



Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2014
Sample Paper

Mathematics
(Project Maths – Phase 3)

Paper 1

Higher Level

Time: 2 hours, 30 minutes

300 marks

Examination number

Centre stamp

Running total	
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For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper:

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

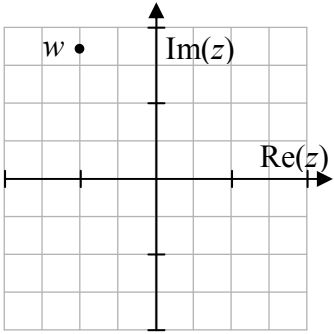
Section A	Concepts and Skills	150 marks
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Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) $w = -1 + \sqrt{3}i$ is a complex number, where $i^2 = -1$.

(i) Write w in polar form.



(ii) Use De Moivre's theorem to solve the equation $z^2 = -1 + \sqrt{3}i$. Give your answer(s) in rectangular form.

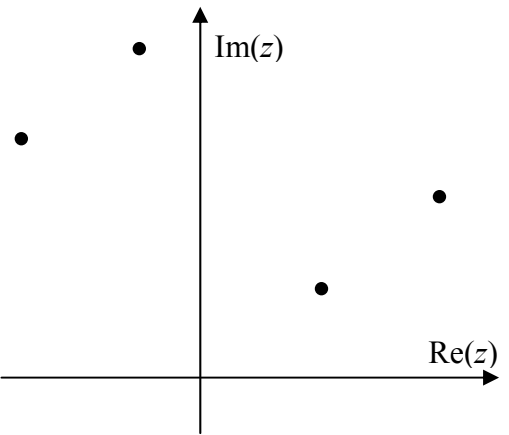
(b) Four complex numbers z_1, z_2, z_3 and z_4 are shown on the Argand diagram. They satisfy the following conditions:

$z_2 = iz_1$
 $z_3 = kz_1$, where $k \in \mathbb{R}$
 $z_4 = z_2 + z_3$.

The same scale is used on both axes.

- (i) Identify which number is which, by labelling the points on the diagram.
- (ii) Write down the approximate value of k .

