



Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2018

Mathematics

Paper 2

Higher Level

Monday, 11 June – Morning 9:30 to 12:00

300 marks

Examination number

Centre stamp

Running total	
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For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are two sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You may lose marks if your solutions do not include supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **all six** questions from this section.

Question 1**(25 marks)**

In a competition Mary has a probability of $\frac{1}{20}$ of winning, a probability of $\frac{1}{10}$ of finishing in second place, and a probability of $\frac{1}{4}$ of finishing in third place. If she wins the competition she gets €9000. If she comes second she gets €7000 and if she comes third she gets €3000. In all other cases she gets nothing. Each participant in the competition must pay €2000 to enter.

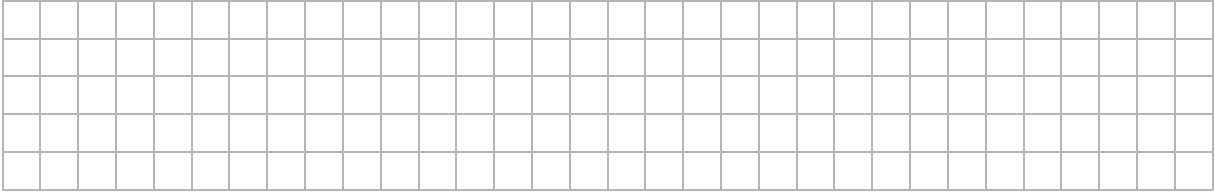
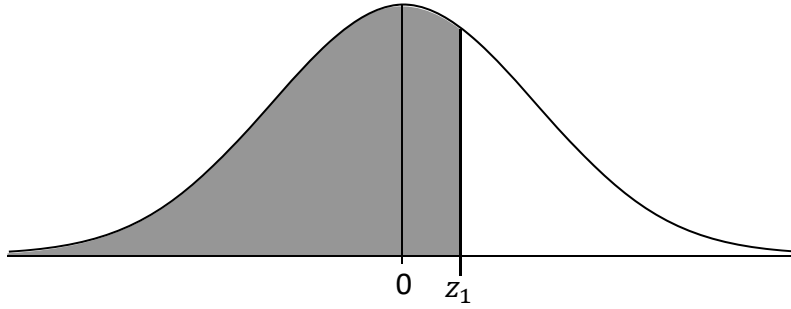
- (a) Find the **expected value** of Mary's loss if she enters the competition.

- (b) Each of the 3 prizes in the competition above is increased by the same amount (€ x) but the entry fee is unchanged.
For example, if Mary wins the competition now, she would get €(9000 + x).
Mary now expects to break even.
Find the value of x .

Question 2

(25 marks)

- (a) The diagram shows the standard normal curve. The shaded area represents 67% of the data. Find the value of z_1 .



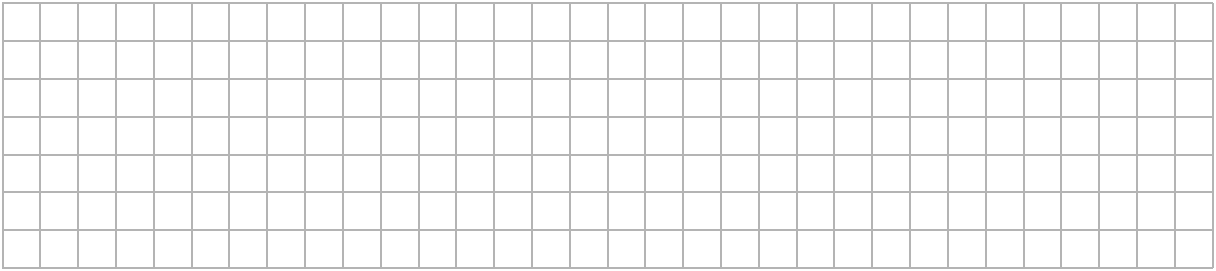
- (b) The percentage results in a Maths exam for a class had a mean mark of 70 with a standard deviation of 15. The percentage results in an English exam for the same class had a mean mark of 72 with a standard deviation of 10. The results in both exams were normally distributed.

- (i) Mary got 65 in Maths and 68 in English. In which exam did Mary do better relative to the other students in the class? Justify your answer.

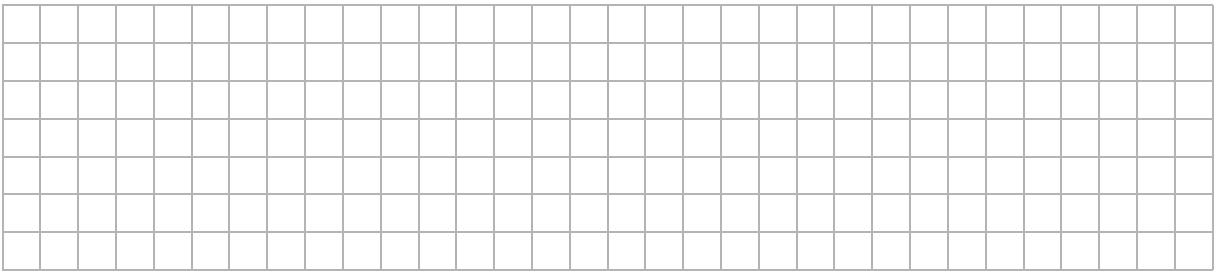
Answer: _____

Justification: _____

- (ii) In English the top 15% of students were awarded an A grade.
Find the least whole number mark that merited the award of an A grade in English.



- (iii) Using the empirical rule, or otherwise, estimate the percentage of students in the class who scored between 52 and 82 in the English test.



Question 4

(25 marks)

(a) Find **all** the values of x for which $\cos(2x) = -\frac{\sqrt{3}}{2}$, where $0^\circ \leq x \leq 360^\circ$.

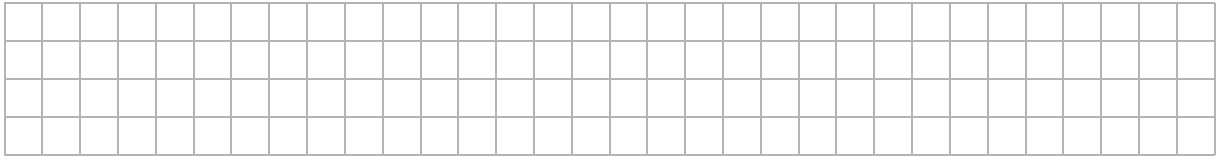
(b) Let $\cos A = \frac{y}{2}$, where $0^\circ < A < 90^\circ$. Write $\sin(2A)$ in terms of y .

