Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination

Mathematics (Project Maths)

Paper 2

Higher Level

Monday 14 June   Morning 9:30 – 12:00

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions, as follows:

In Section A, answer all six questions.

In Section B, answer:

Question 7

Question 8

either Question 9A **or** Question 9B.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

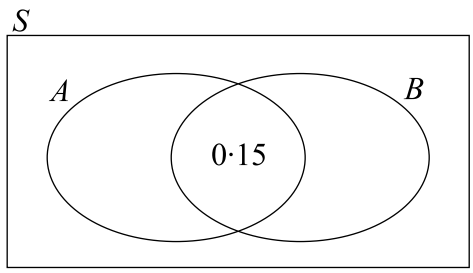
Section A	Concepts and Skills	150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

Two events A and B are such that $P(A) = 0{\cdot}2$, $P(A \cap B) = 0{\cdot}15$ and $P(A' \cap B) = 0{\cdot}6$.

(a) Complete this Venn diagram.



(b) Find the probability that neither A nor B happens.

(c) Find the conditional probability $P(A \mid B)$.

(d) State whether A and B are independent events and justify your answer.

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Question 2 **(25 marks)**

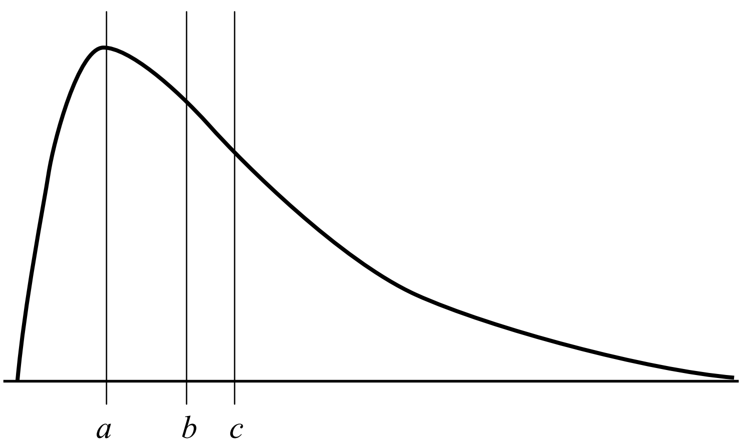
(a) The back-to-back stem-and-leaf diagram below shows data from two samples. The corresponding populations are assumed to be identical in shape and spread. Use the *Tukey quick test* to test, at the 5% significance level, the hypothesis that the populations have the same average.

				6	3							
8	6	6	3	0	4	4						
	5	3	2	0	5	1	5	5	7	7	8	9
		2	0	0	6	1	4	9				
					7	2	4					
					8	1						

key: 6 | 3 | means 36 key: | 8 | 1 means 81

Conclusion:

(b) The diagram below shows a skewed frequency distribution. Vertical lines have been drawn through the mean, mode and median. Identify which is which by inserting the relevant letters in the spaces below.