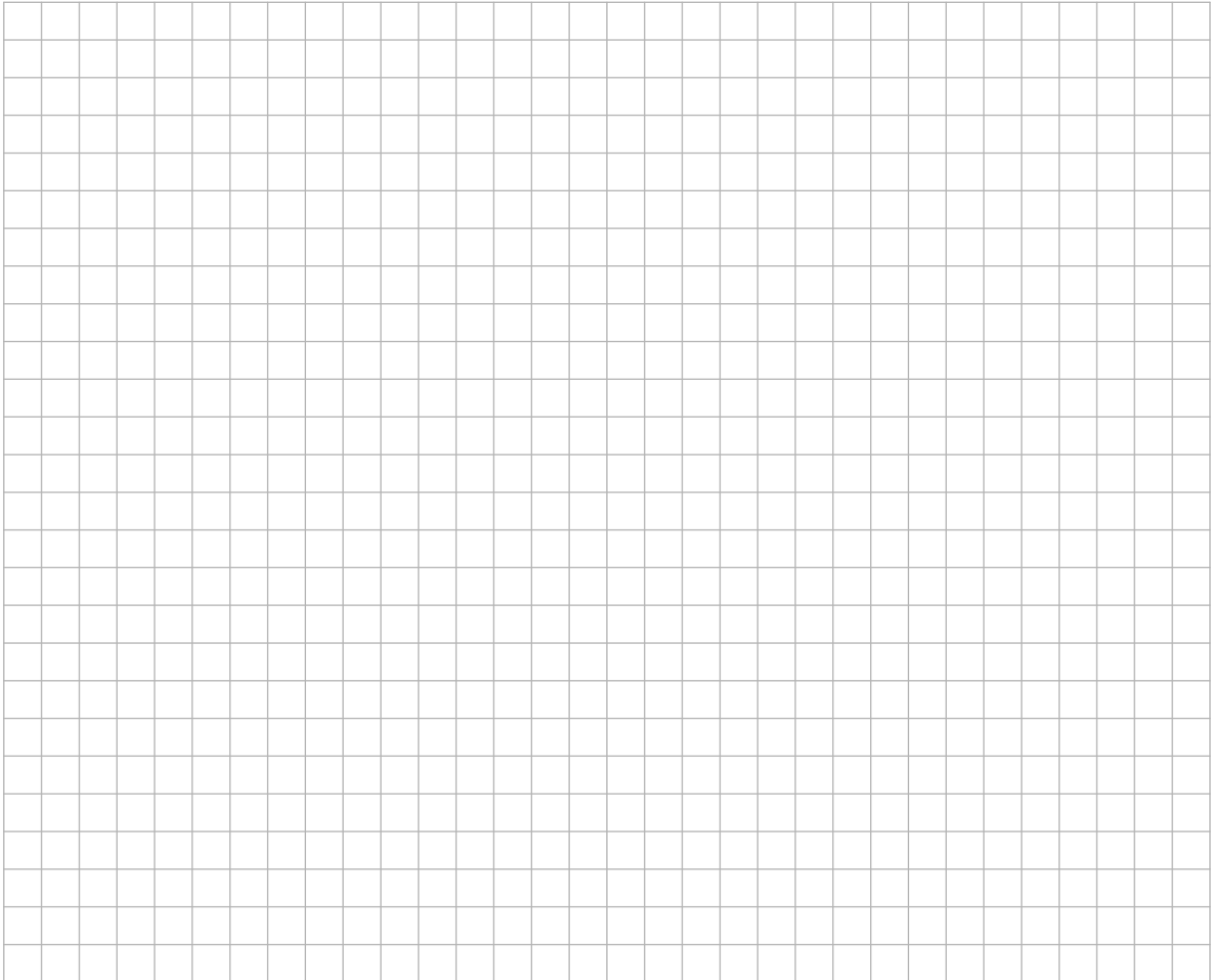
Answer: _____



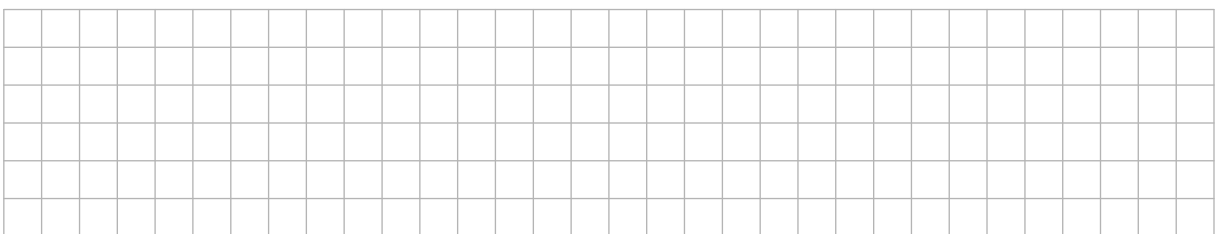
Question 2

(25 marks)


- (a) (i) Prove by induction that, for any n , the sum of the first n natural numbers is $\frac{n(n+1)}{2}$.



- (ii) Find the sum of all the natural numbers from 51 to 100, inclusive.



- (b) Given that $p = \log_c x$, express $\log_c \sqrt{x} + \log_c (cx)$ in terms of p .



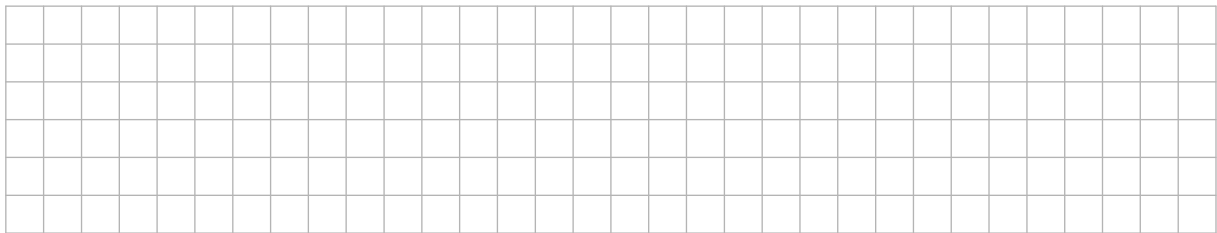
Question 3

(25 marks)