Question 5

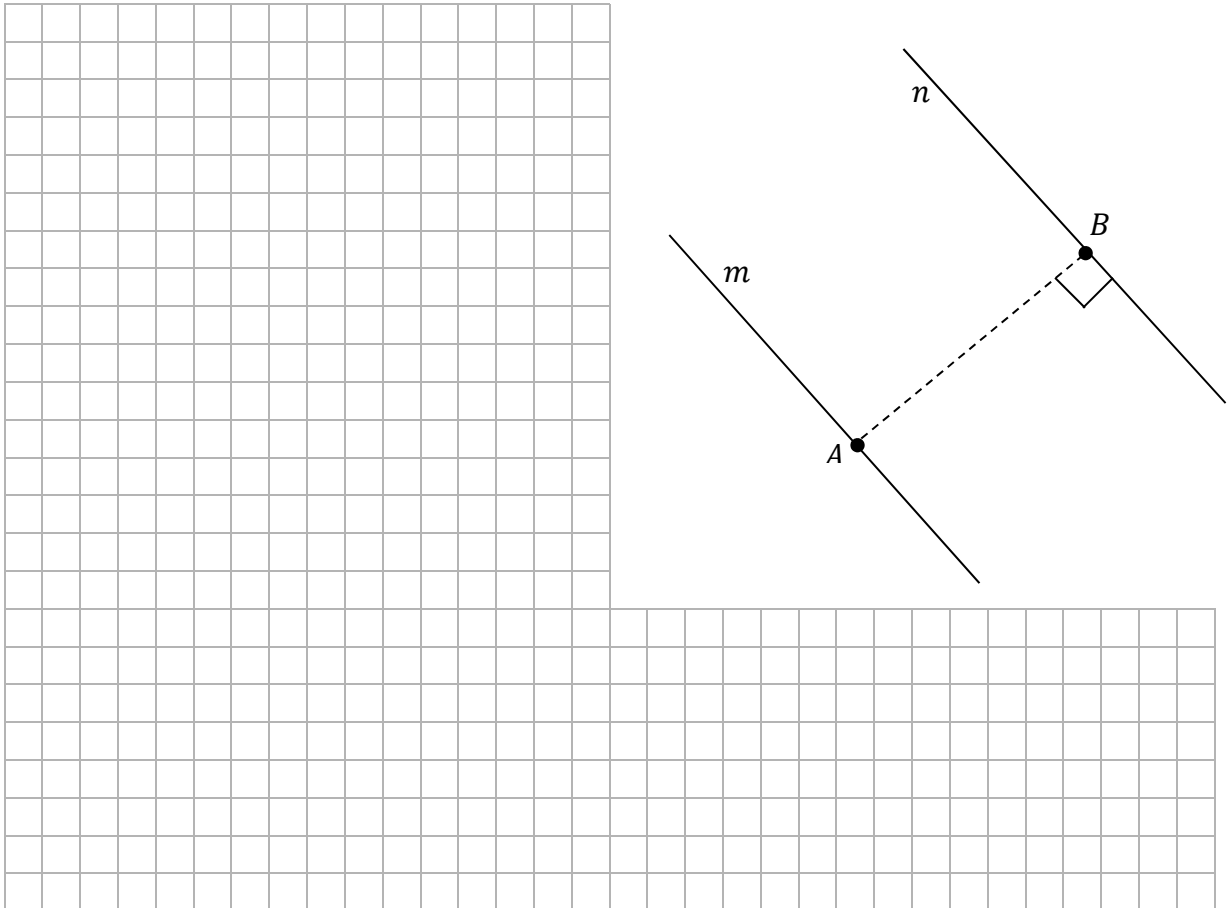
(25 marks)

The line $m: 2x + 3y + 1 = 0$ is parallel to the line $n: 2x + 3y - 51 = 0$.

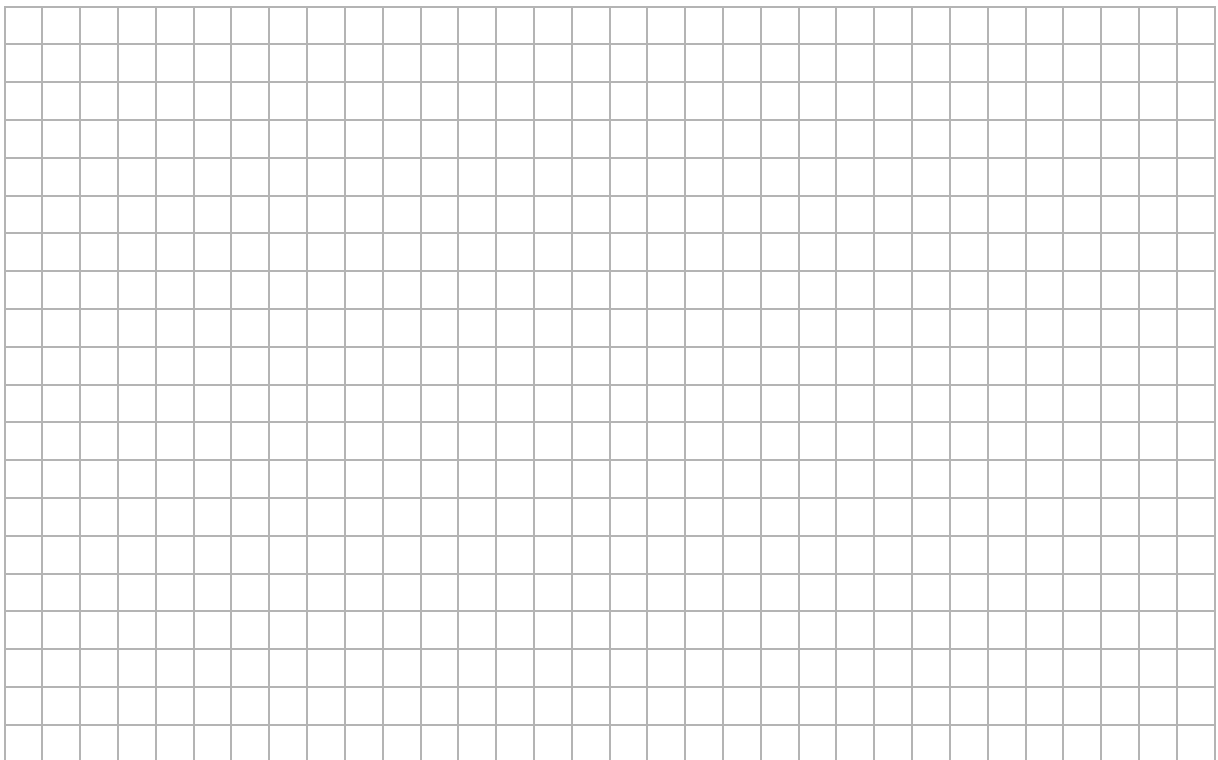
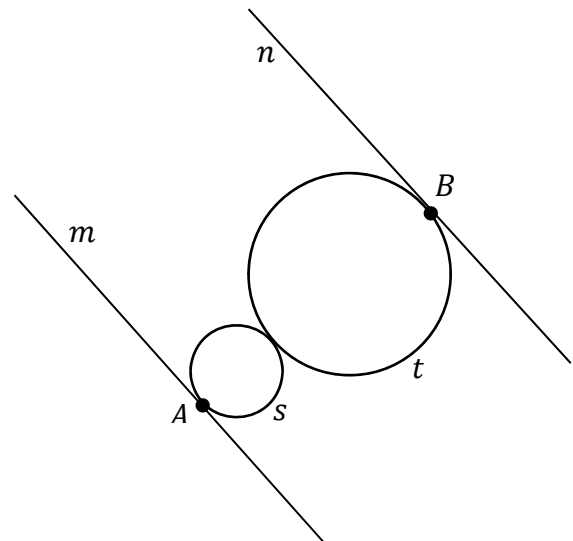
(a) Verify that $A(-2, 1)$ is on m .



(b) Find the coordinates of B , the point on the line n closest to A , as shown below.



- (c) Two touching circles, s and t , are shown in the diagram. m is a tangent to s at A and n is a tangent to t at B . The ratio of the radius of s to the radius of t is $1 : 3$. Find the equation of s .



Question 6

(25 marks)

- (a) Let $\triangle ABC$ be a triangle. Prove that if a line l is parallel to BC and cuts $[AB]$ in the ratio $s : t$, where $s, t \in \mathbb{N}$, then it also cuts $[AC]$ in the same ratio.

