\dfrac{34020}{51030}$ or $\dfrac{2}{3}$
	or	
$1-\dfrac{1}{3}$	=	$\dfrac{2}{3}$

* Accept candidate's answer from (b) (i) and (ii)

Blunders (-3)

B1 Correct answer without work shown (✍)

B2 $\dfrac{\text{Volume of cone}}{\text{Volume of cylinder}}$

Slips (-1)

S1 Arithmetic slips to a maximum of *(-3)*

S2 π not cancelled

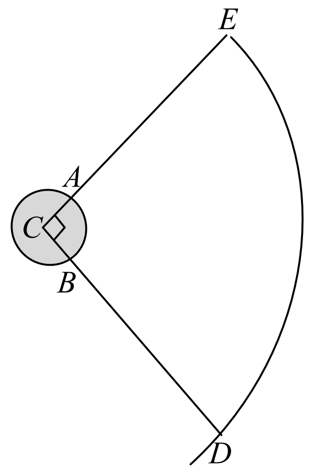
Attempts (2 marks)

A1 Effort at Remainder

Part (c) **20 (5,5,10) marks** **Att (2,2,3)**

The diagram, not to scale, represents a shot-put zone in an athletics stadium.
The area of *CDE* is a quarter of the area of a disc of centre *C* and of radius 100 m.

(i) ✍ Calculate the area of *CDE*, correct to two decimal places.



The shot-put zone consists of a throwing zone and a landing zone.
The throwing zone (shaded) is a disc of centre *C* and of radius 1 m.

(ii) ✍ Calculate the area of the throwing zone, correct to two decimal places.

The landing zone is the unshaded area *ABDE*, which is part of *CDE*.

(iii) ✍ Calculate the total area of the shot-put zone, correct to two decimal places.

(c) (i) **5 marks** **Att 2**

$$\text{Area of } CDE = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(100)^2$$
$$= 2500\pi$$
$$= 7853{\cdot}9816$$
$$= 7853{\cdot}98 \text{ m}^2$$

* Value of π used, other than value of π from calculator, giving an answer in the range 7850 - 7857·14 incurs −1

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect relevant formula
B3 Incorrect substitution into correct formula
B4 Value of π which affects the accuracy of the answer
B5 Answer in terms of π
B6 Incorrect *r*
B7 Incorrect squaring

Slips (-1)
S1 Arithmetic slips to a maximum of *(-3)*
S2 Answer not rounded or incorrectly rounded

Attempts (2 marks)
A1 Correct formula with some correct substitution

Worthless (0)
W1 Length formula

(c) (ii) **5 marks** **Att 2**

Area of throwing zone	=	πr^2	=	$\pi(1)^2$
			=	1π
			=	3·1416
			=	3·14 m^2

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect relevant formula
B3 Incorrect substitution into correct formula
B4 Value of π which affects the accuracy of the answer
B5 Answer in terms of π
B6 Incorrect r
B7 Incorrect squaring

Slips (-1)
S1 Arithmetic slips to a maximum of *(-3)*
S2 Answer not rounded or incorrectly rounded

Attempts (2 marks)
A1 Correct formula with some correct substitution

Worthless (0)
W1 Length formula

(c) (iii) **10 marks** **Att 3**

Area of the shot-put zone	=	$\frac{3}{4}$(Area of throwing zone) + Area *CDE*
$\frac{3}{4}$(Area of throwing zone)	=	$\frac{3}{4}$(3·14) = 2·355
Area of shot-put zone	=	2·355 + 7853·98
	=	7856·335
	=	7856·34
	or	
$\frac{3}{4}$(Area of throwing zone)	=	0·75π
Area of shot put zone	=	0·75π+2500π = 2500·75π
	=	7856·337828
	=	7856·34 m^2

or

$\frac{1}{4}$(Area of throwing zone)	=	$\frac{1}{4}(3{\cdot}14)$	=	$0{\cdot}785$
Area of shot put zone	=	$7853{\cdot}98 + 3{\cdot}14 - 0{\cdot}785$		
	=	$7856{\cdot}335$		
	=	$7856{\cdot}34$		

or

$\frac{1}{4}$(Area of throwing zone)	=	$0{\cdot}25\pi$		
Area of shot put zone	=	$2500\pi + 1\pi - 0{\cdot}25\pi$	=	$2500{\cdot}75\pi$
	=	$7856{\cdot}337828$		
	=	$7856{\cdot}34\ m^2$		

* Accept candidate's answers from (c) (i) and (ii)

Blunders (-3)

B1 Correct answer without work shown (✍)

B2 Area $CDE - \frac{1}{4}$(Area of throwing zone)

B3 Area CDE + Area of throwing zone

B4 Value of π which affects the accuracy of the answer

B5 Answer in terms of π

Slips (-1)

S1 Arithmetic slips to a maximum of *(-3)*

S2 Answer not rounded or incorrectly rounded

Attempts (3 marks)

A1 Indication of $\frac{3}{4}$ or $\frac{1}{4}$ of area of throwing zone

A2 Correct formula with some correct substitution

Model Solutions
(Questions 2 – 15)

Note that the model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.

Question 2

The percentage distribution of blood groups in the Irish population is given in the table below. The table also gives information about which types of blood can be safely used when people need to be given blood during an operation.

Blood Group	Percentage in Irish population	Blood groups to which transfusions can be safely given.	Blood groups from which transfusions can be safely received.
O-	8	All	O-
O+	47	O+, AB+, A+, B+	O+ and O-
A-	5	A-, A+, AB+, AB-	A- and O-
A+	26	A+ and AB+	A+, O-, O+, A-
B-	2	B-, B+, AB-, AB+	B- and O-
B+	9	B+ and AB+	B+, B-, O-, O+
AB-	1	AB- and AB+	AB-, O-, A-, B-
AB+	2	AB+	all

Source: Irish Blood Transfusion Service

(a) If an Irish person is chosen at random, what is the probability that that person will have blood group AB-?

$$\frac{1}{100}$$

(b) Mary has blood group B-. If a person is chosen at random from the population, what is the probability that Mary could safely receive blood from that person?

