Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2014
Sample Paper

Mathematics
(Project Maths – Phase 3)

Paper 1

Ordinary Level

Time: 2 hours, 30 minutes

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **all six** questions from this section.

Question 1 (25 marks)

(a) Write 6^{-2} and $81^{\frac{1}{2}}$ without using indices.

$6^{-2} =$ | $81^{\frac{1}{2}} =$

(b) Express 2^{24} in the form $a \times 10^n$, where $1 \le a < 10$ and $n \in \mathbb{Z}$, correct to three significant figures.

(c) Show that $\dfrac{\left(a\sqrt{a}\right)^3}{a^4}$ simplifies to $\sqrt{a}$.

(d) Solve the equation $49^x = 7^{2+x}$ and verify your answer.

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Question 2 **(25 marks)**

(a) A sum of €5,000 is invested in an eight-year government bond with an annual equivalent rate (AER) of 6%. Find the value of the investment when it matures in eight years' time.
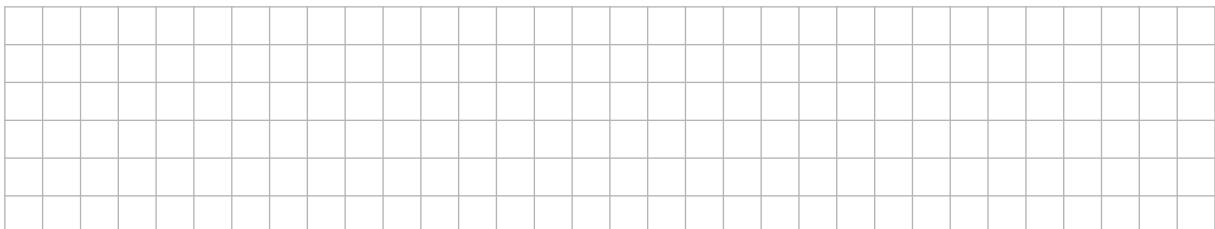
(b) A different investment bond gives 20% interest after 8 years.
Calculate the AER for this bond.
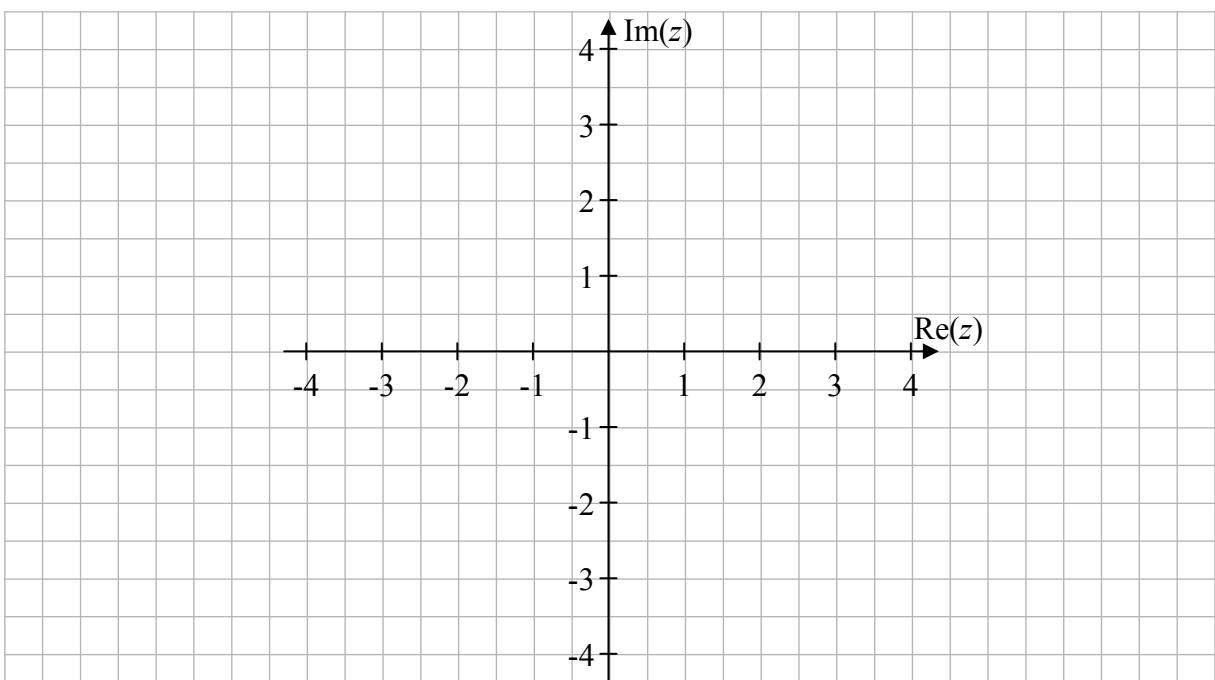
Question 3   (25 marks)

Two complex numbers are $u = 3 + 2i$ and $v = -1 + i$, where $i^2 = -1$.
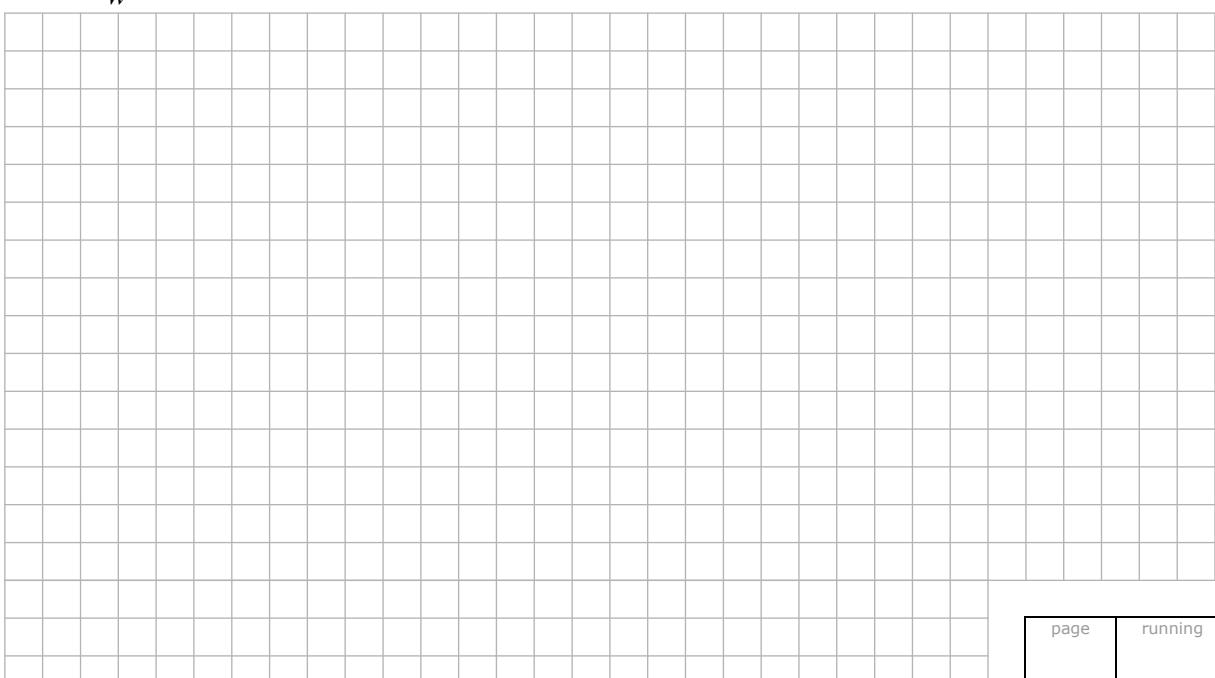
(a)   Given that $w = u - v - 2$, evaluate w.