Diagram:

Given:

To Prove:

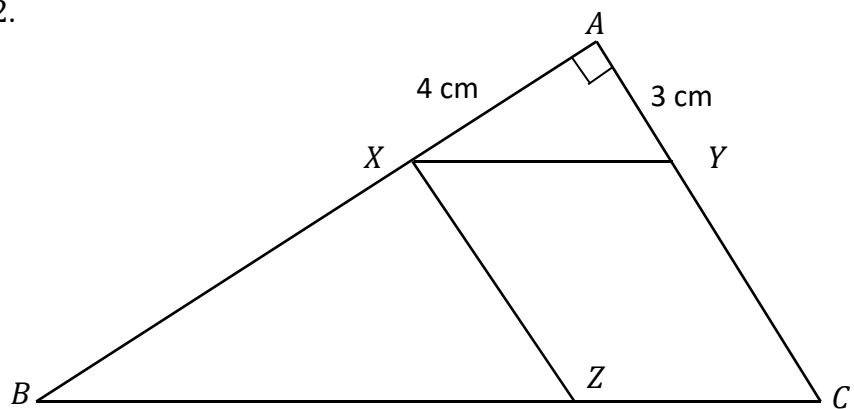
Construction:

Proof:

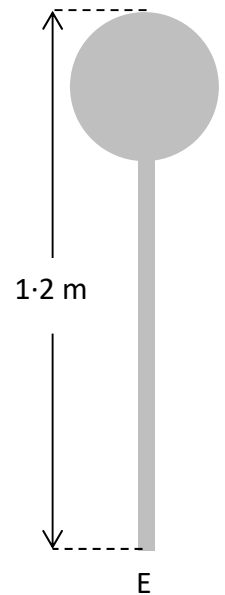
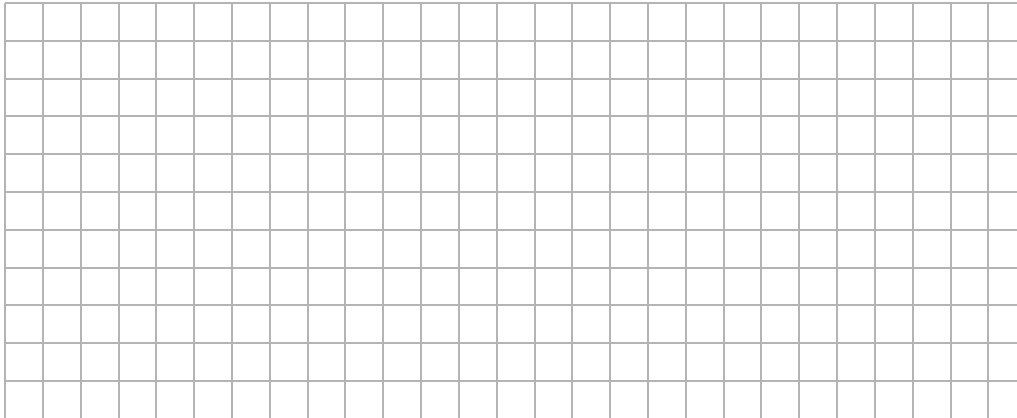
(b) In the triangle ABC shown below:

$|\angle CAB| = 90^\circ$, $|AX| = 4 \text{ cm}$, $|AY| = 3 \text{ cm}$, $XY \parallel BC$, $XZ \parallel AC$,
and $|AX| : |XB| = 1 : 2$.

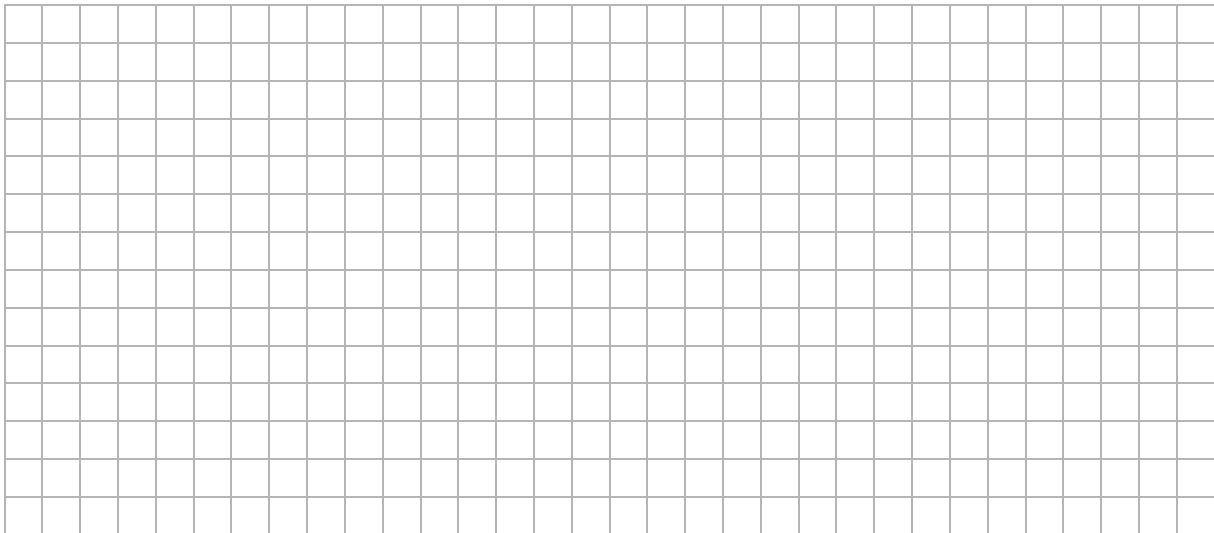
Find $|BZ|$.



- (b) (i) The **surface area** of sphere E can be taken to be 503 cm^2 .
 The height of the railing at E (i.e. the sum of the heights of bar E and sphere E) is 1.2 metres.
 Find the height of bar E, in cm, correct to 1 decimal place.



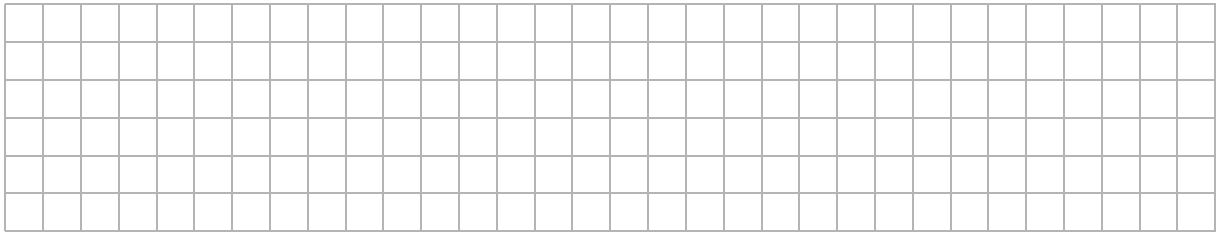
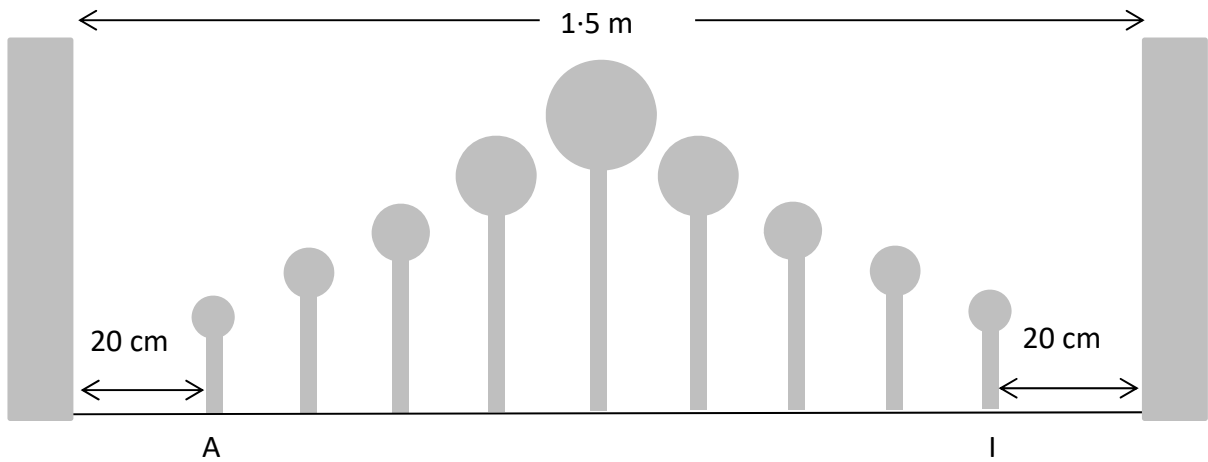
- (ii) The radius of each bar is 1 cm. The volume of bar A is $71.3\pi \text{ cm}^3$.
 The heights of the bars A, B, C, D, and E form an arithmetic sequence.
 Find, in cm, the height of each bar.