B- (2%) + O-(8%) = 10%

OR B- (2/100) + O-(8/100) = 10/100

$$\frac{10}{100}$$

(c) Aaron has blood group O+ and donates blood. What is the probability that his blood can be given to a person randomly chosen from the population?

O+(47%) + AB+(2%) + A+(26%) + B+(9%) = 84%

$$\frac{84}{100}$$

(d) The *Irish Blood Transfusion Service* recently asked that people with blood group O- should give blood as regularly as possible. Give a reason why this might be the case.

O- can only receive blood from other O- people. This is only 8% of the population, therefore this category needs to be encouraged to donate blood.

or

O- can safely give blood to all other groups and so is the best to have if there is any shortage of blood.

Question 3

The colour of 500 cars that pass a particular set of traffic lights during a two hour period is recorded by a group of students.

Colour	Frequency	Relative frequency	Daily frequency (Part **(e)** below)
Red	70	$^{70}/_{500}$ or 0·14	336
Blue	100	$^{100}/_{500}$ or 0·2	480
Yellow	45	$^{45}/_{500}$ or 0·09	216
White	55	$^{55}/_{500}$ or 0·11	264
Black		$^{90}/_{500}$ or 0·18	432
Silver	140	$^{140}/_{500}$ = 0·28	672
Total	500	$^{500}/_{500}$ or 1	2400

(a) Calculate the number of black cars and write it into the table.

$$\begin{aligned} &500 - (70+100+45+55+140) \\ &= 500 - 410 \\ &= 90 \text{ [black cars]} \end{aligned}$$

(b) Calculate the relative frequency of each colour and write these into the table.

Done in table

(c) Suggest a method to check that your relative frequency calculations are correct. Perform this check.

Method: The sum of the relative frequencies should total to 1
OR The percentages should sum to 100%
Check: Candidate to show his/her check

(d) What is the probability that the next car to pass the lights is red?

$$\frac{70}{500} = 0{\cdot}14 = 14\%$$

(e) Use the information to estimate the frequency of each colour if 2400 cars pass the lights in a full day. Write this information into the table.

$$\frac{70}{500} \times 2400 = 336 \qquad \frac{55}{500} \times 2400 = 264$$

$$\frac{100}{500} \times 2400 = 480 \qquad \frac{90}{500} \times 2400 = 432$$

$$\frac{45}{500} \times 2400 = 216 \qquad \frac{140}{500} \times 2400 = 672$$

OR

$2400 \div 500 = 4.8$
$70 \times 4{\cdot}8 = 336$
$100 \times 4{\cdot}8 = 480$
$45 \times 4{\cdot}8 = 216$
$55 \times 4{\cdot}8 = 264$
$90 \times 4{\cdot}8 = 432$
$140 \times 4{\cdot}8 = 672$

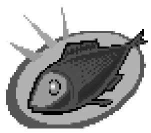
(f) The data collected by the students is not a random sample of the cars passing throughout the day. Do you think that this makes your estimates in **(e)** above unreliable? Give a reason for your answer.

No. A test is reliable if repeated runs of the test would give the same results. There is no reason to say that if this test was run again it would be different because of the sample not being random. The colour of a vehicle is random and running the test at different times of the day or on different days would not necessarily make the test any more reliable.

Question 4

A restaurant advertises its lunch menu using the sign below.

3 course lunch for €15



Choose from our range of
starters, main courses and desserts

180 different lunches to choose from!

(a) The menu has a choice of five starters and nine main courses. How many items must appear on the dessert menu to justify the above claim of 180 different lunches?

5 x 9 = 45 180÷45 = 4 [dessert choices]	OR 180 ÷ 5 = 36 36 ÷ 9 = 4 [dessert choices]	OR 180 ÷ 5 = 36 36 ÷ 9 = 4 [dessert choices]

(b) On a particular day one of the starters and one of the main courses is not available. How many different three course lunches is it possible to have on that day?

$S \times M \times D$
$4 \times 8 \times 4 = 128$ [different 3-course lunches]

Question 5
The table below shows the distances travelled by seven paper airplanes after they were thrown.

Airplane	A	B	C	D	E	F	G
Distance (cm)	188	200	250	30	380	330	302

(a) Find the median of the data. Median = 250 OR C

(b) Find the mean of the data.

$$\frac{188+200+250+30+380+330+302}{7} = \frac{1680}{7} = 240[cm]$$

(c) Airplane D is thrown again and the distance it travels is measured and recorded in place of the original measurement. The median of the data remains unchanged and the mean is now equal to the median. How far did airplane D travel the second time?

250×7 = 1750
1680–30 =1650
1750–1650=100 cm

OR
1750–1680 =70
70+30 =100 cm

OR

$$\frac{188+200+250+30+380+330+302+x}{7} = 250$$

$$\frac{1650+x}{7} = 250$$

$$1650+x = 1750$$

$$x = 100 \text{ cm}$$

(d) What is the minimum distance that airplane D would need to have travelled in order for the median to have changed?

The minimum distance is anything greater than 250 cm
OR $x > 250$ cm, $x \in R$

Question 6

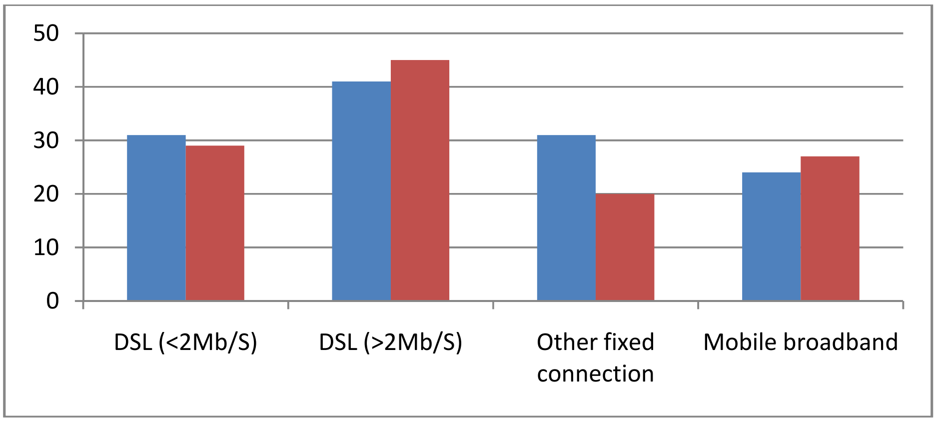
Data on the type of broadband connection used by enterprises in Ireland for 2008 and 2009 is contained in the table below.