(b)   Plot u, v, and w on the Argand diagram below.

(c)   Find $\dfrac{2u + v}{w}$.

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Question 4 (25 marks)

(a) Solve the equation $x^2 - 6x - 23 = 0$. Give your answers in the form $a \pm b\sqrt{2}$, where $a, b \in \mathbb{Z}$.

(b) Solve the simultaneous equations:

$$2r - s = 10$$
$$rs - s^2 = 12.$$

Question 5 **(25 marks)**
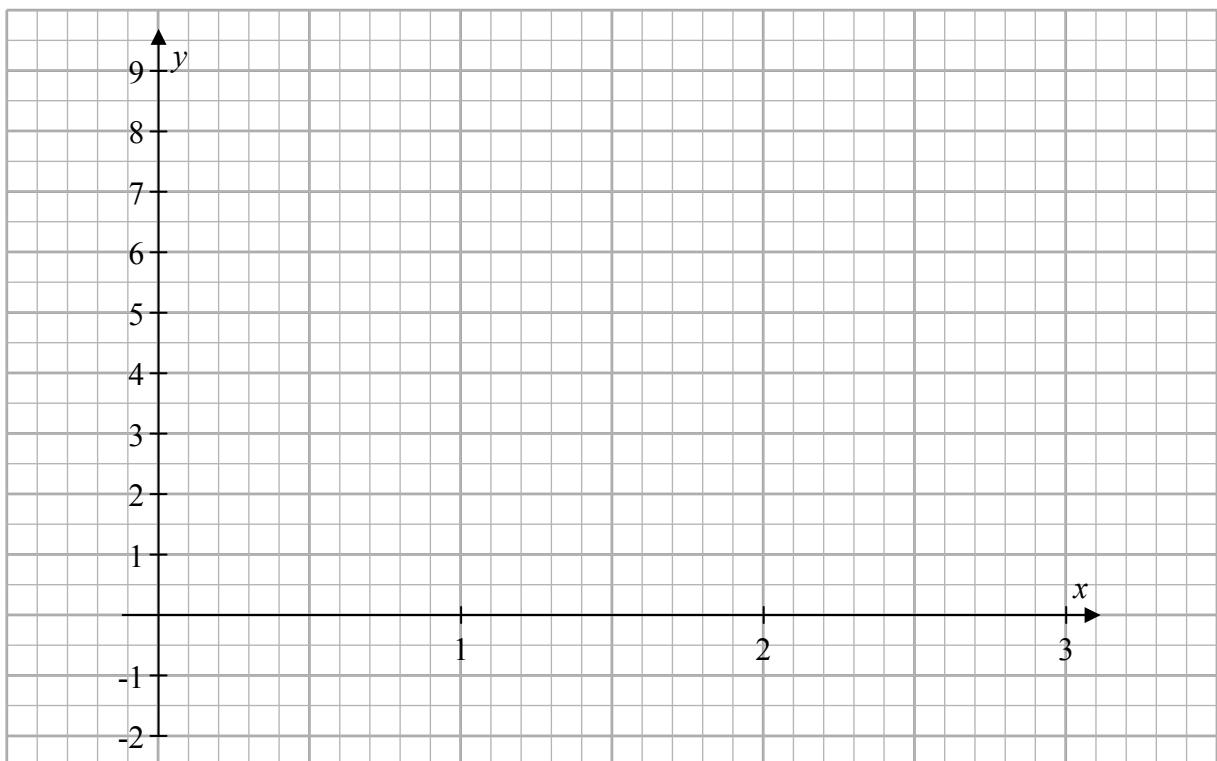
Two functions f and g are defined for $x \in \mathbb{R}$ as follows:

$$f : x \mapsto 2^x$$
$$g : x \mapsto 9x - 3x^2 - 1.$$

**(a)** Complete the table below, and use it to draw the graphs of f and g for $0 \le x \le 3$.
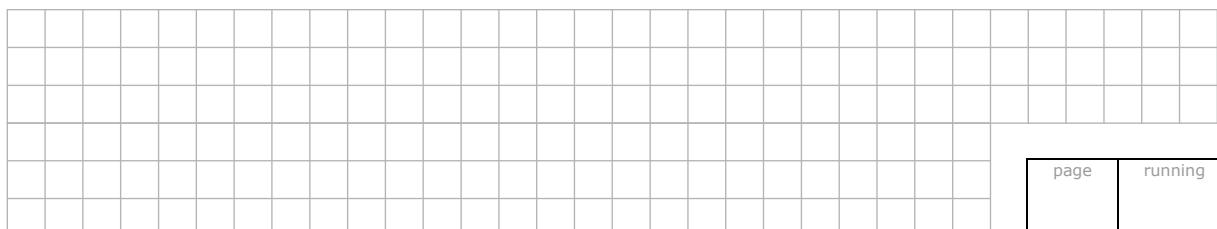
x	0	0·5	1	1·5	2	2·5	3
$f(x)$							
$g(x)$							

(b) Use your graphs to estimate the value(s) of x for which $2^x + 3x^2 - 9x + 1 = 0$.