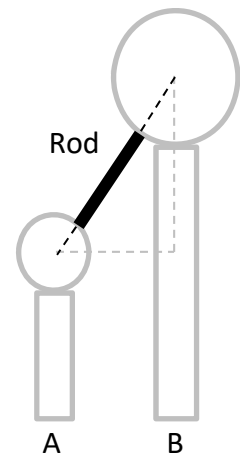
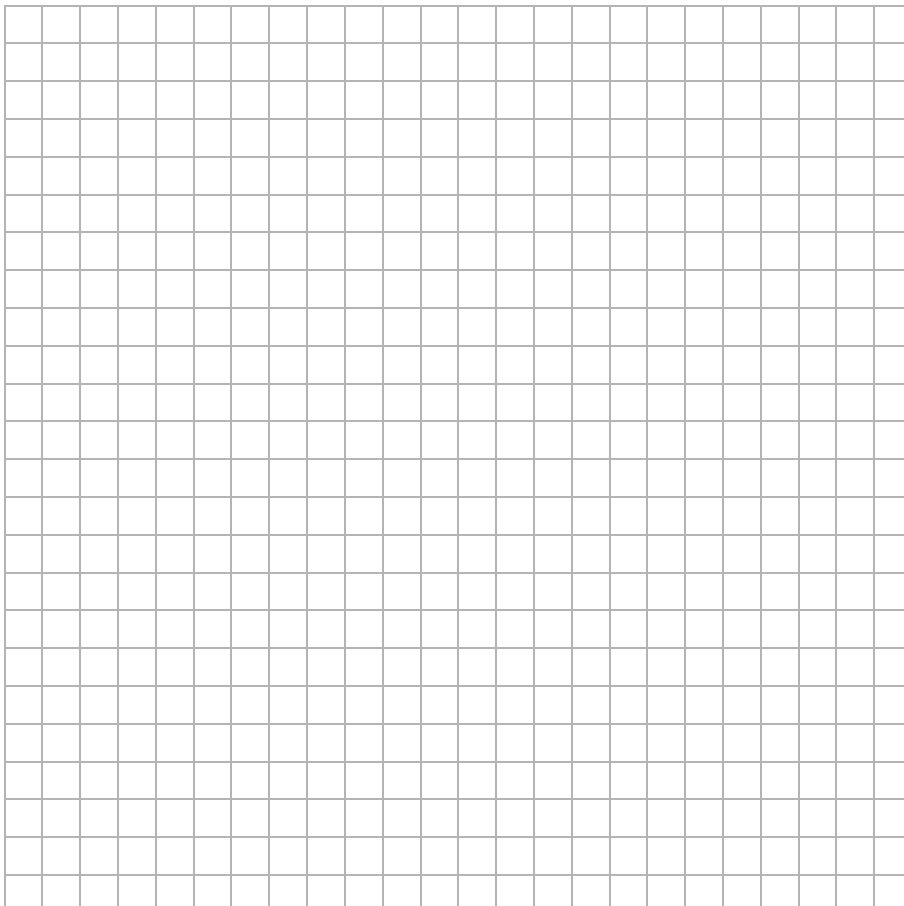
A =	B =	C =	D =	E =
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- (c) There is a wall on each side of the section of railing, as shown in the diagram below which is not to draw to scale. The distance from wall to wall is 1.5 m. The distance from the wall to bar A is 20 cm and similarly from the other wall to bar I is 20 cm. The radius of each bar is 1 cm. The gap between each bar is identical. Find the size of this gap.



- (d) The sphere on bar A and the sphere on bar B are to be joined by a straight rod as shown in the diagram below which is not to draw to scale. Find the length of the shortest rod that will join sphere A to sphere B. Give your answer in cm, correct to 1 decimal place.



Question 8

(60 marks)

Acme Confectionery makes cakes and chocolate bars.

- (a) (i) *Acme Confectionery* has launched a new bar called *Chocolate Crunch*. The weights of these new bars are normally distributed with a mean of 4.64 g and a standard deviation of 0.12 g. A sample of 10 bars is selected at random and the mean weight of the sample is found.

Find the probability that the mean weight of the sample is between 4.6 g and 4.7 g.

- (ii) A company surveyed 400 people, chosen from the population of people who had bought at least one *Chocolate Crunch* bar.

Of those surveyed, 324 of them said they liked the new bar.

Create the 95% confidence interval for the population proportion who liked the new bar.

Give your answer correct to 2 decimal places.

This question continues on the next page

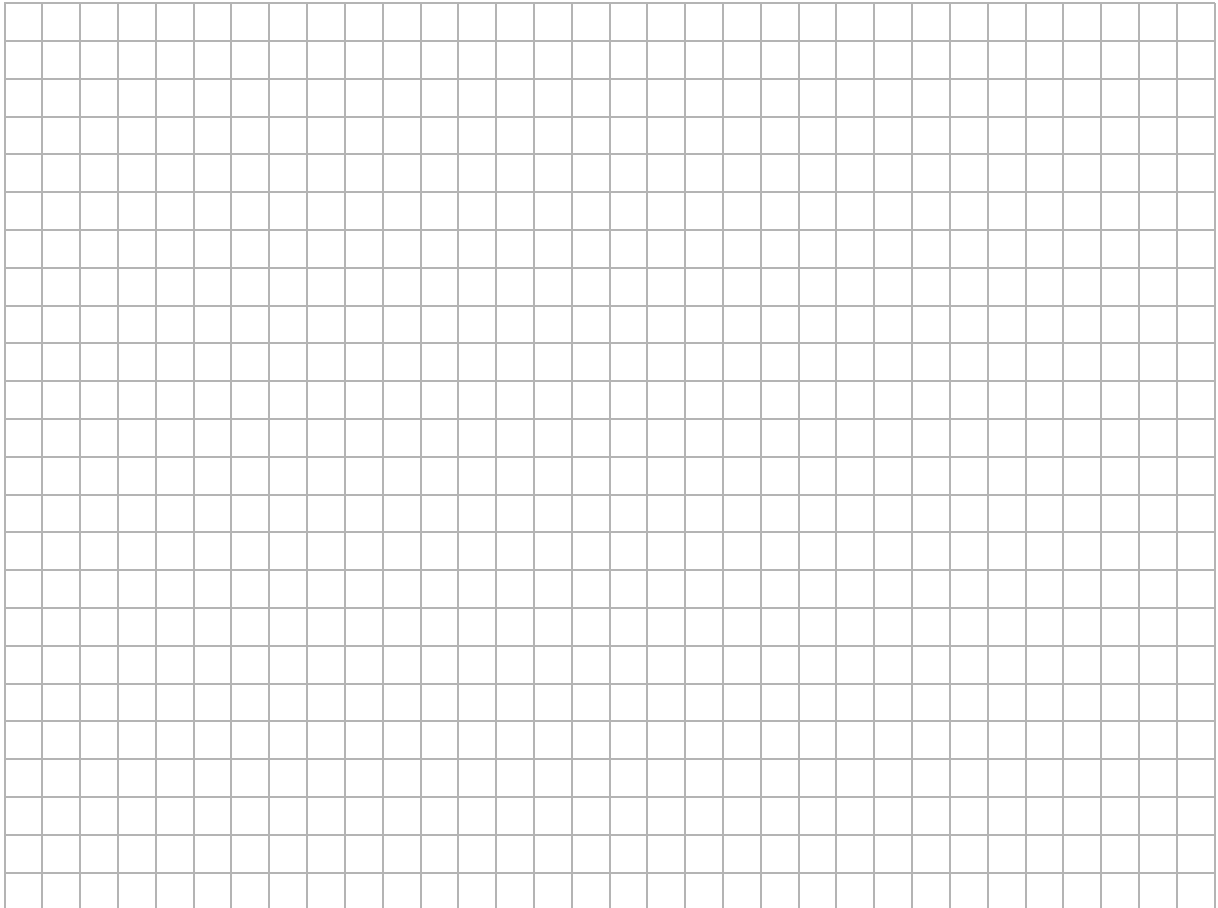
- (b) (i) Put one tick into the table for each statement to indicate whether the statement is **Always True**, **Sometimes True** or **Never True**.
 In the table, n is the size of the sample and \hat{p} is the sample proportion.