	2008	2009
	%	%
Broadband connection	84	84
By type of connection		
DSL (<2Mb/S)	31	29
DSL (>2Mb/S)	41	45
Other fixed connection	31	20
Mobile broadband	24	27

Source: Central Statistics Office

(a) Display the data in a way that allows you to compare the data for the two years.

Bar Charts



(b) Identify any trends that you think are shown by the data.

- The 'fixed connection' went down a lot
- The DSL>2Mb (faster connection) went up
- The DSL<2Mb (slower connection) went down
- No increase in broadband connection
- Mobile broadband went up slightly

These my not be exhaustive.

Question 7

John's third-year Physical Education class did a fitness test. The number of sit-ups that each student did in one minute is recorded below:

59	48	27	53	36	29	52	46	45	37	49	51
33	45	38	52	40	51	37	44	47	45	60	41

The students practiced this exercise for the next three weeks and then repeated the test in the same order. The data for the second test are as follows:

61	52	33	51	39	40	50	49	46	37	59	49
38	48	39	58	44	52	38	44	49	51	62	44

(a) Represent the data from the two tests on a back-to-back stem-and-leaf diagram.

Test 2										Test 1									
									2	*7*	*9*								
			9	*9*	*8*	*8*	*7*	*3*	**3**	*3*	*6*	*7*	*7*	*8*					
9	*9*	*9*	*8*	*6*	*4*	*4*	*4*	*0*	**4**	*0*	*1*	*4*	*5*	*5*	*5*	*6*	*7*	*8*	*9*
		9	*8*	*2*	*2*	*1*	*1*	*0*	**5**	*1*	*1*	*2*	*2*	*3*	*9*				
							2	*1*	**6**	*0*									

The stem-and-leaf diagram need not be sorted.

(b) How many students are in the class? 24

(c) What is the *range* of sit-ups for the class? Test 1 60–27 = 33 Test 2 62–33 = 29

(d) Based on the data and the diagram, do you think that practice improves the ability to do sit-ups? Give a reason for your answer.

- Yes. Only 3 people did worse after practising. 2 did the same and 19 did better.
- Yes higher average
- Yes median higher.
- Yes. General 'shift' of data upwards.
- Most students did better after the exercise

(e) John did 41 sit-ups in Test 1 and 44 in Test 2. How did his performance compare with that of the rest of the class?

- **Compared Favorably:** The class average improvement is 2·67. John's improvement is 3. Therefore he improved by more than the average improvement of his classmates.
- **Compared Unfavorably:** There were 8 people below him before the practice. There were only 7 people below him after the practice. Therefore he moved down relative to his classmates.

Question 8

There are 24 students in a class. On a Friday each student present in class is asked for the number of days they had been absent that week. The results are recorded in the table below.

Number of days absent	None	One	Two	Three	Four	Five
Number of students	9	2	3	4	1	0

(a) How many students were absent on that Friday? 5

(b) On the following Monday all of the students were present in class and the table was updated to include the entire class. Which number from the above table could not have changed? Give a reason for your answer.

The '9' students (or 'none') who missed no days would not change. The 5 who were absent on the Friday would fall under one of the other five categories, since they had missed at least one day (the Friday).

(c) The total number of days that were missed during the week will depend on the answers given by the students who were absent on Friday. Complete the tables below to show how the largest possible and smallest possible number of days missed would arise.

Smallest possible number of days missed						
Number of days absent	None	One	Two	Three	Four	Five
Number of students	**9**	**7**	**3**	**4**	**1**	**0**

Largest possible number of days missed						
Number of days absent	None	One	Two	Three	Four	Five
Number of students	**9**	**2**	**3**	**4**	**1**	**5**

(d) Cathal decides to draw a pie chart of the actual data collected on Monday. He calculates the number of degrees for each sector of the pie chart. Use this data to calculate the mean number of absences per pupil for the previous week correct to one place of decimals.

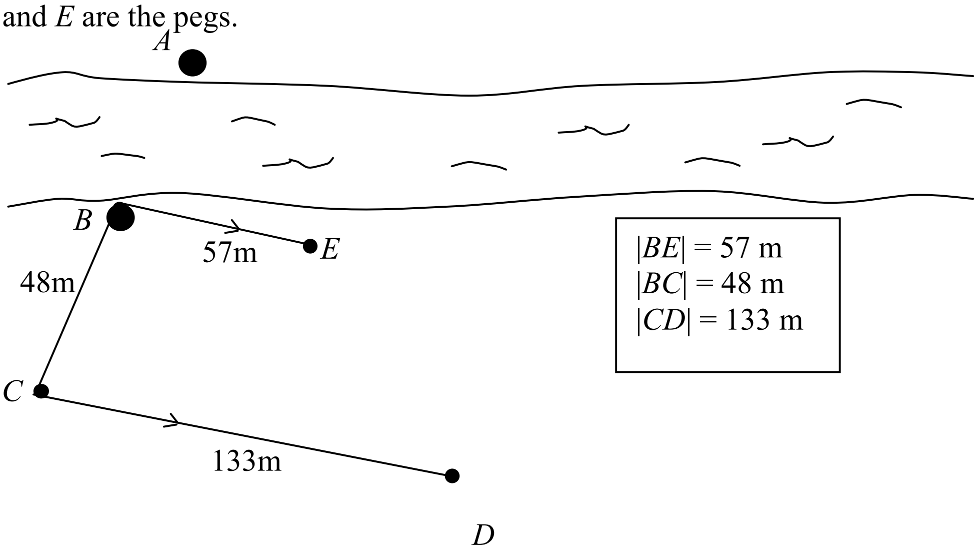
Number of days absent	None	One	Two	Three	Four	Five
Number of students	**9**	**2**	**5**	**4**	**3**	**1**
Number of degrees	135°	30°	75°	60°	45°	15°

$$\frac{135}{360}\times 24 = 9$$

$$\frac{9\times 0+2\times 1+5\times 2+4\times 3+3\times 4+1\times 5}{24}$$
$$=\frac{0+2+10+12+12+5}{24}$$
$$=\frac{41}{24}=1{\cdot}7\ [days]$$

Question 9

A group of students were trying to find the distance between two trees on opposite sides of a river using pegs, a measuring tape and a large amount of string. They align the pegs in a particular way, take several measurements and sketch this diagram. On the diagram, *A* and *B* are the trees and *C*, *D* and *E* are the pegs.