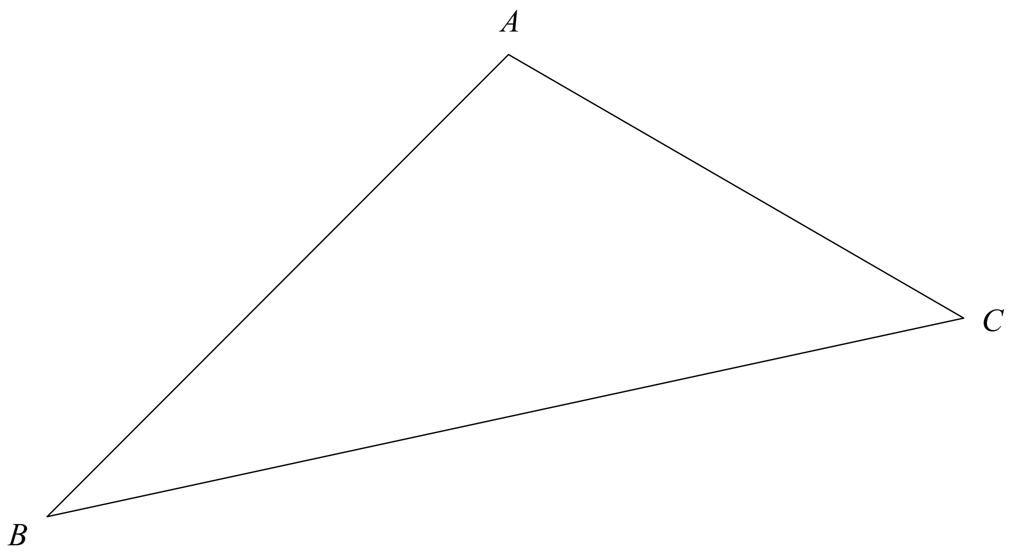


mean = ________ mode = ________ median = ______

Question 3 **(25 marks)**

(a) Construct the incircle of the triangle *ABC* below using only a compass and straight edge.

Show all construction lines clearly.



(b) An equilateral triangle has sides of length 2 units.
Find the area of its incircle.

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Question 4 **(25 marks)**

(a) The centre of a circle lies on the line $x-2y-1=0$. The x-axis and the line $y=6$ are tangents to the circle. Find the equation of this circle.

(b) A different circle has equation $x^2+y^2-6x-12y+41=0$. Show that this circle and the circle in part **(a)** touch externally.

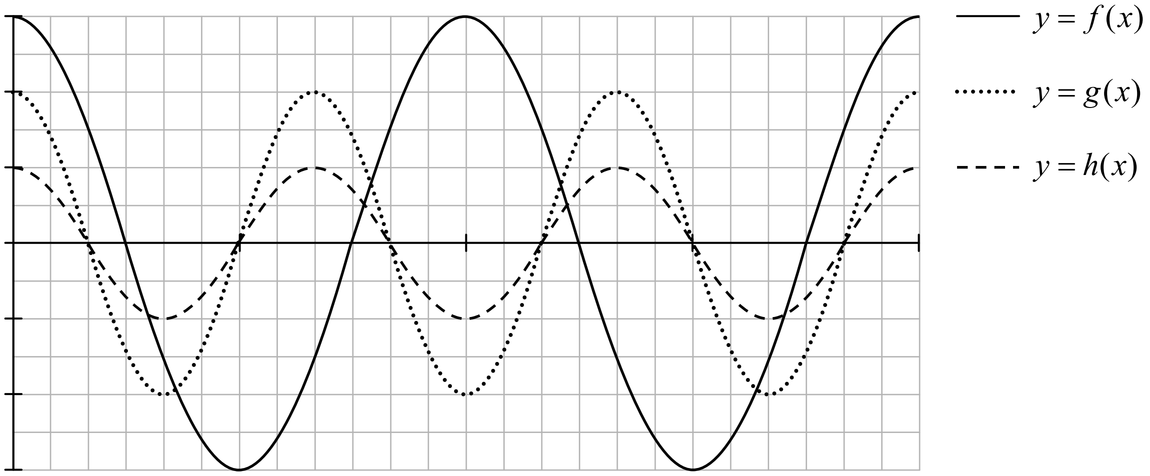
Question 5 **(25 marks)**

(a) Solve the equation $\cos 3\theta = \frac{1}{2}$, for $\theta \in \mathbb{R}$, (where θ is in radians).

(b) The graphs of three functions are shown on the diagram below. The scales on the axes are not labelled. The three functions are:

$$x \to \cos 3x$$
$$x \to 2\cos 3x$$
$$x \to 3\cos 2x$$

Identify which function is which, and write your answers in the spaces below the diagram.



$f : x \to$ _______________ $g : x \to$ _______________ $h : x \to$ _______________

(c) Label the scales on the axes in the diagram in part **(b)**.