**(c)** Let k be the number such that $2^k = 6$. Using your graph(s), or otherwise, estimate $g(k)$.
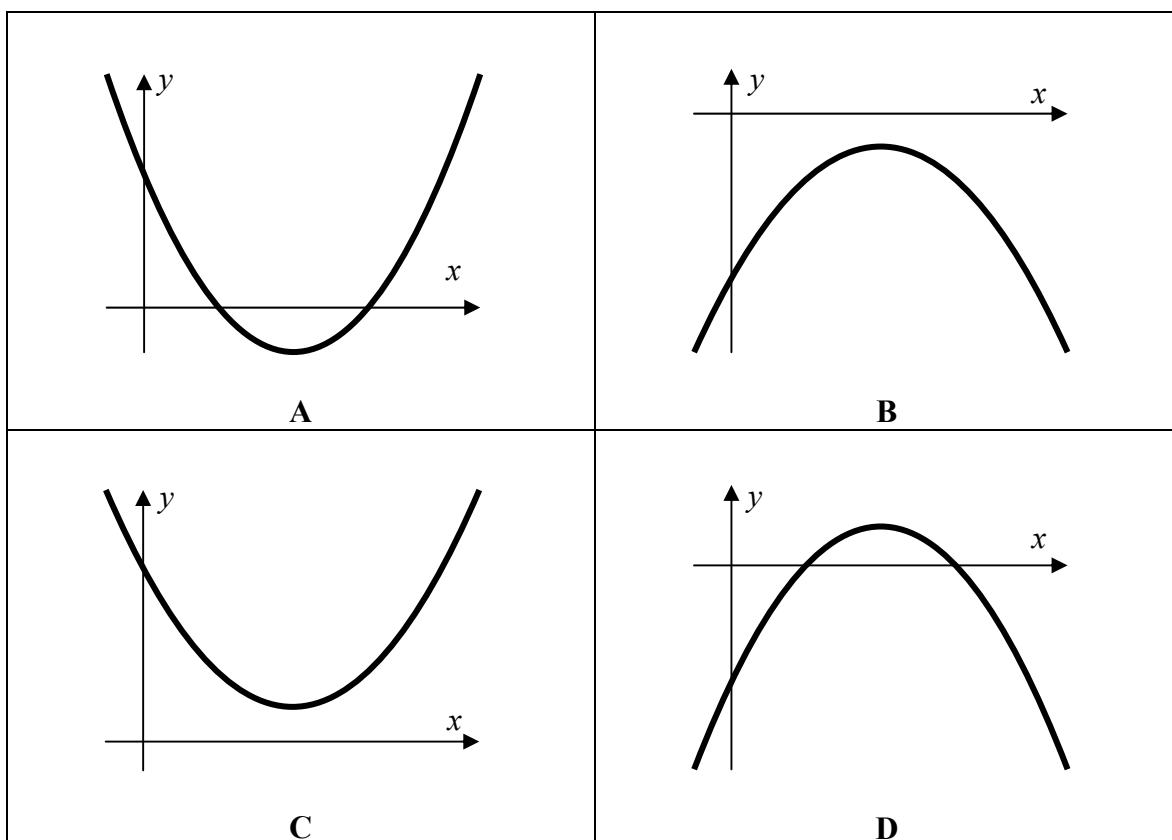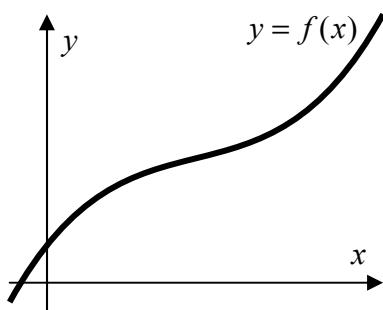
Question 6 **(25 marks)**

The graph of a cubic function *f* is shown on the right.

One of the four diagrams **A**, **B**, **C**, **D** below shows the graph of the derivative of *f*.

State which one it is, and justify your answer.

y = f(x)

A

B

C

D
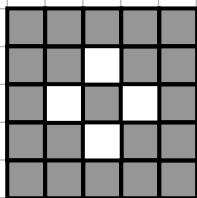
Answer: _____

Justification:

Answer **all three** questions from this section.
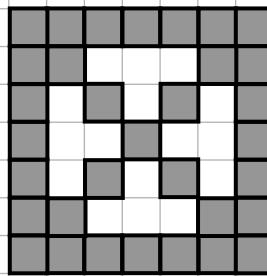
**Question 7**   **(50 marks)**

Síle is investigating the number of grey square tiles needed to make patterns in a sequence. The first three patterns are shown below, and the sequence continues in the same way. In each pattern, the tiles form a square and its two diagonals. There are no tiles in the white areas in the patterns – there are only the grey tiles.
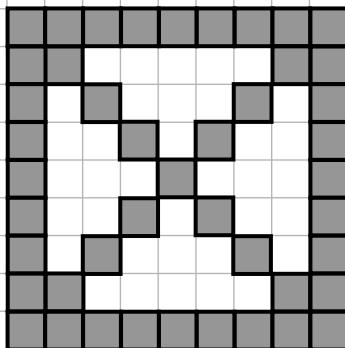
(Questions start overleaf.)

1st pattern

2nd pattern
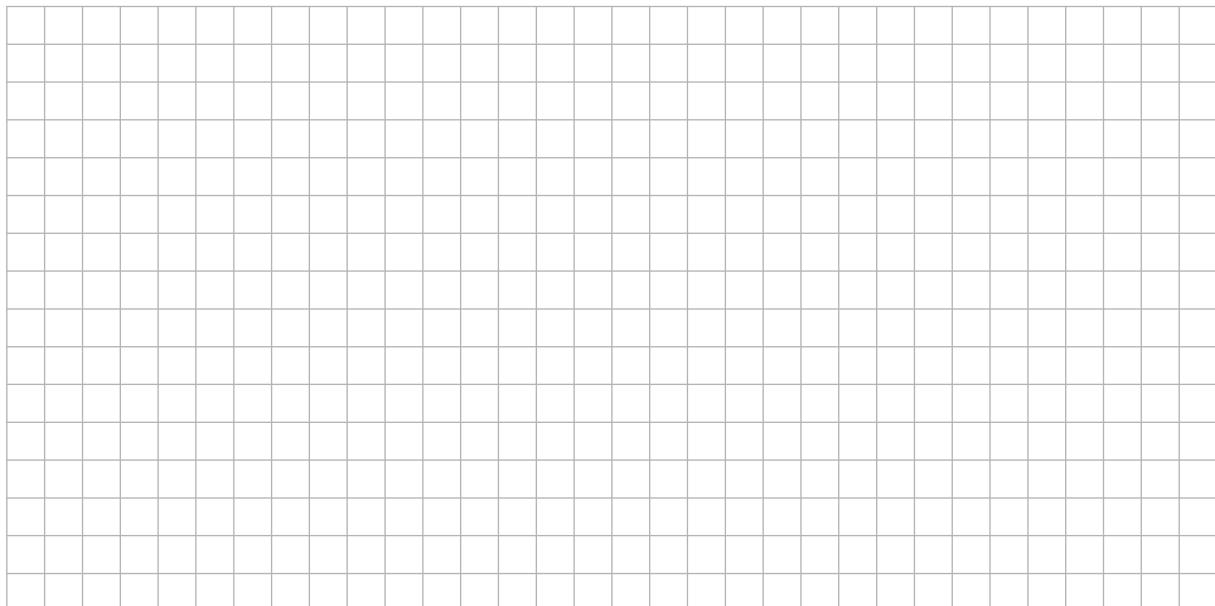
3rd pattern
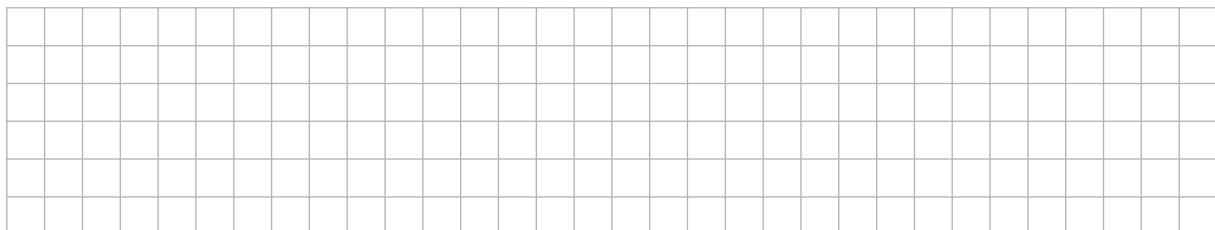
(a) In the table below, write the number of tiles needed for each of the first five patterns.