(a) In what way must the pegs and the trees be aligned if the students are to use these measurements to calculate $|AB|$.

Peg C must be collinear with the two trees, A and B. Pegs E and D must be collinear with each other and the tree A. Also [BE] must be parallel to [CD]

(b) Calculate the distance between the trees.

$$\frac{|BA|}{|AC|}=\frac{|BE|}{|CD|}$$

$$\frac{|AB|}{48+|AB|}=\frac{57}{133}$$

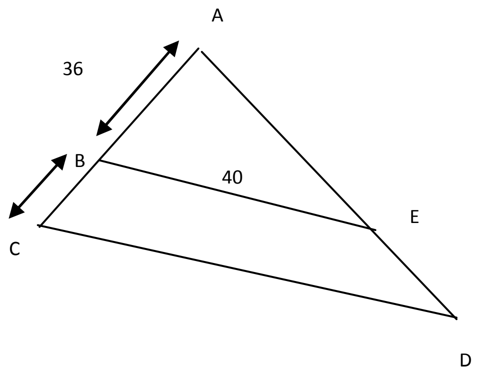
=> 133|AB| = 2736+57|AB|

=> 76|AB| = 2736

=> |AB| = 36m

(c) Another group of students repeats the activity. They have a similar diagram but different measurements. Their measurements are $|BE| = 40$ m and $|BC| = 9$ m. Based on the value of $|AB|$ that the first group got, what measurement will this second group have for $|CD|$.

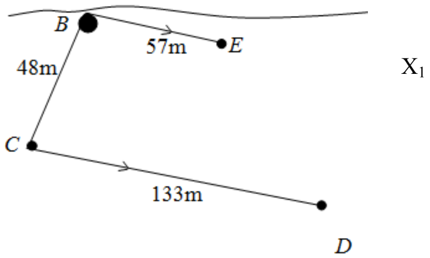
$$\frac{|CD|}{40} = \frac{45}{36}$$
$$|CD| = 50 \text{ m}$$



(d) Suggest how the group of students might have ensured that $[BE]$ was parallel to $[CD]$.

Create a parallelogram CBEX using strings,
where |CB|=|XE|= 48m
And |BE|=|CX|= 57m
Then extend [CX] until D is collinear with E and A

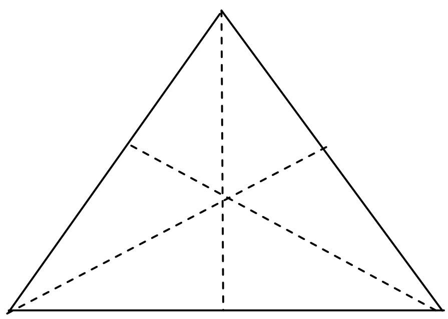
OR
Create a parallelogram BX_1DC, where
$|BX_1|=|CD|=133$m and
$|BC=|X_1D|=48$m



Question 10

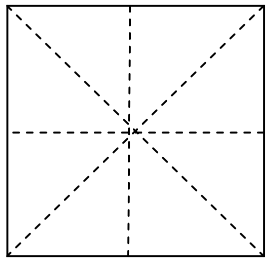
(a) Draw a shape below which has exactly three axes of symmetry. Show the axes on the diagram.

Equilateral triangle showing the three axes of symmetry.



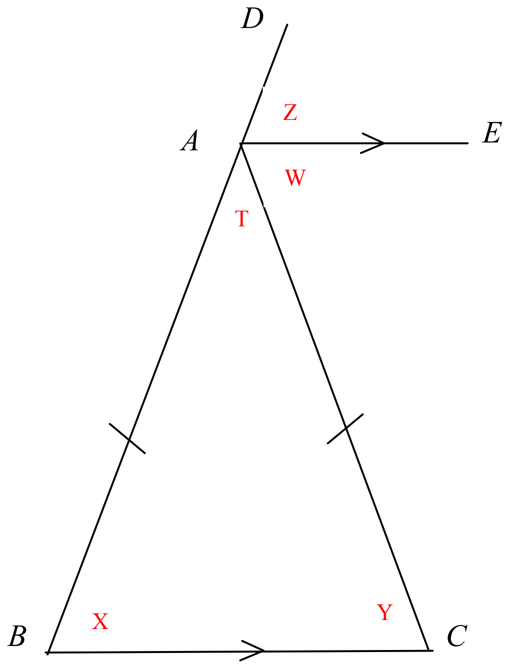
(b) Draw a shape below which has exactly four axes of symmetry. Show the axes on the diagram.

A square showing the four axes of symmetry



Question 11

ABC is an isosceles triangle with $|AB| = |AC|$.
[*BA*] is produces to *D*.
AE is parallel to *BC*.



(a) Prove that [*AE* bisects $\angle DAC$.