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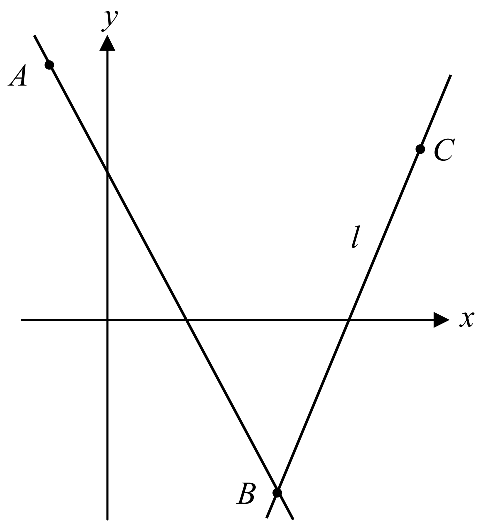
Question 6 **(25 marks)**

Three points A, B and C have co-ordinates:
$A(-2,9)$, $B(6,-6)$ and $C(11,6)$.

The line l passes through B and has equation $12x-5y-102=0$.



(a) Verify that C lies on l.

(b) Find the slope of AB, and hence find $\tan(\angle ABC)$, as a fraction.

(c) Find the vectors $\overrightarrow{BC}$ and $\overrightarrow{BA}$ in terms of $\vec{i}$ and $\vec{j}$.

(d) Use the dot product to find $\cos(\angle ABC)$ and show that the answer is consistent with the answer to part **(b)**.

You may use this space for extra work.

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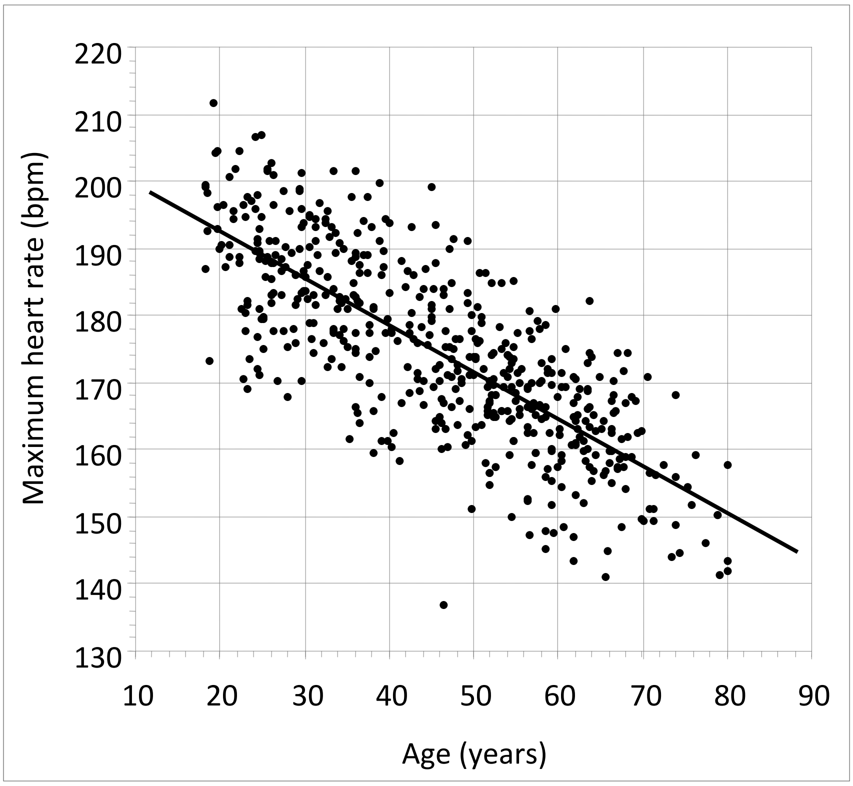
Section B	Contexts and Applications	150 marks

Answer Question 7, Question 8, and **either** Question 9A **or** Question 9B.