Pattern	1	2	3	4	5
No. of tiles	21	33			

(b) Find, in terms of n, a formula that gives the number of tiles needed to make the nth pattern.
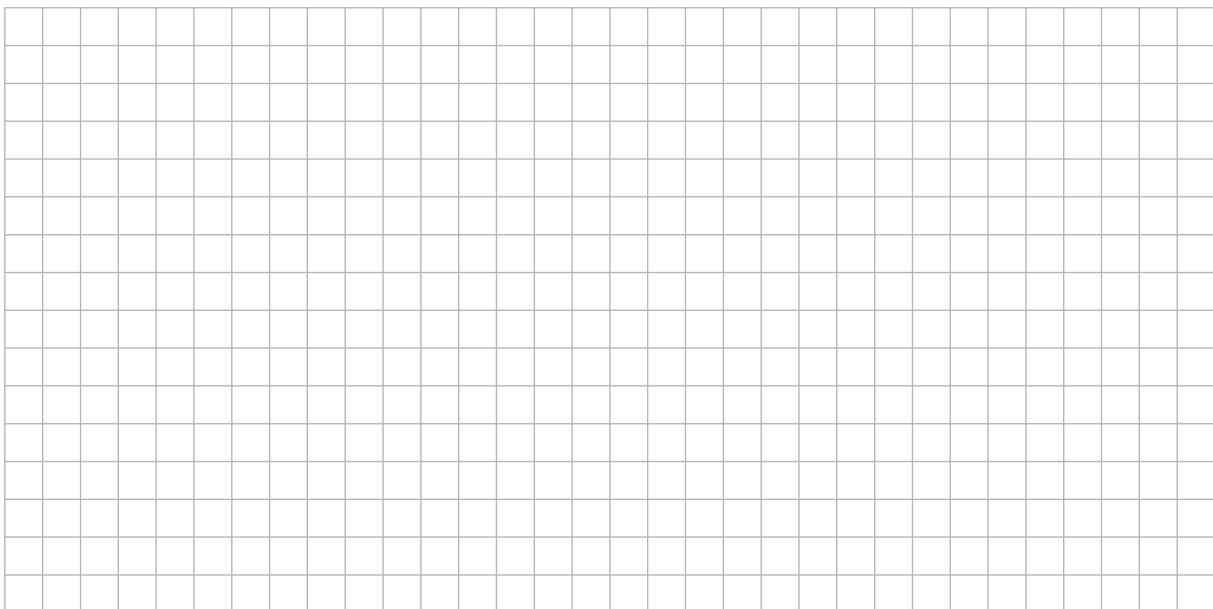
(c) Using your formula, or otherwise, find the number of tiles in the tenth pattern.
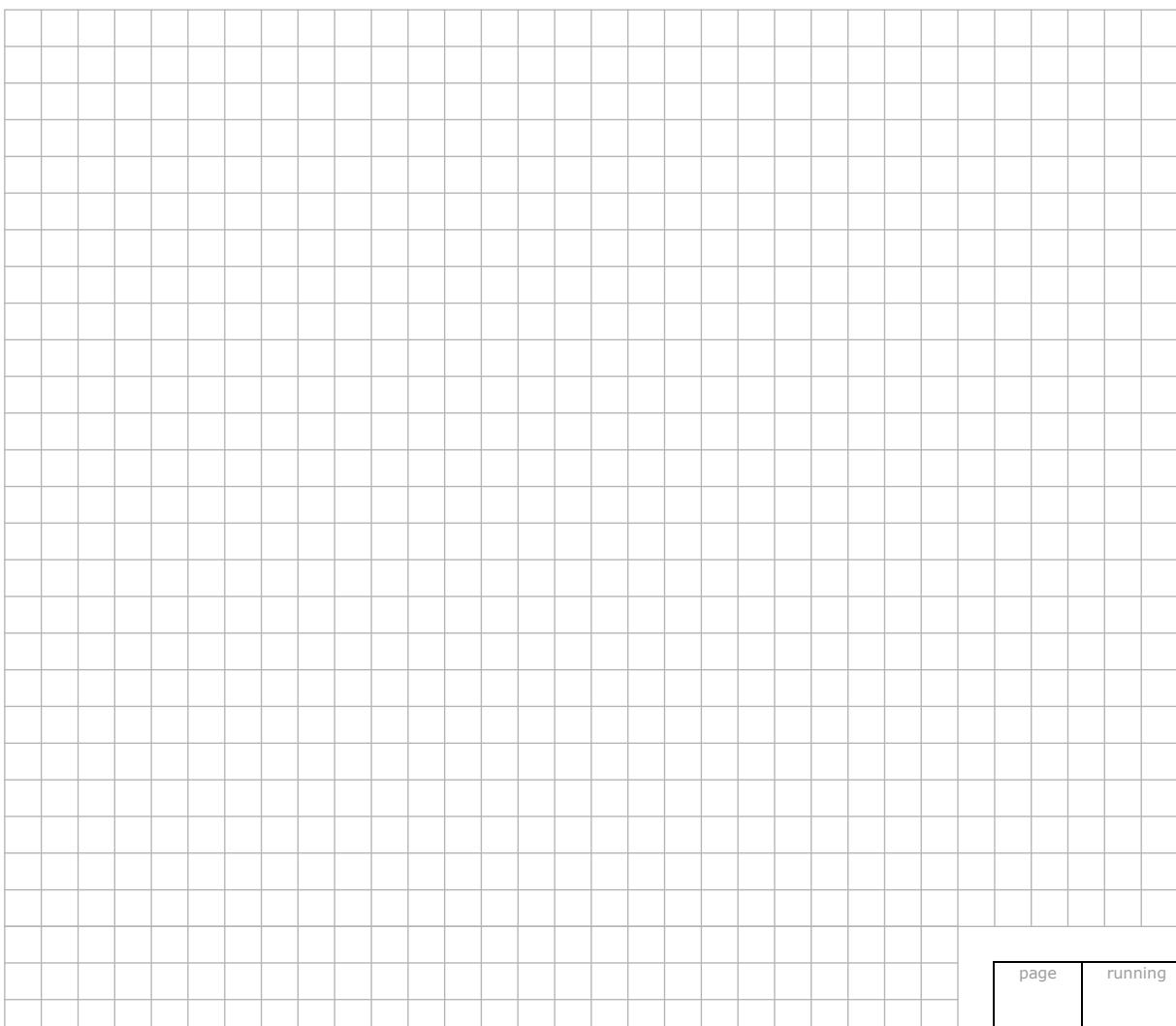
(d) Síle has 399 tiles. What is the biggest pattern in the sequence that she can make?

(e) Find, in terms of n, a formula for the total number of tiles in the first n patterns.

(f) Síle starts at the beginning of the sequence and makes as many of the patterns as she can. She does not break up the earlier patterns to make the new ones. For example, after making the first two patterns, she has used up 54 tiles $(21 + 33)$. How many patterns can she make in total with her 399 tiles?