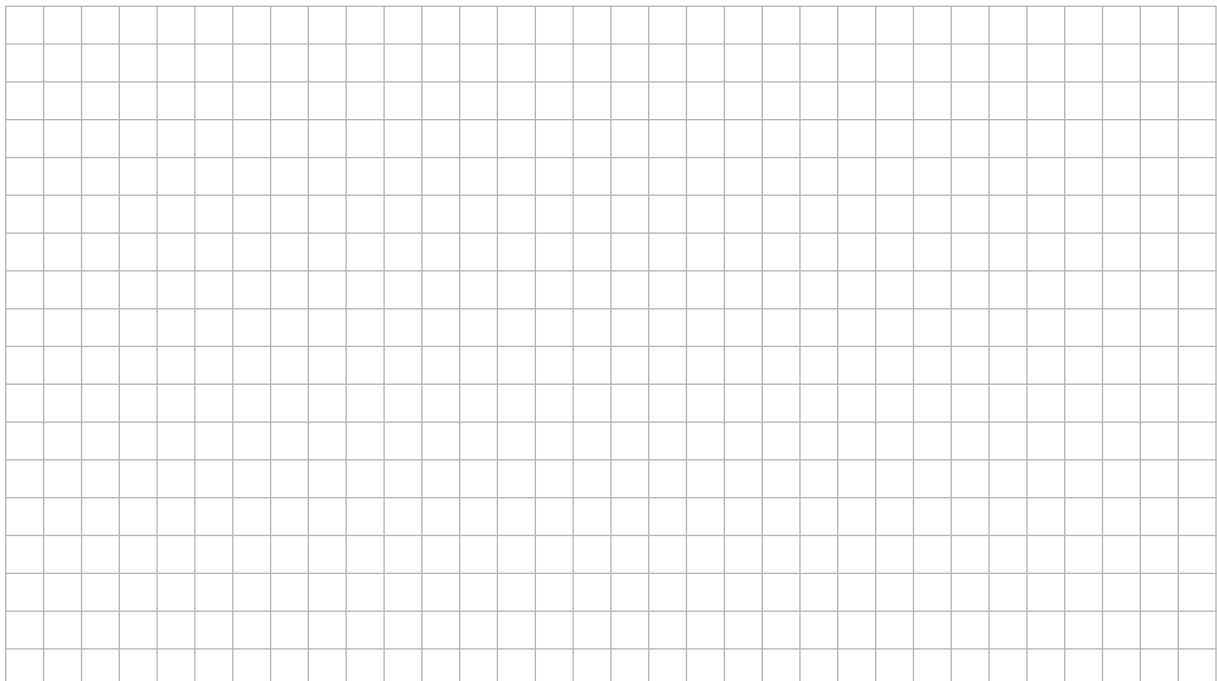
A cubic function f is defined for $x \in \mathbb{R}$ as