Question 7 **Probability and Statistics** **(50 marks)**

A person's *maximum heart rate* is the highest rate at which their heart beats during certain extreme kinds of exercise. It is measured in beats per minute (bpm). It can be measured under controlled conditions. As part of a study in 2001, researchers measured the maximum heart rate of 514 adults and compared it to each person's age. The results were like those shown in the scatter plot below.



Source: Simulated data based on:Tanaka H, Monaghan KD, and Seals DR. *Age-predicted maximal heart rate revisited*, J. Am. Coll. Cardiol. 2001;37;153-156.

(a) From the diagram, estimate the correlation coefficient. Answer:

(b) Circle the *outlier* on the diagram and write down the person's age and maximum heart rate.

Age = Max. heart rate =

(c) The line of best fit is shown on the diagram. Use the line of best fit to estimate the maximum heart rate of a 44-year-old person.

Answer:

(d) By taking suitable readings from the diagram, calculate the slope of the line of best fit.

(e) Find the equation of the line of best fit and write it in the form: $MHR = a - b \times (\text{age})$, where MHR is the maximum heart rate.

(f) The researchers compared their new rule for estimating maximum heart rate to an older rule. The older rule is: $MHR = 220 - \text{age}$. The two rules can give different estimates of a person's maximum heart rate. Describe how the level of agreement between the two rules varies according to the age of the person. Illustrate your answer with two examples.

(g) A particular exercise programme is based on the idea that a person will get most benefit by exercising at 75% of their estimated *MHR*. A 65-year-old man has been following this programme, using the old rule for estimating *MHR*. If he learns about the researchers' new rule for estimating *MHR*, how should he change what he is doing?

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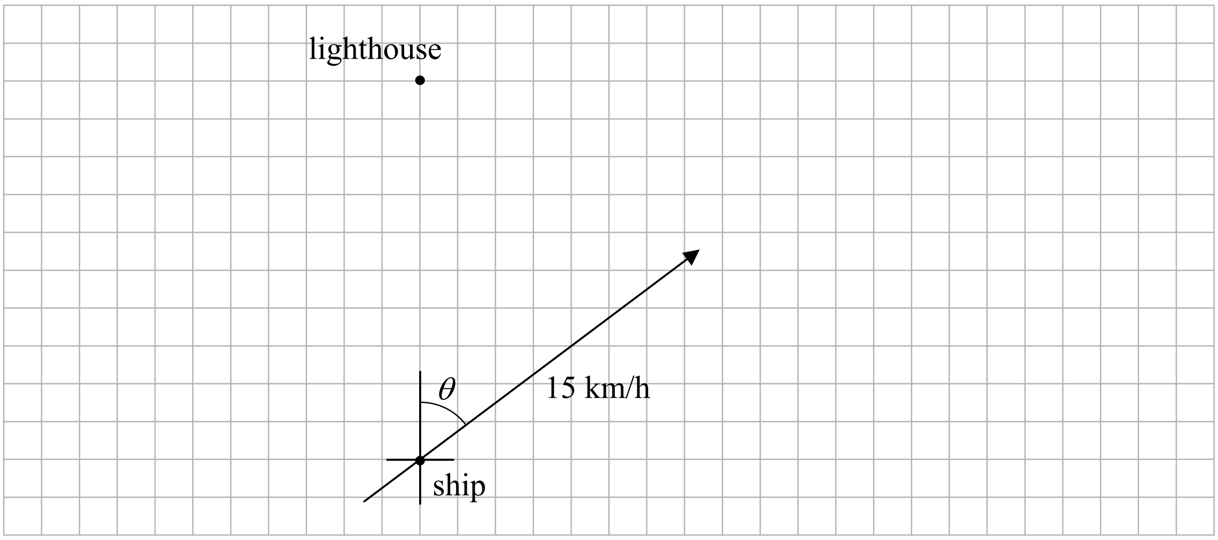
Question 8 **Geometry and Trigonometry** **(50 marks)**

A ship is 10 km due South of a lighthouse at noon.