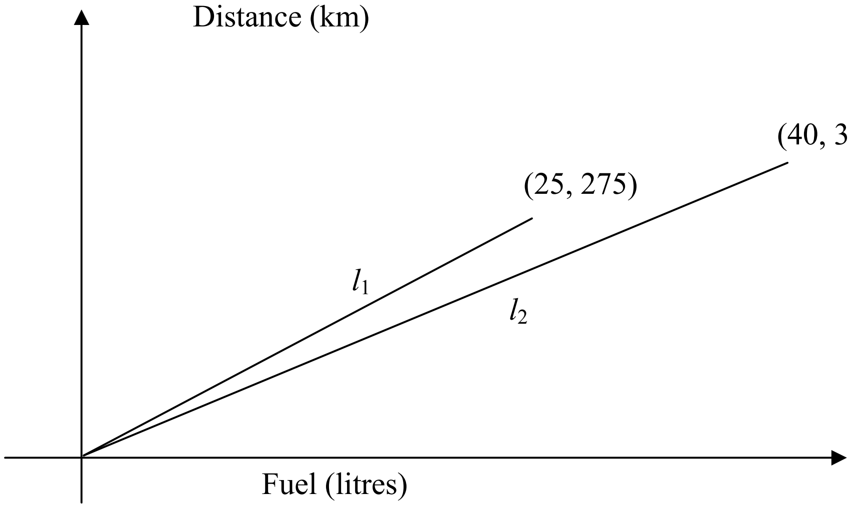
To Prove: [*AE* bisects $\angle DAC$.

Proof:
$|\angle Y| = |\angle W|$ Alternate
$|\angle Y| = |\angle X|$ isosceles
$|\angle X| = |\angle Z|$ Corresponding
=> $|\angle W| = |\angle Z|$
[Therefore [AE bisects $\angle$DAC]

(b) Would the result in part **(a)** still apply if $|AB|$ and $|AC|$ were not equal? Give a reason for your answer.

No the result in (a) would not still apply. Angle Y would not be equal to angle X.

Question 12
The graphs below show the relationship between distances travelled and fuel consumption for John's car. The segments l_1 and l_2 represent the fuel consumption at steady speeds of 60 km/h and100 km/h respectively.



(a) Find the slopes of l_1 and l_2.

$\frac{275-0}{25-0}$

Slope of $l_1 = 11$

$\frac{340-0}{40-0}$

Slope of $l_2 = 8.5$

OR

$$Slope = \frac{Rise}{Run} = \frac{275}{25} = 11$$

Slope $l_1 = 11$

$$Slope = \frac{Rise}{Run} = \frac{340}{40} = 8{\cdot}5$$

Slope $l_2 = 8{\cdot}5$

(b) What do these slopes tell you about the fuel consumption of the car at these speeds?

- The higher slope for l_1 indicates that you get more km per litre at the lower speed
- OR More fuel is used at the higher speed

(c) Fuel costs 149·9 cent per litre. John drives a distance of 200 km at a steady speed. How much cheaper is the journey at 60 km/h than at 100 km/h?

l_1:

y=mx
200=11x
=> $x = 18{\cdot}18$ litres
$18{\cdot}18 \times 149{\cdot}9 =$ €27·25

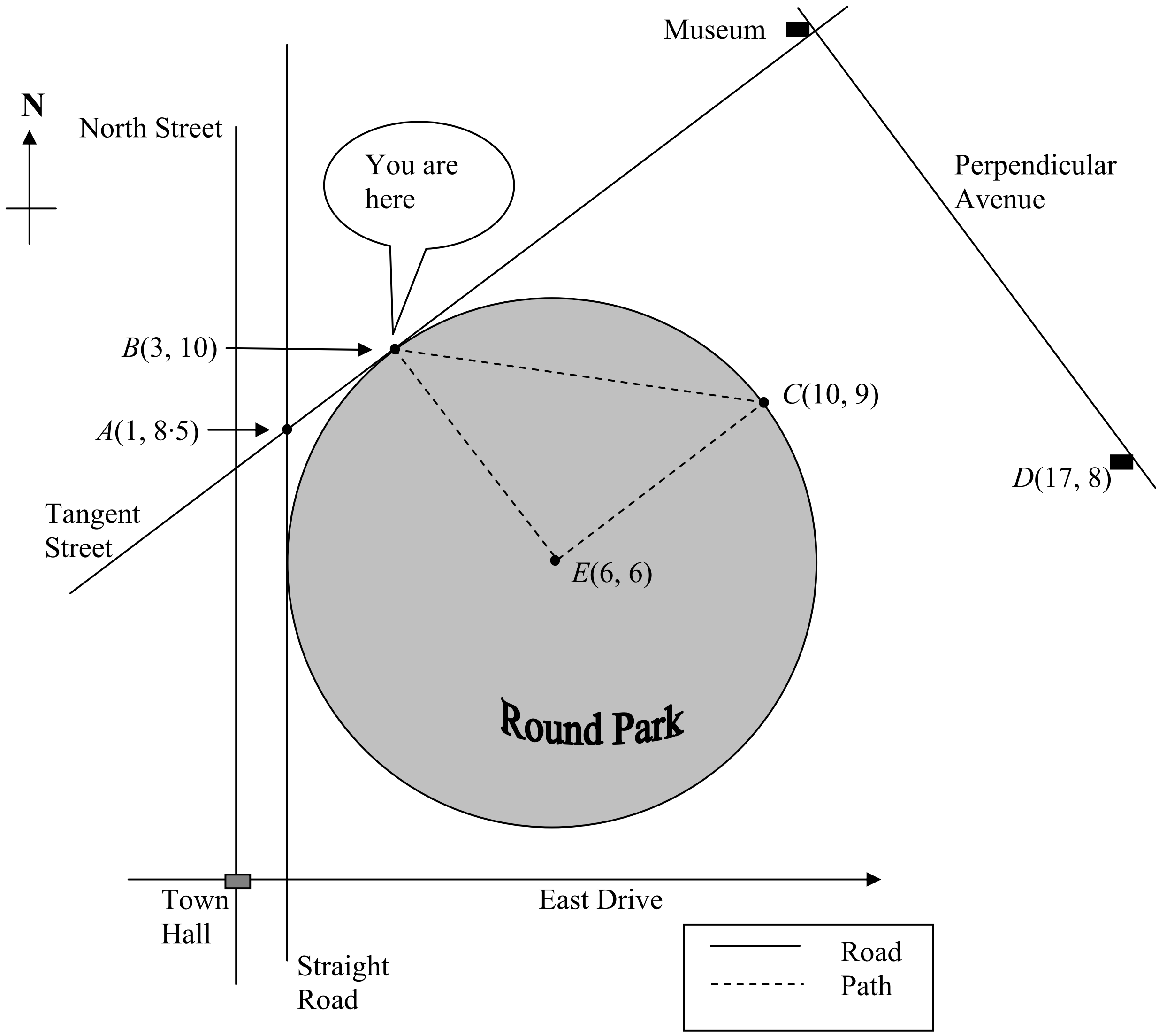
l_2:

y=mx
200=8·5x
=> $x = 23{\cdot}53$ litres
$23{\cdot}53 \times 149{\cdot}9 =$ €35·27

35·27–27·25 = €8·02 OR 802c

Question 13

The map below shows part of a town containing a park and some streets. Distances are measured (in kilometres) horizontally and vertically from the Town Hall and shown in co-ordinate form.