$$f : x \mapsto x^3 + (1 - k^2)x + k, \text{ where } k \text{ is a constant.}$$

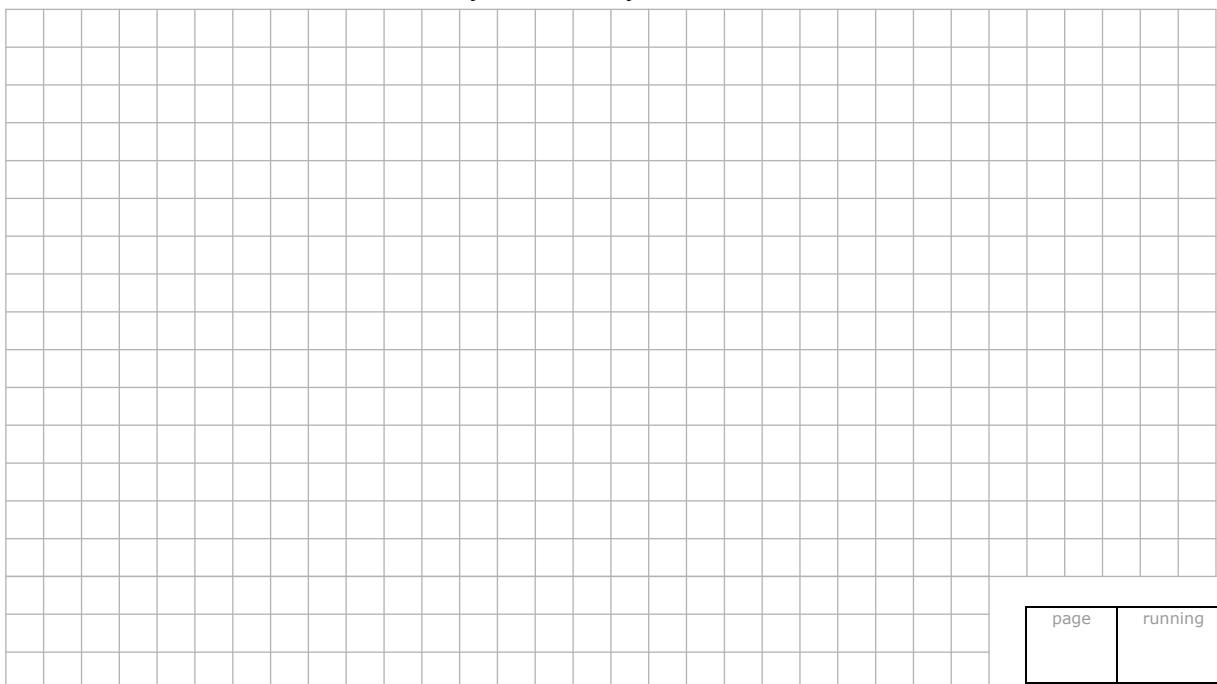
- (a)** Show that $-k$ is a root of f .



- (b)** Find, in terms of k , the other two roots of f .



- (c)** Find the set of values of k for which f has exactly one real root.



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Question 4

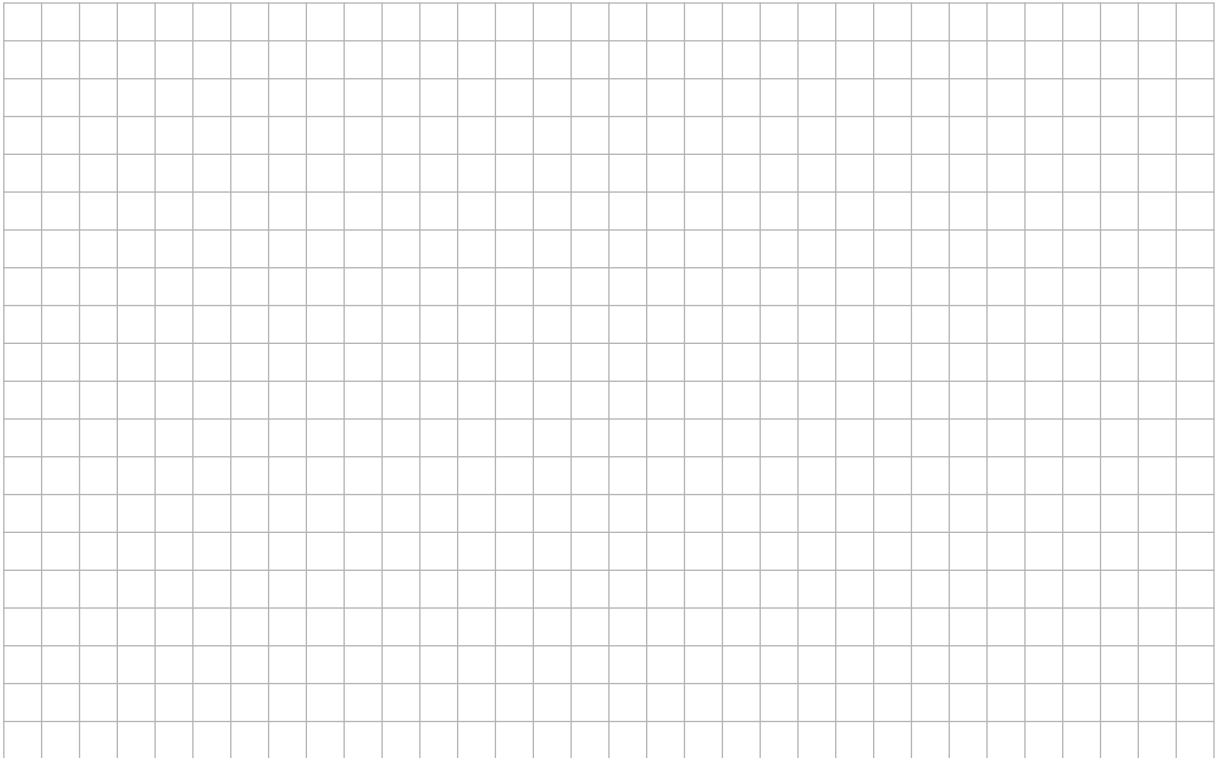
(25 marks)