Statement	Always True	Sometimes True	Never True
1. When forming confidence intervals (for fixed n and \hat{p}), an increased confidence level implies a wider interval.			
2. As the value of \hat{p} increases (for fixed n), the estimated standard error of the population proportion increases.			
3. As the value of $\hat{p}(1 - \hat{p})$ increases (for fixed n), the estimated standard error of the population proportion increases.			
4. As n , the number of people sampled, increases (for fixed \hat{p}), the estimated standard error of the population proportion increases.			

- (ii) Using calculus or otherwise, find the maximum value of $\hat{p}(1 - \hat{p})$.

- (iii) Hence, find the largest possible value of the radius of the 95% confidence interval for a population proportion, given a random sample of size 800.

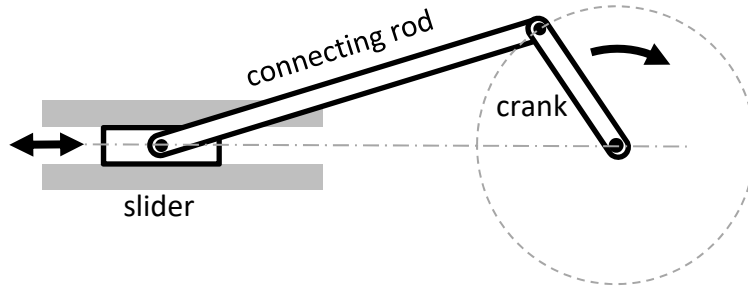
- (c) *Acme Confectionery* has an employee pension plan. For an employee who qualifies for the full pension, *Acme Confectionery* will pay a sum of €20 000 on the day of retirement. It will then pay a sum on the same date each subsequent year for the next 25 years. Each year the employee is paid a sum that is 1% more than the amount paid in the previous year. What sum of money must the company have set aside on the day of retirement in order to fund this pension? Assume an annual interest rate (AER) of 2.4%.



Question 9

(40 marks)

In engineering, a crank-and-slider mechanism can be used to change circular motion into motion back and forth in a straight line.

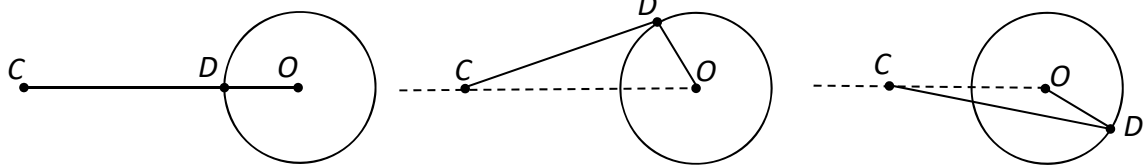


In the diagrams below, the crank $[OD]$ rotates about the fixed point O . The point C slides back and forth in a horizontal line. $[CD]$ is the rod that connects C to the crank. The diagrams below show three of the possible positions for C and D . $|OD| = 10$ cm and $|DC| = 30$ cm.

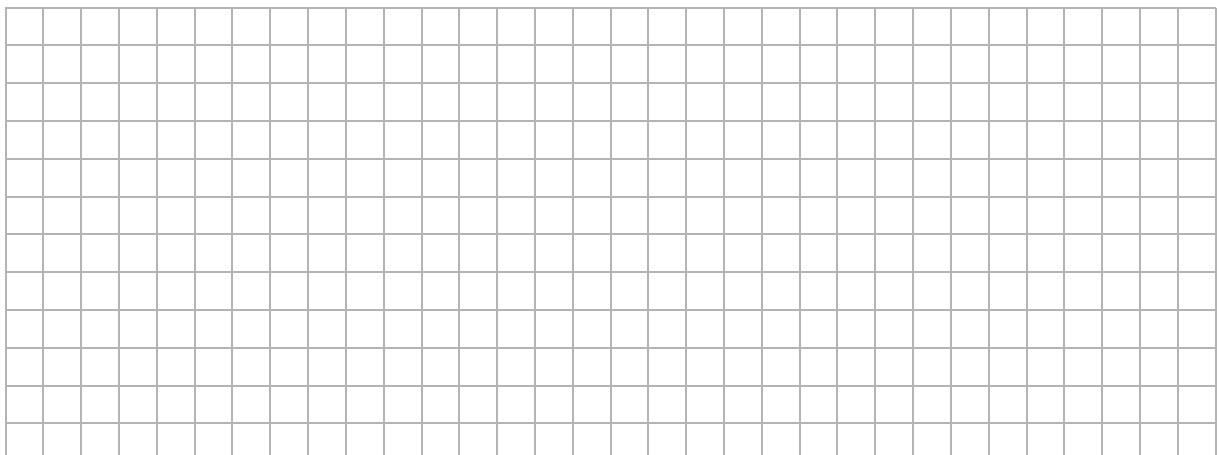
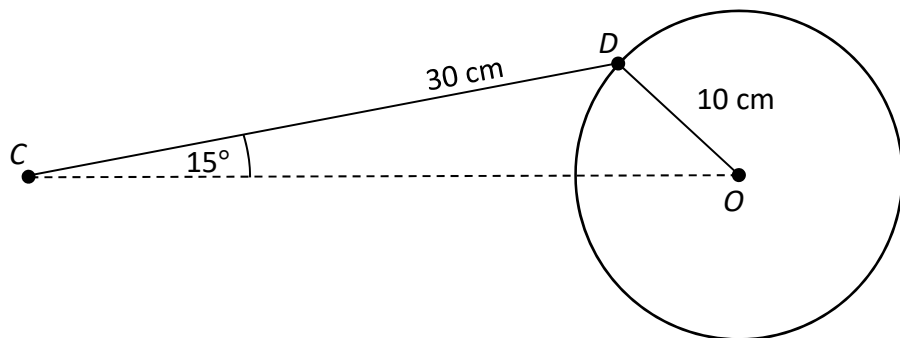
Diagram 1
(Starting position)

Diagram 2

Diagram 3



- (a) The diagram below shows a particular position of the mechanism with $|\angle DCO| = 15^\circ$. Find $|\angle COD|$, correct to the nearest degree.



(b) As D moves in a circle around O , the angle α in the diagram below increases. The distance $|CX|$ can be considered to be a function of α and written as $f(\alpha)$.

(i) Write down the period and range of f .