(a) How long is the path from $B(3, 10)$ to $C(10, 9)$? Give your answer correct to three significant figures.

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\sqrt{(10-3)^2+(9-10)^2}$

$\sqrt{(7)^2+(-1)^2}$

$\sqrt{49+1}$

$\sqrt{50}$

$7{\cdot}07km$

(b) $E(6,\ 6)$ is the centre of Round Park. How much shorter is it to walk directly from B to C rather than take the path to E and then on to C? Give your answer correct to the nearest km.

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\sqrt{(10-6)^2+(9-6)^2}$

$\sqrt{(4)^2+(3)^2}$

$\sqrt{9+16}$

$\sqrt{25}$

5

$5+5=10$

$10-7{\cdot}07=3$ km

(c) The points $A(1,\ 8{\cdot}5)$ and $B(3,\ 10)$ are on Tangent Street. Find the equation of Tangent Street.

$m=\dfrac{y_2-y_1}{x_2-x_1}$

$=\dfrac{10-8{\cdot}5}{3-1}=\dfrac{1{\cdot}5}{2}\left(\text{or } \dfrac{3}{4}\right)$

$y-y_1=m(x-x_1)$

$y-10=\dfrac{1{\cdot}5}{2}(x-3)$ or $y-8{\cdot}5=\dfrac{1{\cdot}5}{2}(x-1)$ $\left(\text{or } \dfrac{3}{4} \text{ used as slope}\right)$

$3x-4y+31=0$ Equation of Tangent Street

(d) Perpendicular Avenue is perpendicular to Tangent Street and passes through $D(17,\ 8)$. Find its equation.

Perpendicular slope $=\dfrac{-2}{1.5}\left(\text{or } \dfrac{-4}{3}\right)$

$y-y_1=m(x-x_1)$

$y-8=\dfrac{-2}{1{\cdot}5}(x-17)$ $\left(\text{or } \dfrac{-4}{3} \text{ used as slope}\right)$

$4x+3y-92=0$

(e) The museum is located at the intersection of Tangent Street and Perpendicular Avenue. Find the co-ordinates of the museum.

$3x - 4y + 31 = 0$

$4x + 3y - 92 = 0$

Museum at (11, 16)

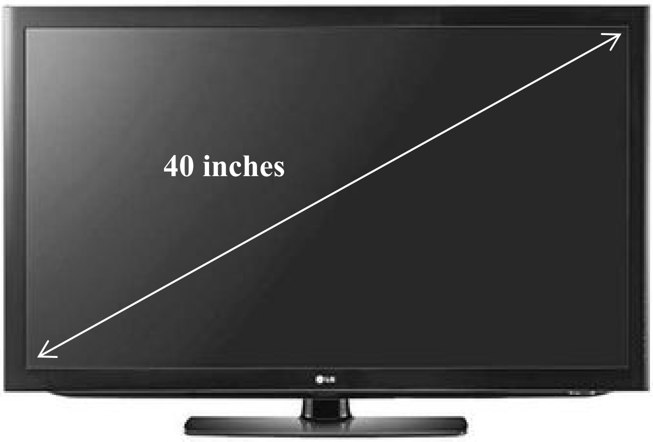
(f) John is at the Town Hall and wants to get to the museum. Give one possible route he might take and calculate the total distance he must travel if he takes that route.

- North to Tangent Street (7·75 km) and then on to the Museum (13·75 km)
  i.e. Distance from (0, 7·75) to (11, 16)
  7·75 + 13·75 = 21·5 km

- East for 1 km to Straight Road. Then North to A (8·5 km).
  Then from A to the Museum i.e. Distance from (1, 8·5) to (11, 16) = 12·5 km
  1 + 8·5 + 12·5 = 22km

Question 14

Mary is thinking of buying a new television. The television is advertised as having a "40 inch" screen. This refers to the diagonal measurement of the screen. The *aspect ratio* of a television screen is the ratio of its width to its height. For this television, the aspect ratio is 16:9 (sixteen units wide for every nine units in height).



(a) Convert 40 inches to centimetres if 1 inch = 2·54 cm.

$40 \times 2{\cdot}54 = 101{\cdot}6$cm

(b) Find the width and the height of the screen, in centimetres. Give your answers correct to the nearest cm.

$(9x)^2 + (16x)^2 = 101{\cdot}6^2$

$81x^2 + 256x^2 = 10322{\cdot}56$

$337x^2 = 10322{\cdot}56$

$x^2 = 30{\cdot}63$

$x = 5{\cdot}534$

=> length = $16 \times 5{\cdot}534$ = 88·55 = 89 to nearest cm

=> height = $9 \times 5{\cdot}534$ = 49·81 = 50 to nearest cm

(c) A different 40 inch television screen has an aspect ratio of 4:3. Which of the two television screens has the greatest area, and by how much?