(a) Solve the simultaneous equations:

$$2x + 8y - 3z = -1$$

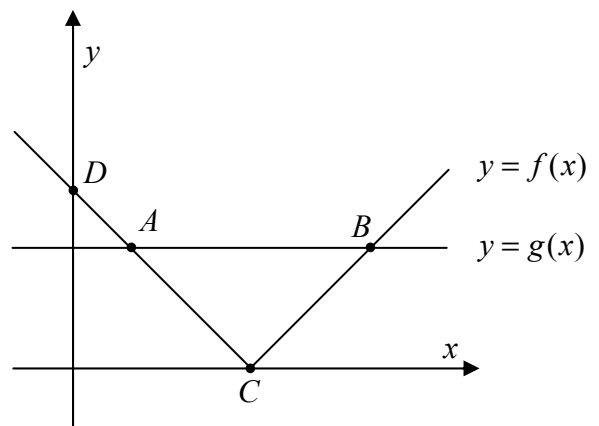
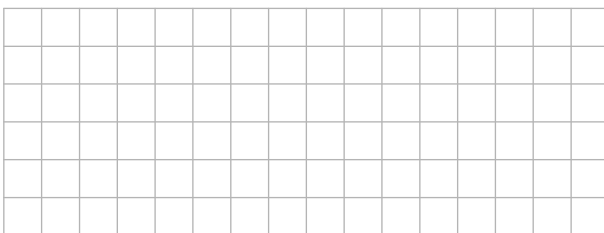
$$2x - 3y + 2z = 2$$

$$2x + y + z = 5.$$



(b) The graphs of the functions $f : x \mapsto |x - 3|$ and $g : x \mapsto 2$ are shown in the diagram.

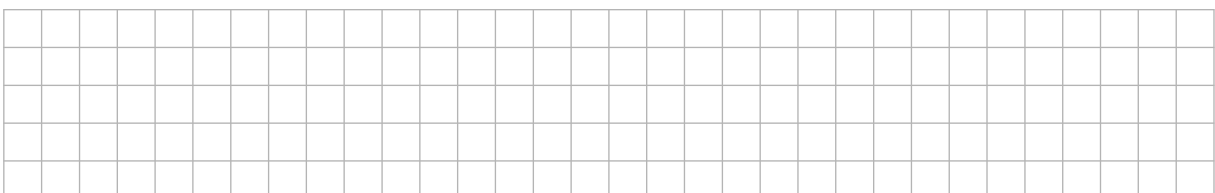
(i) Find the co-ordinates of the points A , B , C and D .



$$A = (\quad , \quad) \quad B = (\quad , \quad)$$

$$C = (\quad , \quad) \quad D = (\quad , \quad)$$

(ii) Hence, or otherwise, solve the inequality $|x - 3| < 2$.



Question 5

(25 marks)

A is the closed interval $[0, 5]$. That is, $A = \{x \mid 0 \leq x \leq 5, x \in \mathbb{R}\}$.

The function f is defined on A by:

$$f : A \rightarrow \mathbb{R} : x \mapsto x^3 - 5x^2 + 3x + 5.$$

- (a)** Find the maximum and minimum values of f .

- (b)** State whether f is injective. Give a reason for your answer.