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Question 8 (50 marks)
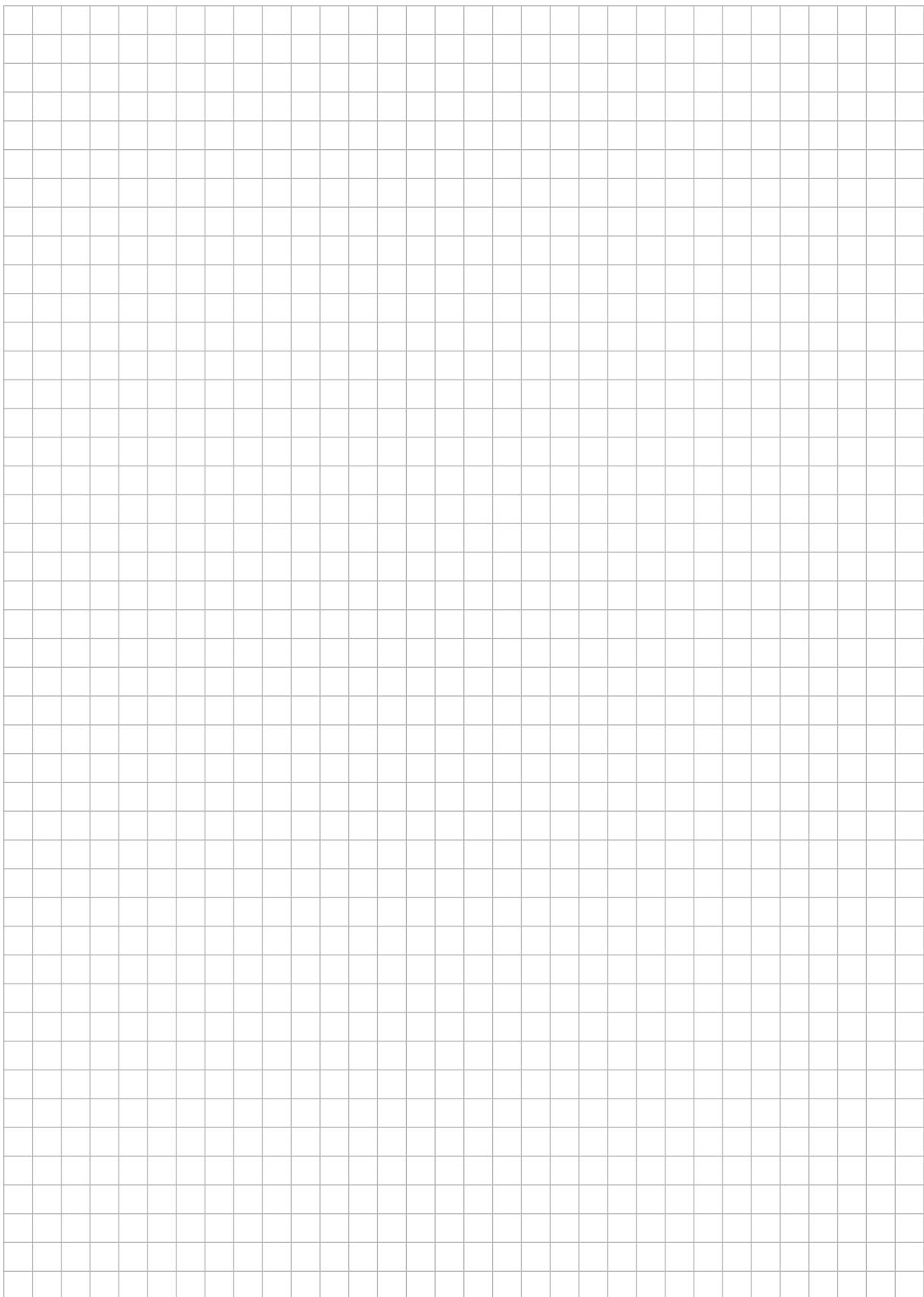
John is given two sunflower plants. One plant is 16 cm high and the other is 24 cm high. John measures the height of each plant at the same time every day for a week. He notes that the 16 cm plant grows 4 cm each day, and the 24 cm plant grows 3·5 cm each day.

(a) Draw up a table showing the heights of the two plants each day for the week, starting on the day that John got them.

(b) Write down two formulas – one for each plant – to represent the heights of the two plants on any given day. State clearly the meaning of any letters used in your formulas.

(c) John assumes that the plants will continue to grow at the same rates. Draw graphs to represent the heights of the two plants over the first *four weeks*.

(Questions continue overleaf.)

(d) (i) From your diagram, write down the point of intersection of the two graphs.

Answer: _____
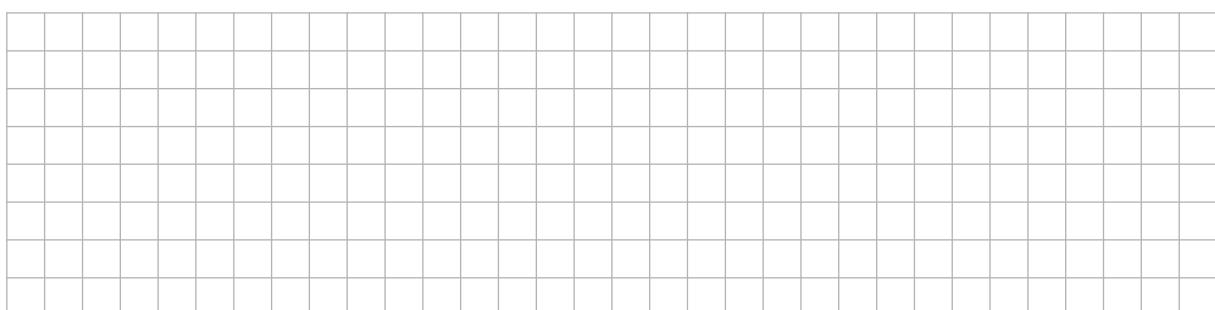
(ii) Explain what the point of intersection means, with respect to the two plants. Your answer should refer to the meaning of *both* co-ordinates.
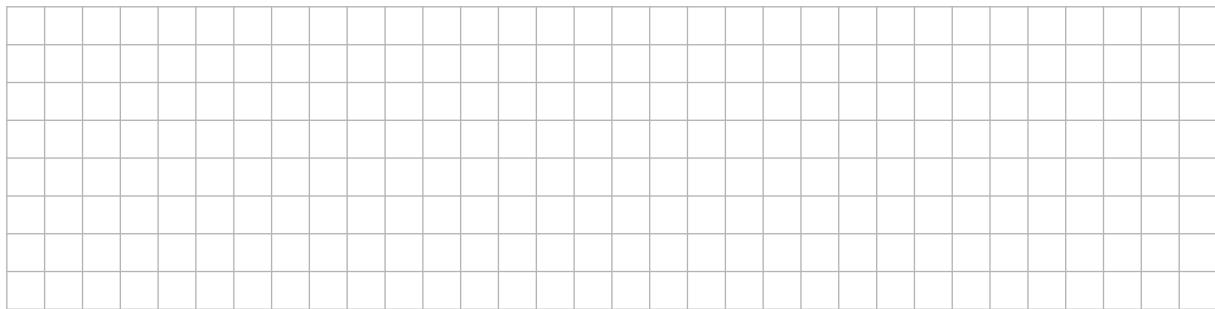
(e) Check your answer to part **(d)(i)** using your formulas from part **(b)**.

(f) The point of intersection can be found either by reading the graph or by using algebra. State one advantage of finding it using algebra.