The ship is travelling at 15 km/h on a bearing of θ, as shown below, where $\theta = \tan^{-1}\left(\frac{4}{3}\right)$.



(a) On the diagram above, draw a set of co-ordinate axes that takes the lighthouse as the origin, the line East-West through the lighthouse as the *x*-axis, and kilometres as units.

(b) Find the equation of the line along which the ship is moving.

(c) Find the shortest distance between the ship and the lighthouse during the journey.

(d) At what time is the ship closest to the lighthouse?

(e) Visibility is limited to 9 km. For how many minutes in total is the ship visible from the lighthouse?

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Question 9A **Probability and Statistics** **(50 marks)**

A factory manufactures aluminium rods. One of its machines can be set to produce rods of a specified length. The lengths of these rods are normally distributed with mean equal to the specified length and standard deviation equal to 0·2 mm.

The machine has been set to produce rods of length 40 mm.

(a) What is the probability that a randomly selected rod will be less than 39·7 mm in length?

(b) Five rods are selected at random. What is the probability that at least two of them are less than 39·7 mm in length?

(c) The operators want to check whether the setting on the machine is still accurate. They take a random sample of ten rods and measure their lengths. The lengths in millimetres are:

39·5	40·0	39·7	40·2	39·8
39·7	40·2	39·9	40·1	39·6

Conduct a hypothesis test at the 5% level of significance to decide whether the machine's setting has become inaccurate. You should start by clearly stating the null hypothesis and the alternative hypothesis, and finish by clearly stating what you conclude about the machine.

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Question 9B **Geometry and Trigonometry** **(50 marks)**

(a) Prove that if three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal line.

Diagram:

Given:

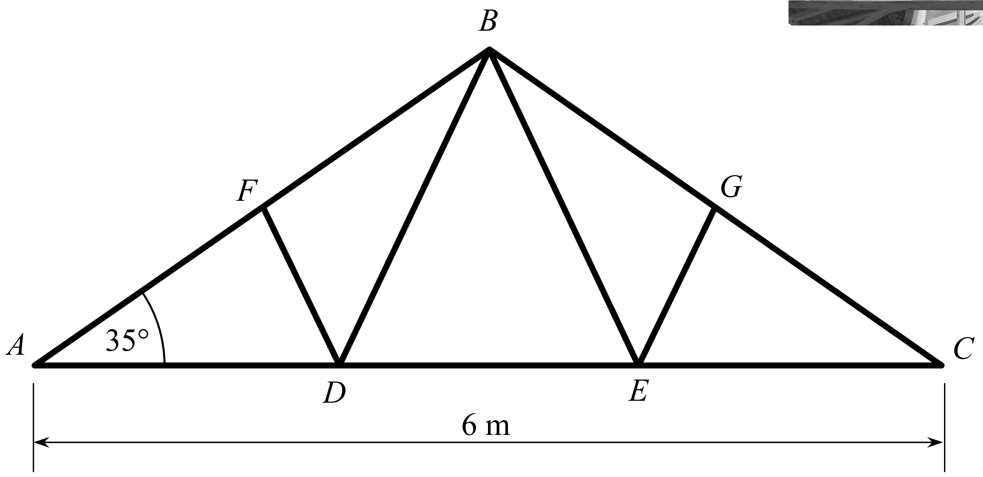
To prove:

Construction:

Proof:

(b) Roofs of buildings are often supported by frameworks of timber called *roof trusses*.

A quantity surveyor needs to find the total length of timber needed in order to make the triangular truss shown below.



The length of [AC] is 6 metres, and the pitch of the roof is 35°, as shown.
$|AD| = |DE| = |EC|$ and $|AF| = |FB| = |BG| = |GC|$.

(i) Calculate the length of [AB], in metres, correct to two decimal places.

(ii) Calculate the total length of timber required to make the truss.

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You may use this page for extra work

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