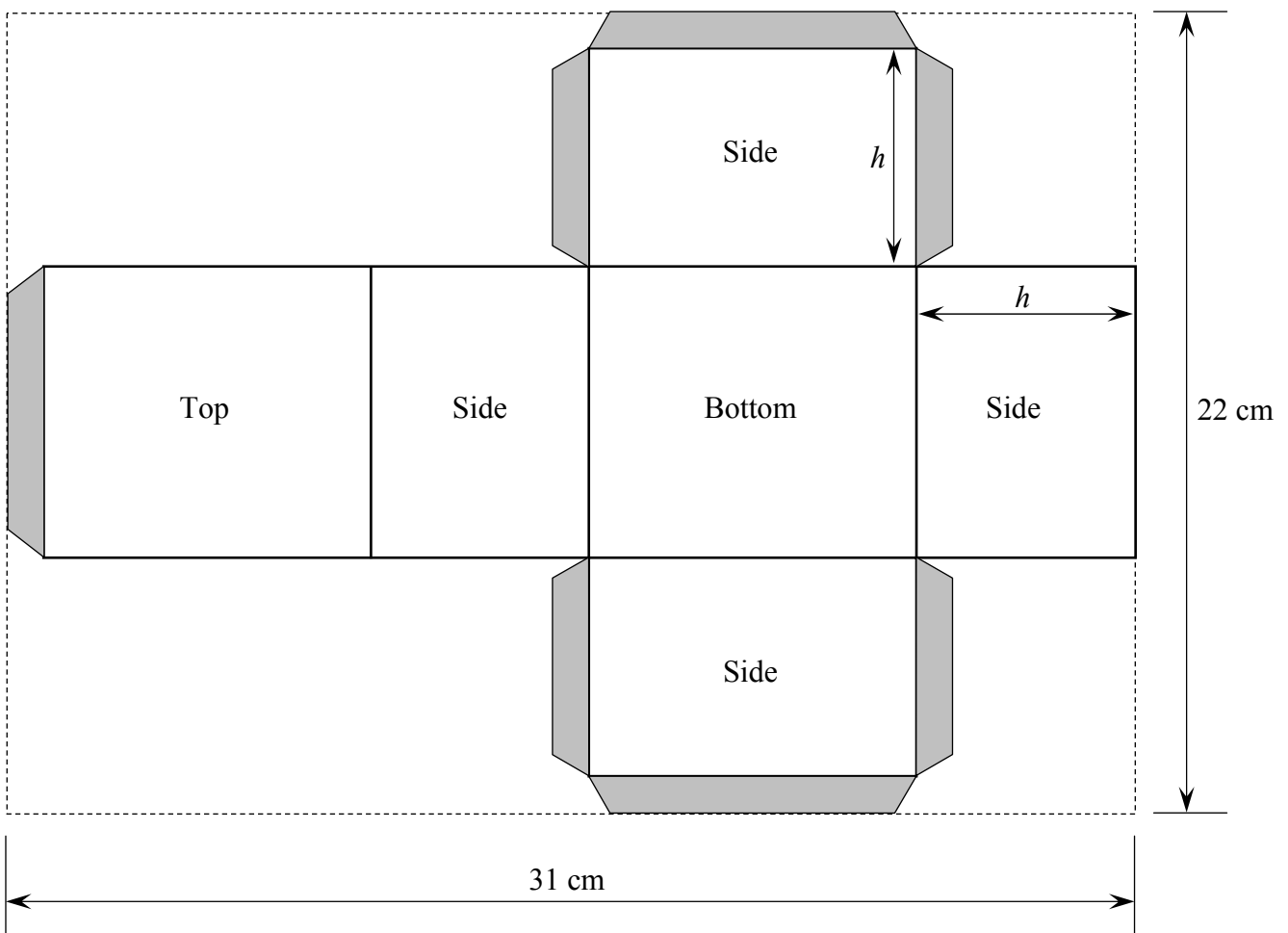
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Answer **all three** questions from this section.

Question 7**(50 marks)**

A company has to design a rectangular box for a new range of jellybeans. The box is to be assembled from a single piece of cardboard, cut from a rectangular sheet measuring 31 cm by 22 cm. The box is to have a capacity (volume) of 500 cm^3 .

The net for the box is shown below. The company is going to use the full length and width of the rectangular piece of cardboard. The shaded areas are flaps of width 1 cm which are needed for assembly. The height of the box is h cm, as shown on the diagram.