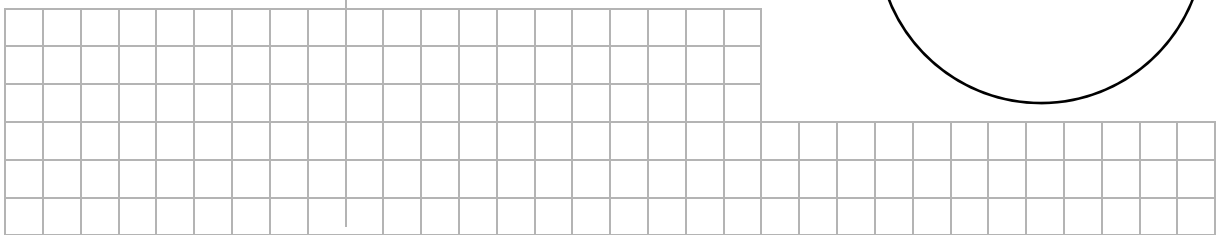
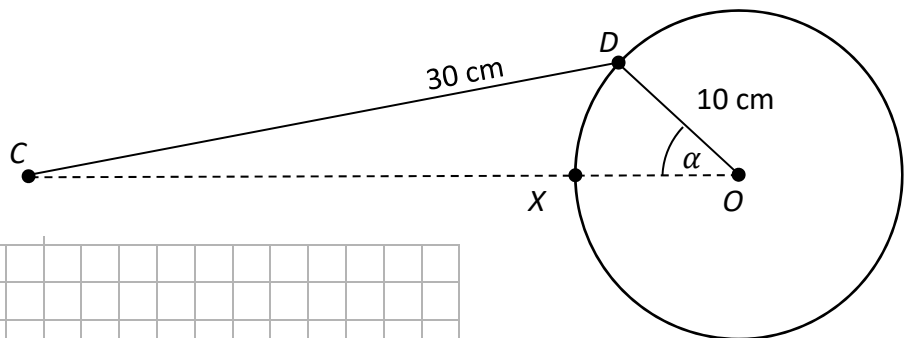
Period =	Range =
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(ii) Complete the table below for $f(\alpha)$.

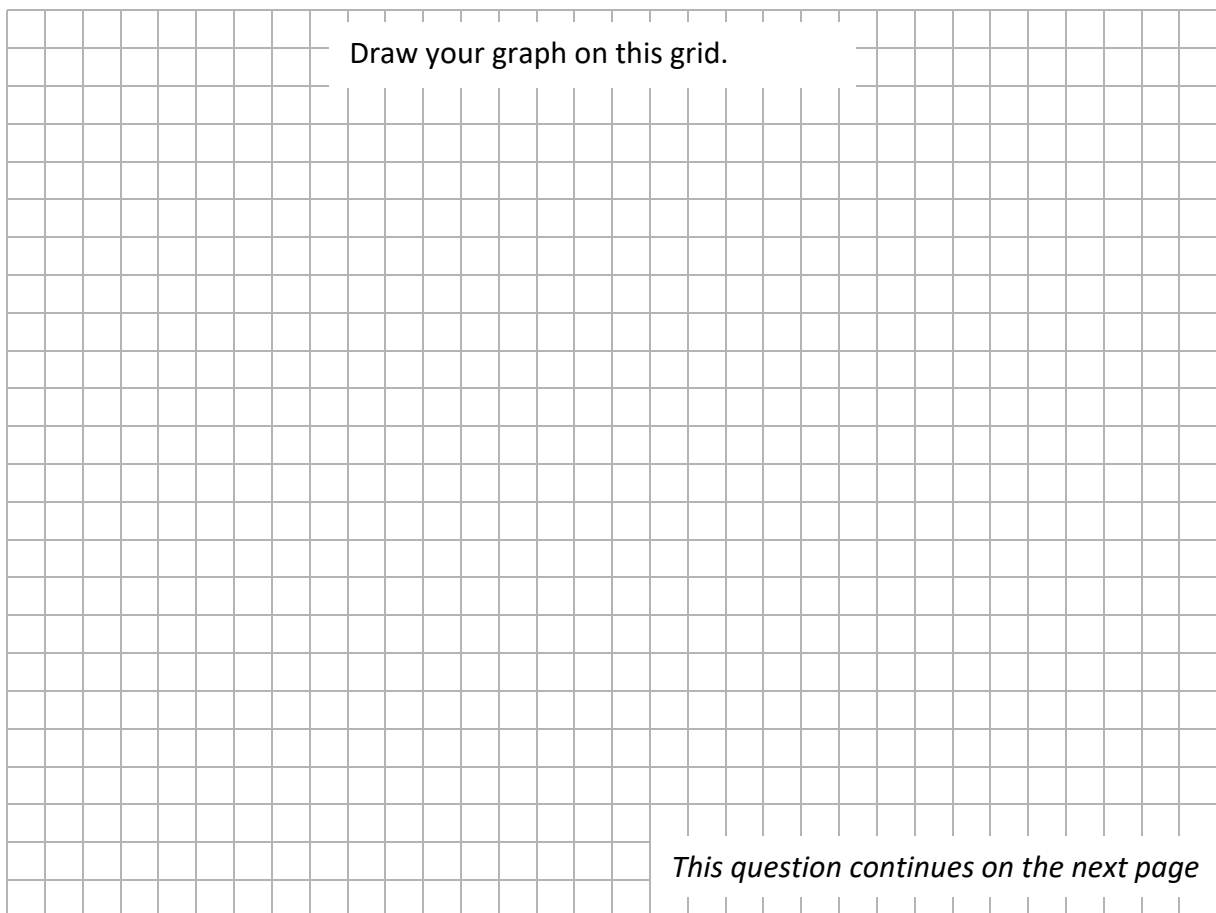
Give your answers correct to 2 decimal places where appropriate.

(Note: **Diagram 1** at the start of this question represents $\alpha = 0^\circ$).

α	0°	90°	180°	270°	360°
$f(\alpha)$ (cm)	30				



(iii) Use your values from the table to draw a rough sketch of f in the domain $0^\circ \leq \alpha \leq 360^\circ$.



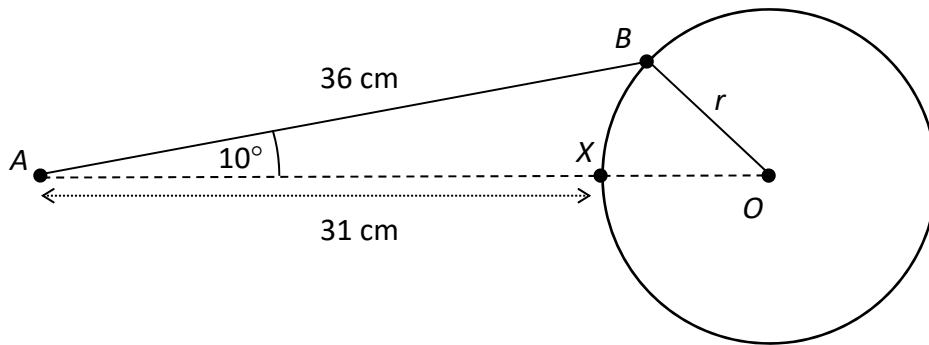
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- (iv) Referring to **Diagrams 1, 2, and 3** near the start of this question, for which of the three positions of the mechanism will a 1 degree change in α cause the greatest change in the position of C ? Explain your answer.