$(4x)^2 + (3x)^2 = 101{\cdot}6^2$

$16x^2 + 9x^2 = 10322{\cdot}56$

$25x^2 = 10322{\cdot}56$

$x^2 = 412{\cdot}9024$

$x = 20{\cdot}32$

=> length = $4 \times 20{\cdot}32 = 81{\cdot}28 = 81$ to nearest cm

=> height = $3 \times 20{\cdot}32 = 60{\cdot}96 = 61$ to nearest cm

$(81 \times 61) - (89 \times 50) = 491\text{cm}^2$

Question 15

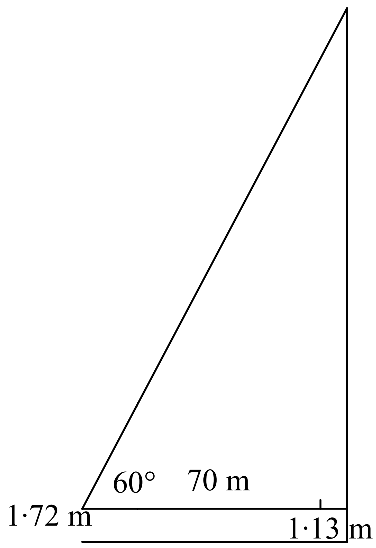
A group of students wish to calculate the height of the Millennium Spire in Dublin. The spire stands on flat level ground. Maria, who is 1·72 m tall, looks up at the top of the spire using a clinometer and records an angle of elevation of 60˚. Her feet are 70 m from the base of the spire. Ultan measures the circumference of the base of the spire as 7.07m

(a) Explain how Ultan's measurement will be used in the calculation of the height of the spire

$2\pi r = 7{\cdot}07$
$6{\cdot}28r = 7{\cdot}07$
$\Rightarrow\ r = 1{\cdot}12579\text{m}$

OR: The circumference can be used to calculate the radius, which will give the full distance that Maria is from the centre of the base of the spire.

(b) Draw a suitable diagram and calculate the height of the spire, to the nearest metre, using the measurements obtained by the students.



$$\tan 60^\circ = \frac{x}{71{\cdot}13}$$

$$x = 123{\cdot}2$$

Spire = $123{\cdot}2 + 1{\cdot}72$ = $124{\cdot}92$ = 125 m

Structure of the marking scheme (Questions 2 – 15)

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C
No of categories	2	3	4
2 mark scale	0, 2	0, 1, 2	
5 mark scale	0, 5	0, 4, 5	0, 3, 4, 5
10 mark scale	0, 10	0, 8, 10	0, 5, 8, 10
15 mark scale		0, 12, 15	0, 7, 14, 15

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, *scale 10C** indicates that 9 marks may be awarded.

Summary of mark allocations and scales to be applied

Question 1
50 marks as per mainstream paper

Question 2
(a) 10A
(b) 10B
(c) 5C
(d) 5B

Question 3
(a) 5B
(b) 5B
(c) 2B
(d) 2A
(e) and (f) 10C

Question 4
(a) 10C
(b) 10B

Question 5
(a) 5B
(b) 10C
(c) And (d) 10C*

Question 6
(a) and (b) 10C

Question 7
(a) 15C
(b) 5A
(c) 10C
(d) and (e) 5C

Question 8
(a) 10A
(b) 5B
(c) and (d) 10C

Question 9
(a) and (b) 2B
(c) and (d) 2B

Question 10
(a) and (b) 5C

Question 11
(a) and (b) 5C

Question 12
(a) and (b) and (c) 15C

Question 13
(a) 5B*
(b) 5C*
(c) 15B
(d) and (e) and (f) 10C

Question 14
(a) 10A*
(b) And (c) 2B*

Question 15
(a) and (b) 5C*

Detailed marking notes

Question 2

(a) Scale 10A

(b) Scale 10B

Partial Credit:

- Identifying either the 2 or the 8 or both

(c) Scale 5C

High Partial Credit:

- Identifies all of the 47, 2, 26, 9. These might be in fraction or decimal form. They might also be in their simplest form (or other)

Low Partial Credit:

- Identifying one or up to any three of the 47, 2, 26, 9. These might be in fraction or decimal form. They might also be in their simplest form (or other)

(d) Scale 5B

Partial Credit:

- Use of 8% but without valid reason given.

Question 3

(a) Scale 5B

Partial Credit:

- 410

(b) Scale 5B

Partial Credit

- One or more relative frequencies right (up to five)

(c) Scale 2B

Partial Credit:

- Answers the "Method" part

NOTE: Accept "The sum of the percentages should be 100%" as the Method part. This could get full credit if the candidate then adds the percentages in this part.

(d) Scale 2A

(e) and (f) Scale 10C
High Partial Credit if either (e) **or** (f) fully correct
Low Partial Credit if any partial credit on (e) **or** (f) **or both**.

(e)

Partial Credit:

- One or more right (up to a maximum of five)
- Finds the common ratio

(f)

Partial Credit:

- Valid reason but without answer.
- Mentions that the time of day of the survey could bias the results
- Mentions that two hours might not be representative of the whole day

NOTE: The candidate's 'yes' or 'no' must match a convincing argument.

Question 4

(a) Scale 10C

Low Partial Credit:

- $5 \times 9 = 45$
- $180 \div 5 = 36$ or $180 \div 9 = 20$ or both of these

High Partial Credit:
4 without work

(b) Scale 10B

Partial Credit:

- Identifies either the 4 or the 8
- Brings down his/her answer from part (a)

NOTE: 128 without work merits full marks

Question 5

(a) Scale 5B

Partial Credit:

- Writes an ordered list of the distances

(b) Scale 10C

High Partial Credit:

- Adding all of the items to get 1680

Low Partial Credit:

- Indicates adding all of the items

(c) and (d) Scale 10C*

HPC if either (c) **or** (d) fully correct
LPC if any partial credit on (c) **or** (d) **or both**.

(c)

High Partial Credit:

- Getting the 1650 or 70 or (1650 + x)/7 = 250 depending on the method
- Correct answer without work

Low Partial Credit:

- Calculating the 1750
- Bringing down the 1680 or candidate's sum for part (b)

(d)

NOTE: Accept 250 cm for full marks

Question 6

(a) and (b) Scale 10C

HPC if either (a) **or** (b) fully correct
LPC if any partial credit on (a) **or** (b) **or both**.

(a)

Partial Credit:

- One set of bar charts
- One or more connections compared to a maximum of three.

(b)

High Partial Credit:

- Two valid trends identified

Low Partial Credit:

- One valid trend identified

NOTE: Fully correct if three (or more) valid trends identified.