(g) John's model for the growth of the plants might not be correct. State one limitation of the model that might affect the point of intersection and its interpretation.

Question 9 (50 marks)

(a) A farmer is growing winter wheat. The amount of wheat he will get per hectare depends on, among other things, the amount of nitrogen fertiliser that he uses. For his particular farm, the amount of wheat depends on the nitrogen in the following way:

$$Y = 7000 + 32N - 0.1N^2$$

where Y is the amount of wheat produced, in kg per hectare, and N is the amount of nitrogen added, in kg per hectare.
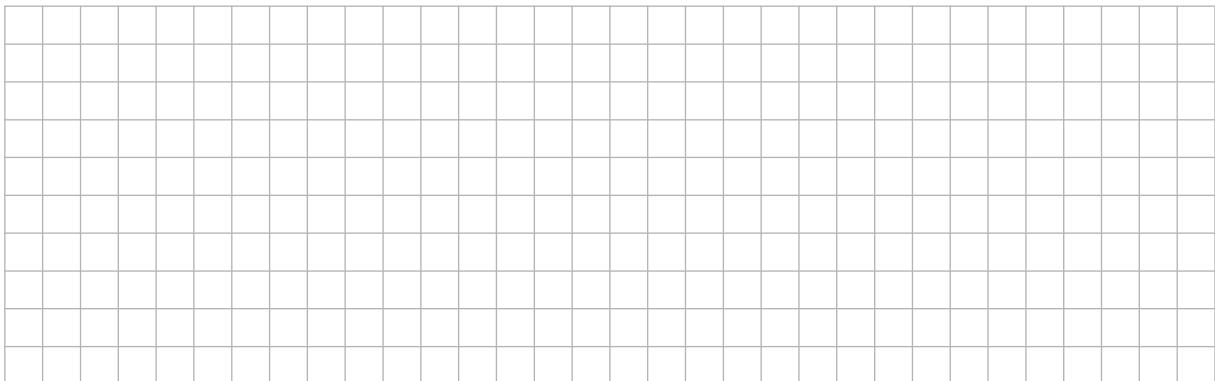
Photo: author: *P177*. Wikimedia Commons. CC BY-SA 3.0

(i) How much wheat will he get per hectare if he uses 100 kg of nitrogen per hectare?
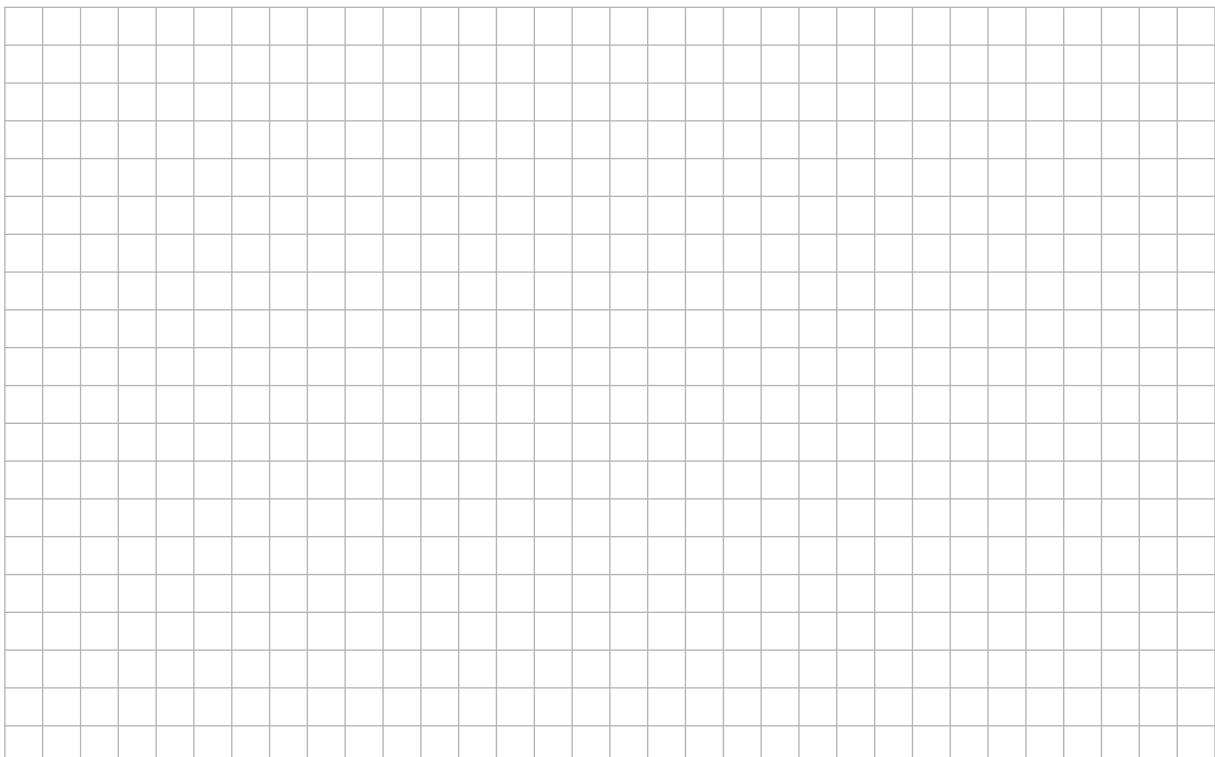
(ii) Find the amount of nitrogen that he must use in order to maximise the amount of wheat produced.

(iii) What is the maximum possible amount of wheat produced per hectare?