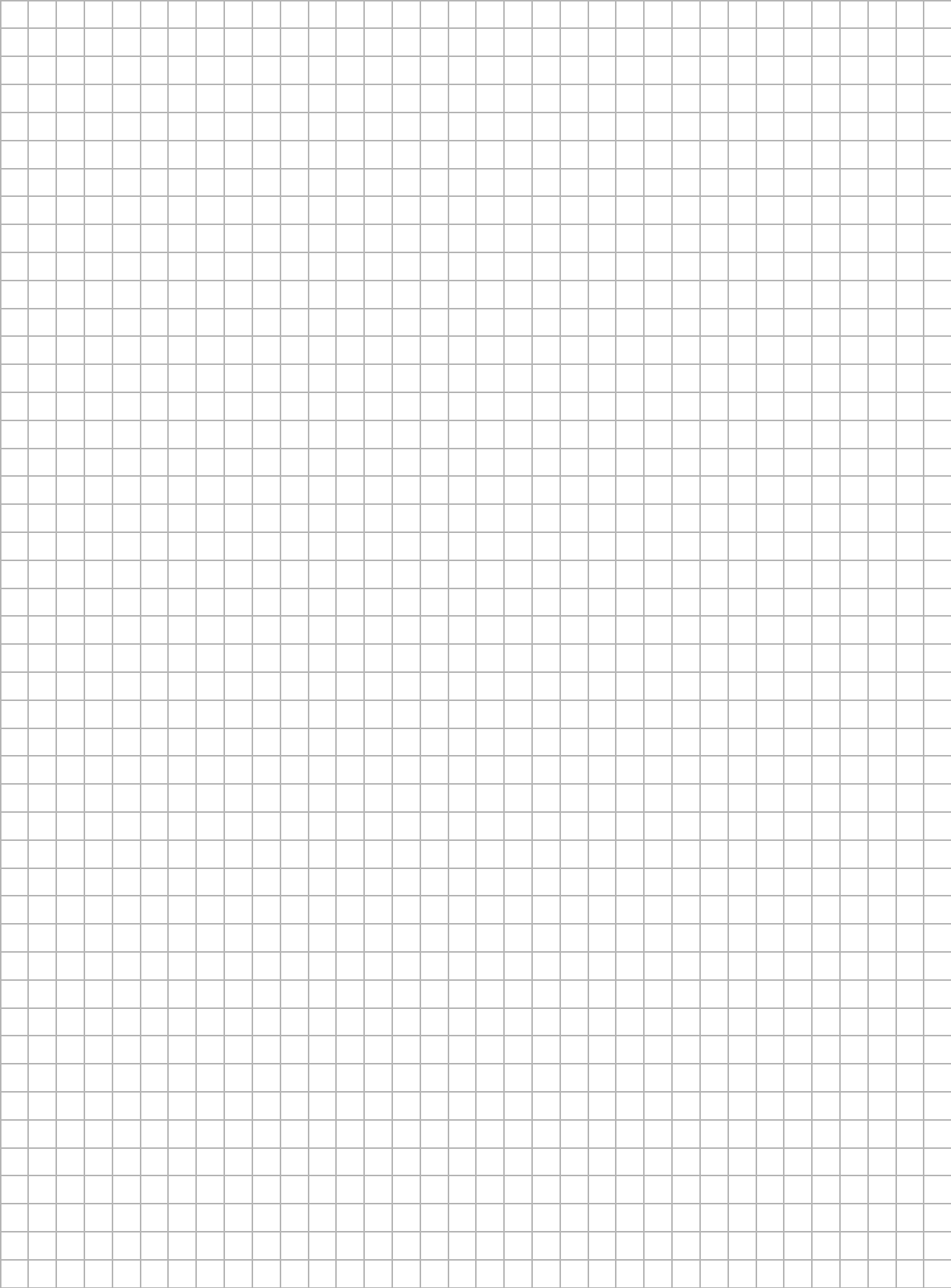
Answer: _____

Explanation: _____

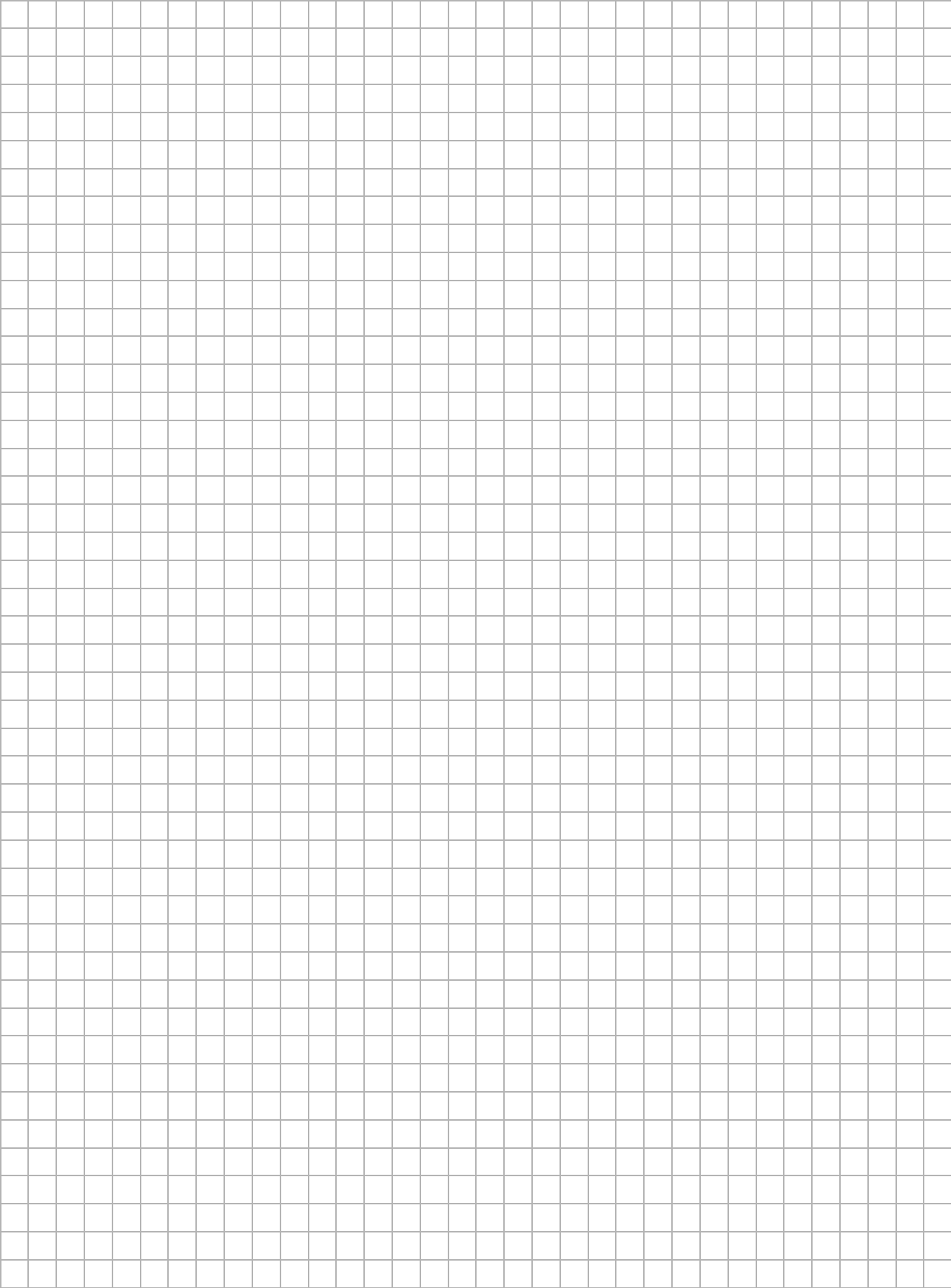
- (c) The diagram below shows another crank-and-slider mechanism with different dimensions. In the diagram, $|AB| = 36$ cm, $|AX| = 31$ cm, and $|\angle BAO| = 10^\circ$.
(Note: $|\angle OBA| \neq 90^\circ$)
 Find r , the length of the crank. Give your answer in cm, correct to the nearest cm.