Question 7

(a) Scale 15C

High Partial Credit:

- Data items omitted (or incorrect) or extra to a maximum of 4

Low Partial Credit:

- One test done only

(b) Scale 5A

(c) Scale 10C

High Partial Credit:

- Doesn't subtract in either test (but shows the subtraction in both tests)

Low Partial Credit:

- Identifies the highest or lowest in any one of the tests or highest and lowest in any one of the tests

(d) and (e) Scale 5C

HPC if either (d) **or** (e) fully correct
LPC if any partial credit on (d) **or** (e) **or both**.

(d)

Partial Credit:

- Individual data item comparison done. E.g. "Person one increased from 59 to 61."
- Mentions that some students did better after the practice.
- One part/section of the data looked at. E.g. "No students in the 20s after the test."

(e)

Partial Credit:

- Calculates mean for Test 1 or Test 2.
- Says something like "John went up by 3." or "The first student went up by only 2 (61 – 59)"

Question 8

(a) Scale 10A

(b) Scale 5B

Partial Credit:

- Gives the 9 but does not give a reason

(c) and (d) Scale 10C

HPC if either (c) **or** (d) fully correct
LPC if any partial credit on (c) **or** (d) **or both**.

(c)

Partial Credit:

- Either table fully correct
- Tables interchanged but fully correct

(d)

High Partial Credit:

- The set-up for the mean is correctly written (This assumes the figures in the table are correct)

Low Partial Credit:

- Any one number in table correct

Question 9

(a) and (b) Scale 2B

Partial Credit if either (a) **or** (b) fully correct or any partial credit on (a) **or** (b) **or both**.

(a)

Partial Credit:

- Mentions that BE must be parallel to CD
- Mentions ABC collinear or AED collinear

NOTE: If candidate identifies ABC collinear <u>and</u> AED collinear = Full marks.

(b)

Partial Credit:

- Correct multiplying across the equation to eliminate the fractions
- Correct substitution into correct ratios
- Identifies at least one correct ratio

(c) and (d) Scale 2B

PC if either (c) **or** (d) fully correct or any partial credit on (a) **or** (b) **or both**.

(c)

Partial Credit:

- Correct set-up with substitution
- Diagram correct
- Some correct substitution. e.g. $9/36 = 40/x$ (or $x/40$)

(d)

Partial Credit:

- Any mention of a parallelogram
- Using resources not given to correctly describe how to get [BE] parallel to [CD]

Question 10

(a) and (b) Scale 5C

HPC if either (a) **or** (b) fully correct
LPC if any partial credit on (a) **or** (b) **or both**.

(a)

Partial Credit:

- Equilateral triangle drawn without the axes of symmetry drawn

NOTE: An isosceles triangle, even with axes of symmetry merits no marks.

(b)

Partial Credit:

- Square drawn without the four axes of symmetry drawn

Question 11

(a) and (b) Scale 5C

HPC if either (a) **or** (b) fully correct
LPC if any partial credit on (a) **or** (b) **or both**.

(a)

High Partial Credit:

- All steps correct but no reasons

Medium Partial Credit:

- A second correct line of proof (with or without the correct reason)

Low Partial Credit:

- Any one correct line of proof (with or without the correct reason)

NOTE: To get the HPC the steps must make full sense. i.e. the last step must be a logical follow-on from the previous two steps which prove the cut.

(b)

Partial Credit:

- It would not be isosceles.

Question 12

(a) and (b) and (c) Scale 15C

HPC if either (a) **or** (c) **or both** fully correct
NOTE: LPC if (b) is the only part which is fully correct
LPC if any partial credit on (a) **or** (b) **or** (c) **or all of them**.

(a)

Partial Credit:

- One slope correct
- Inverts both slopes i.e. calculates 1/11 and 1/8·5 (or 2/17)
- $\frac{Rise}{Run}$
- Some correct substitution into slope formula

(b)

Partial Credit:

- Mentions Km per litre, without further relevant work.
- Mentions using more fuel without being specific.

NOTE: The candidate's answer to part (b) must be consistent with his/her answer to part (a).

(c)

Partial Credit:
- Two correct costs
- One correct cost or reasonable work

Question 13

(a) Scale 5B*

Partial Credit:
- Correct substitution into formula
- Writes 7 and –1

NOTE: The * to be applied once only in Question 13.

(b) Scale 5C*

High Partial Credit:
- Calculating as far as the 10

Low Partial Credit:
- Correct substitution into formula

(c) Scale 15B

Partial Credit:
- Calculates slope
- Correct substitution into slope formula
- Writes $y–10=k(x–3)$, $k \neq 1{\cdot}5/2$ (or ¾)
- Writes $y–8{\cdot}5=k(x–1)$, $k \neq 1{\cdot}5/2$ (or ¾)

(d) and (e) and (f) Scale 10C

HPC if (d) **or** (e) fully correct
NOTE: LPC if (f) is the only part which is fully correct
LPC if any partial credit on (d) **or** (e) **or** (f) **or all of them.**

(d)

Partial Credit:
- Finds perpendicular slope
- Writes $y–8=k_1(x–17)$, $k_1 \neq –1/k$

(e)

Partial Credit:
- Transfers down his/her equation(s) from parts (c) and /or (d)
- Either x or y coordinate found correctly
- Correctly solves one or more incorrect equations

(f)

Partial Credit:

- Describes the journey in words
- Indicates the correct distance for any one segment of the route

Question 14

(a) Scale 10A*

(b) and (c) Scale 2B*

PC if any partial credit on either (b) **or** (c) **or both**.

(b)

Partial Credit:

- Calculates the 5·534
- Ties the ratio 16:9 to Pythagoras' Theorem
- $101{\cdot}6^{2}$

(c)

Partial Credit:

- Ties the ratio to Pythagoras' Theorem
- $101{\cdot}6^{2}$
- Multiplies answers from part (b)

Question 15

(a) and (b) Scale 5C

HPC if either (a) **or** (b) fully correct
LPC if any partial credit on (a) **or** (b) **or both**.

(a)

Partial Credit:

- Mentions that Ultan's measurement is used to get the distance on the ground.

(b)

Partial Credit:

- Correct relevant diagram
- Uses tan 60 = *x*/70

Marcanna breise as ucht freagairt trí Ghaeilge

(Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. 198 marc × 5% = 9·9 ⇒ bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle [300 – bunmharc] × 15%, agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0