(iv) The farmer's total costs for producing the wheat are €1300 per hectare. He can sell the wheat for €160 per tonne. He can also get €75 per hectare for the leftover straw. If he achieves the maximum amount of wheat, what is his profit per hectare?
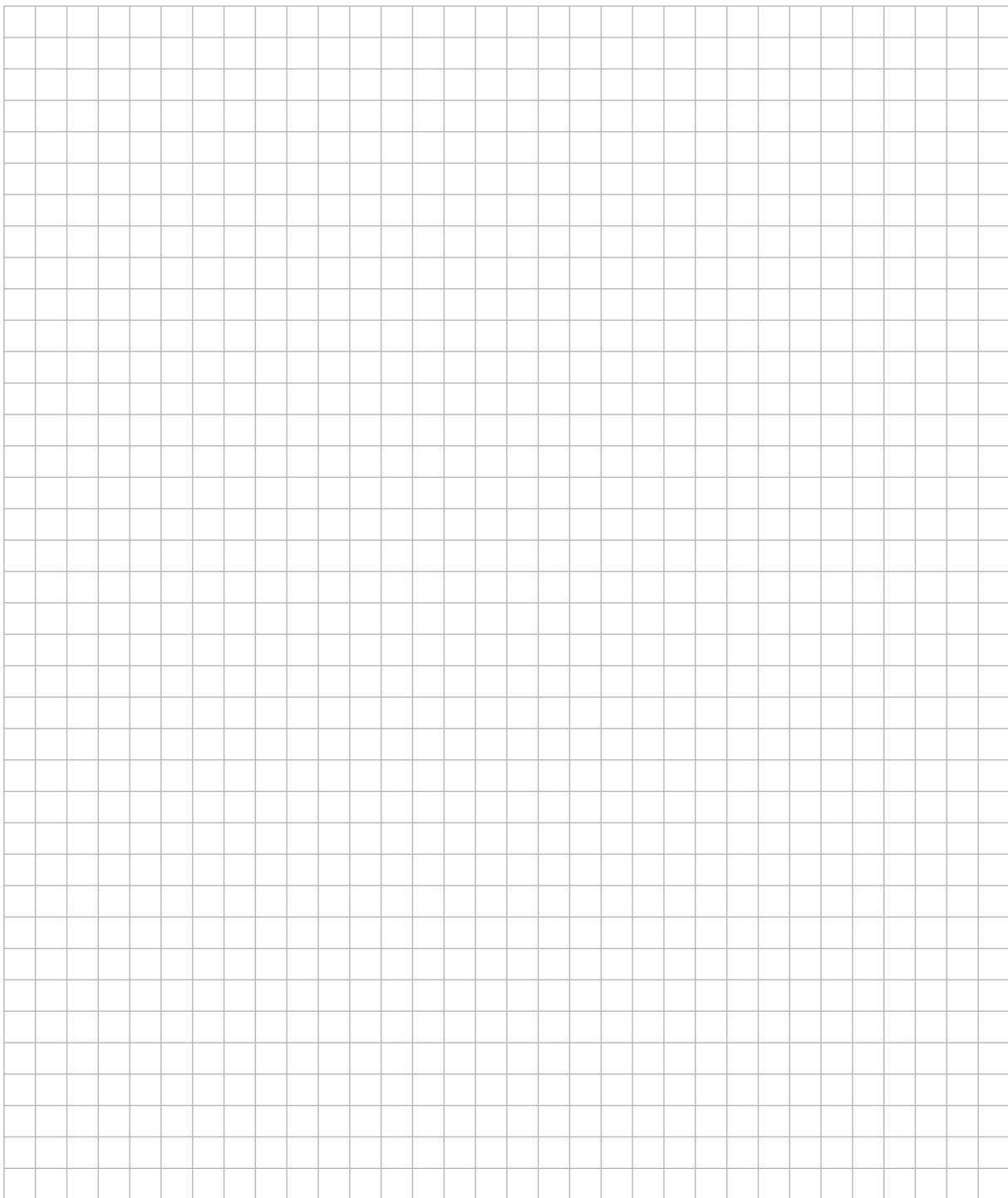
(b) A marble is dropped from the top of a fifteen-storey building. The height of the marble above the ground, in metres, after t seconds is given by the formula:

$$h(t) = 44 \cdot 1 - 4 \cdot 9t^2.$$

Find the speed at which the marble hits the ground.
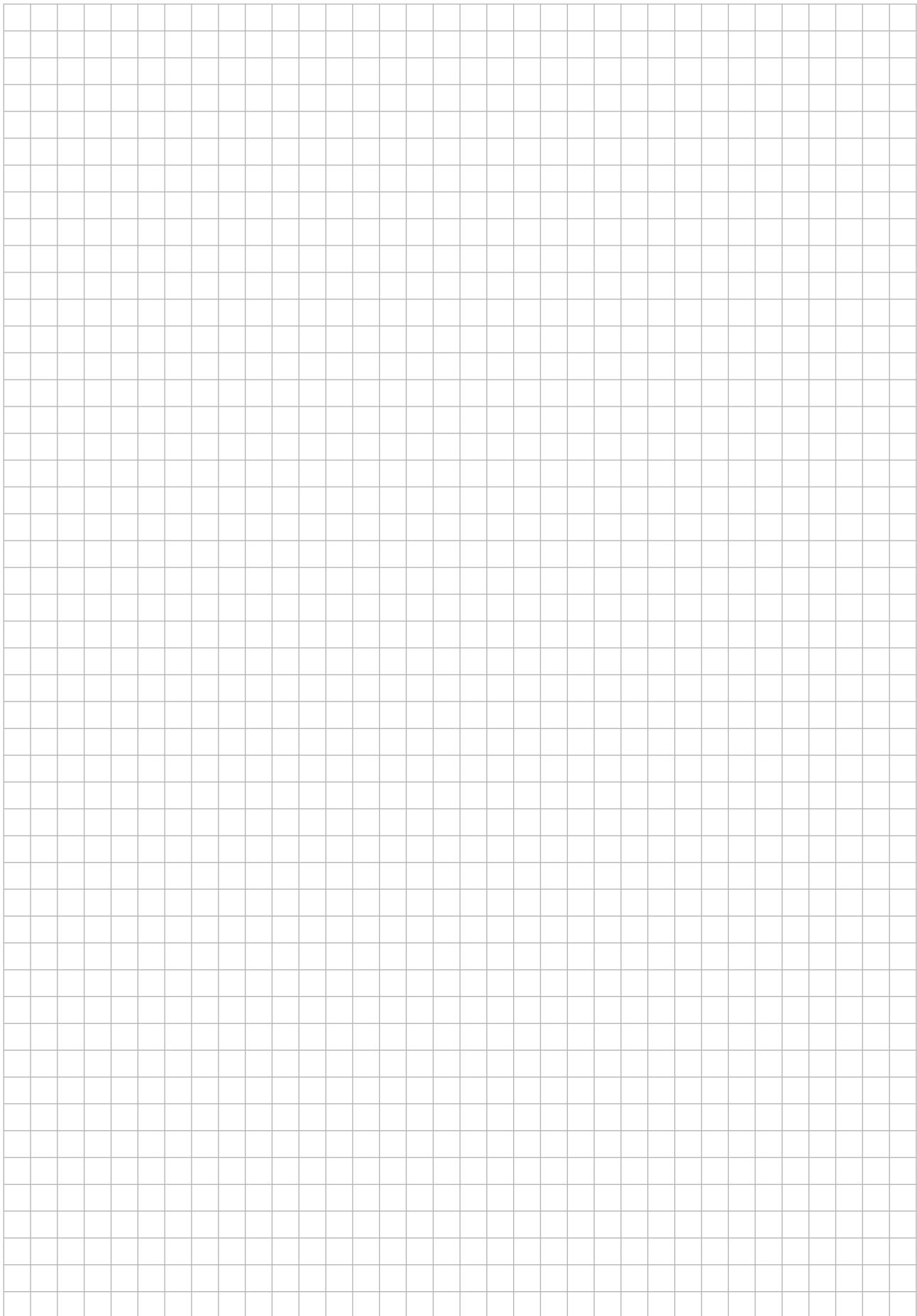
Give your answer   **(i)**   in metres per second, and