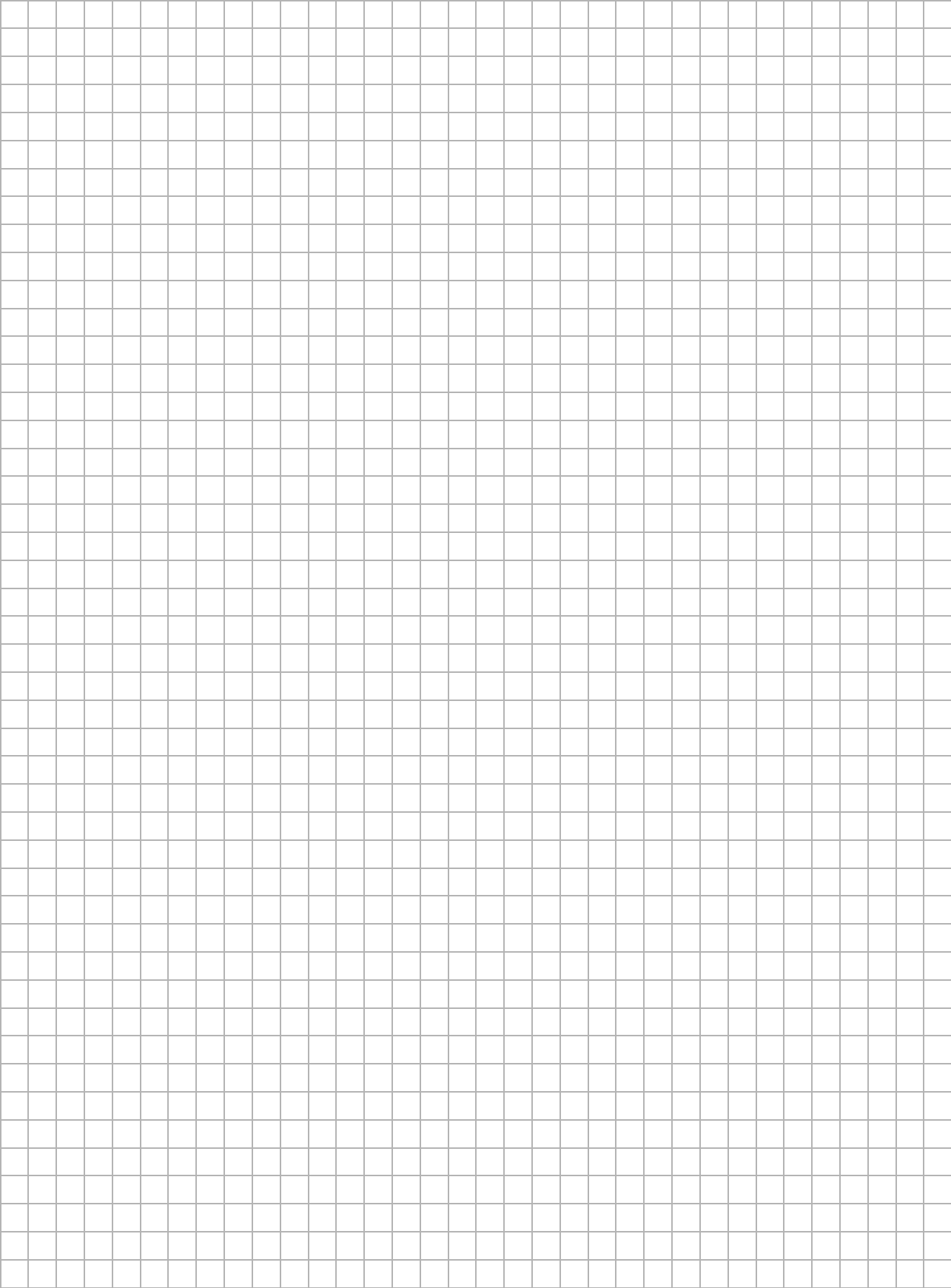
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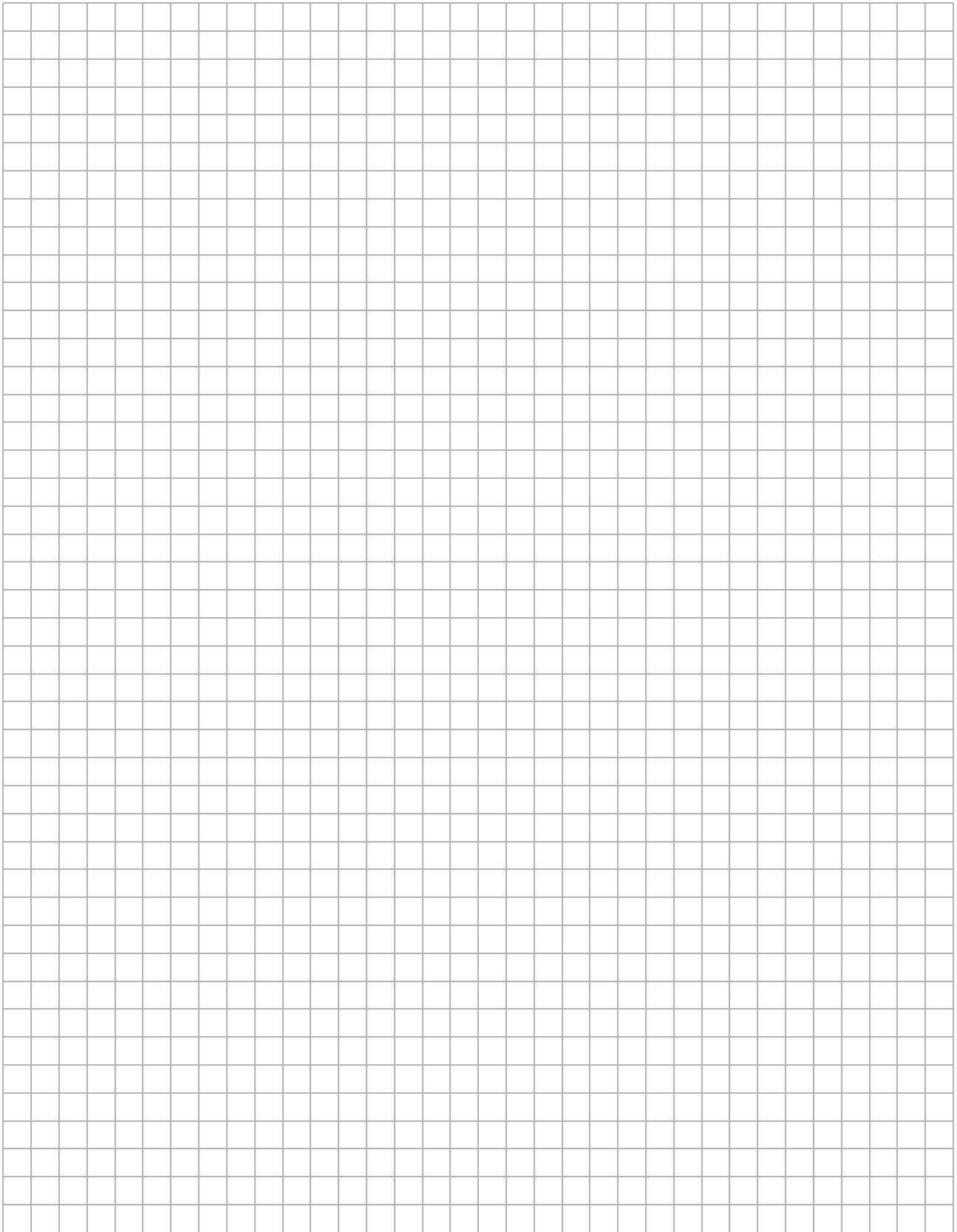


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Morning, 9:30 to 12:00