                          (ii)   in kilometres per hour.
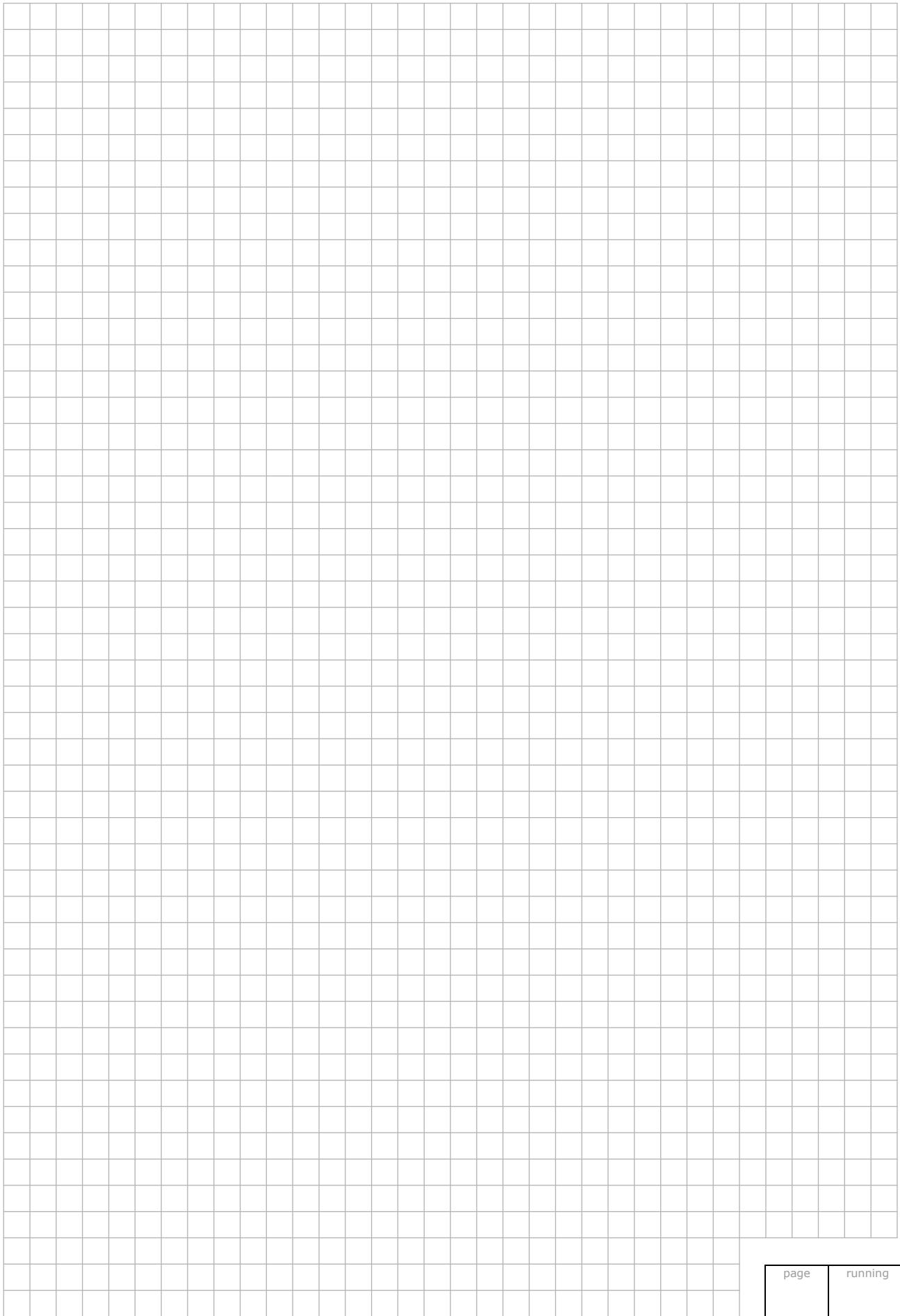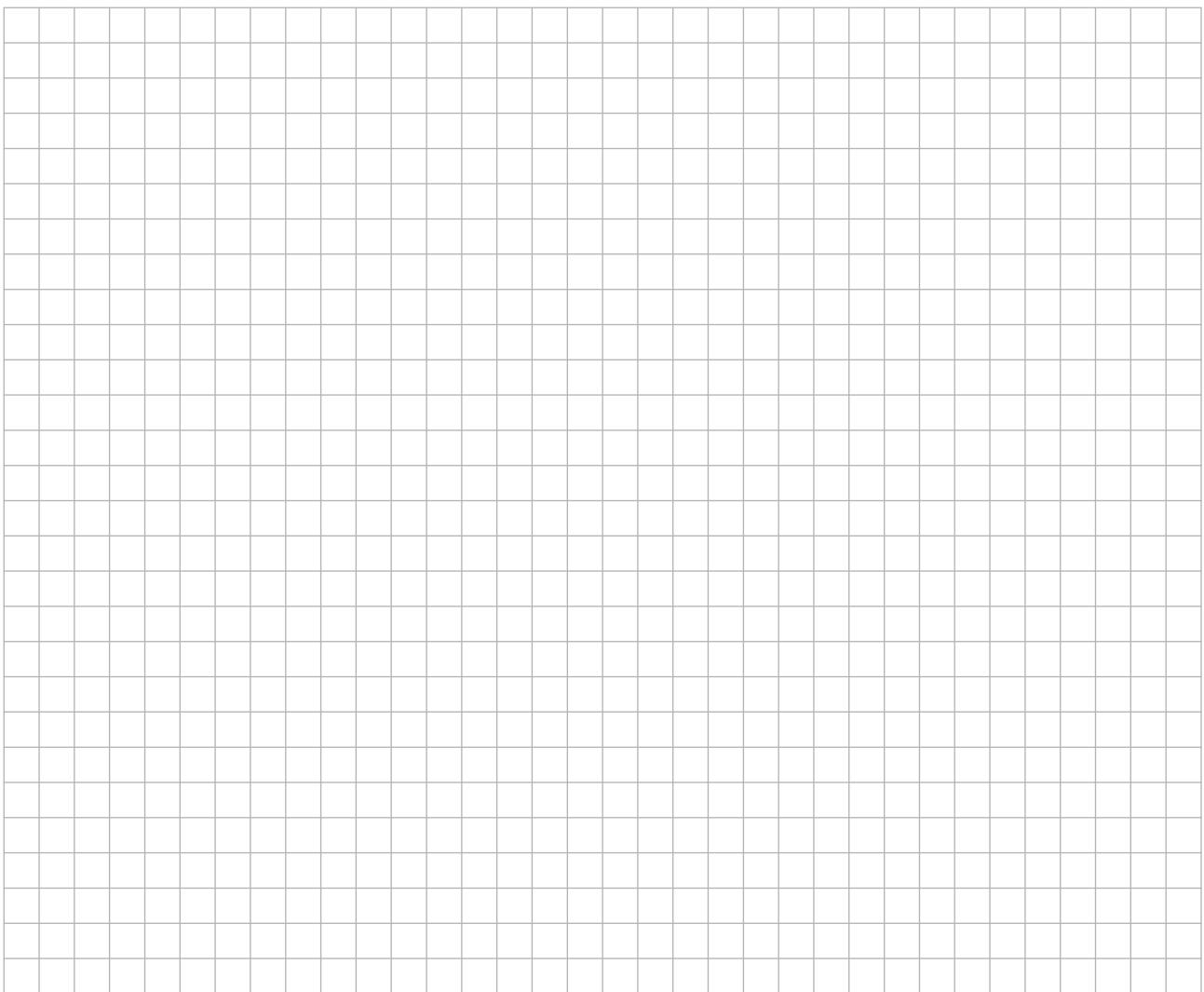
You may use this page for extra work.

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Mathematics (Project Maths – Phase 3) – Paper 1

Sample Paper
Time: 2 hours 30 minutes