- (a) Write the dimensions of the box, in centimetres, in terms of h .

height =	h	cm
length =	_____	cm
width =	_____	cm

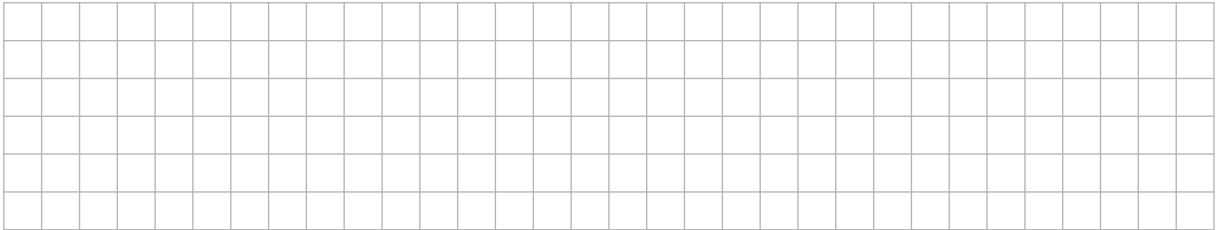
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Question 9

(50 marks)

(a) Let $f(x) = -0.5x^2 + 5x - 0.98$, where $x \in \mathbb{R}$.

(i) Find the value of $f(0.2)$.



(ii) Show that f has a local maximum point at $(5, 11.52)$.



(b) A sprinter's velocity over the course of a particular 100 metre race is approximated by the following model, where v is the velocity in metres per second, and t is the time in seconds from the starting signal:

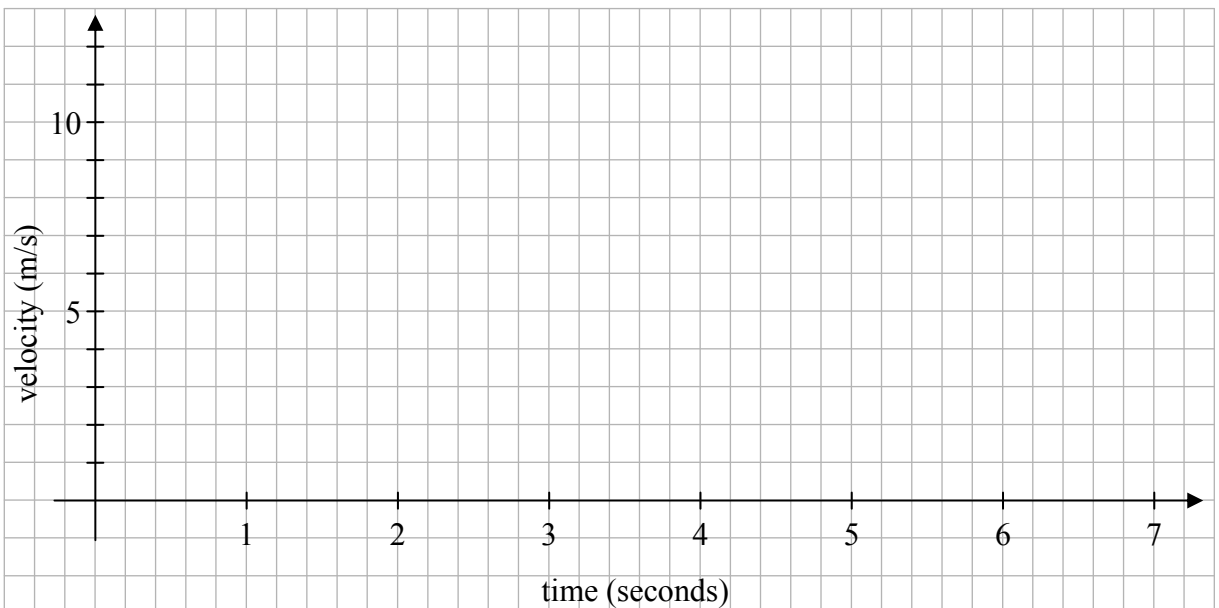
$$v(t) = \begin{cases} 0, & \text{for } 0 \leq t < 0.2 \\ -0.5t^2 + 5t - 0.98, & \text{for } 0.2 \leq t < 5 \\ 11.52, & \text{for } t \geq 5 \end{cases}$$



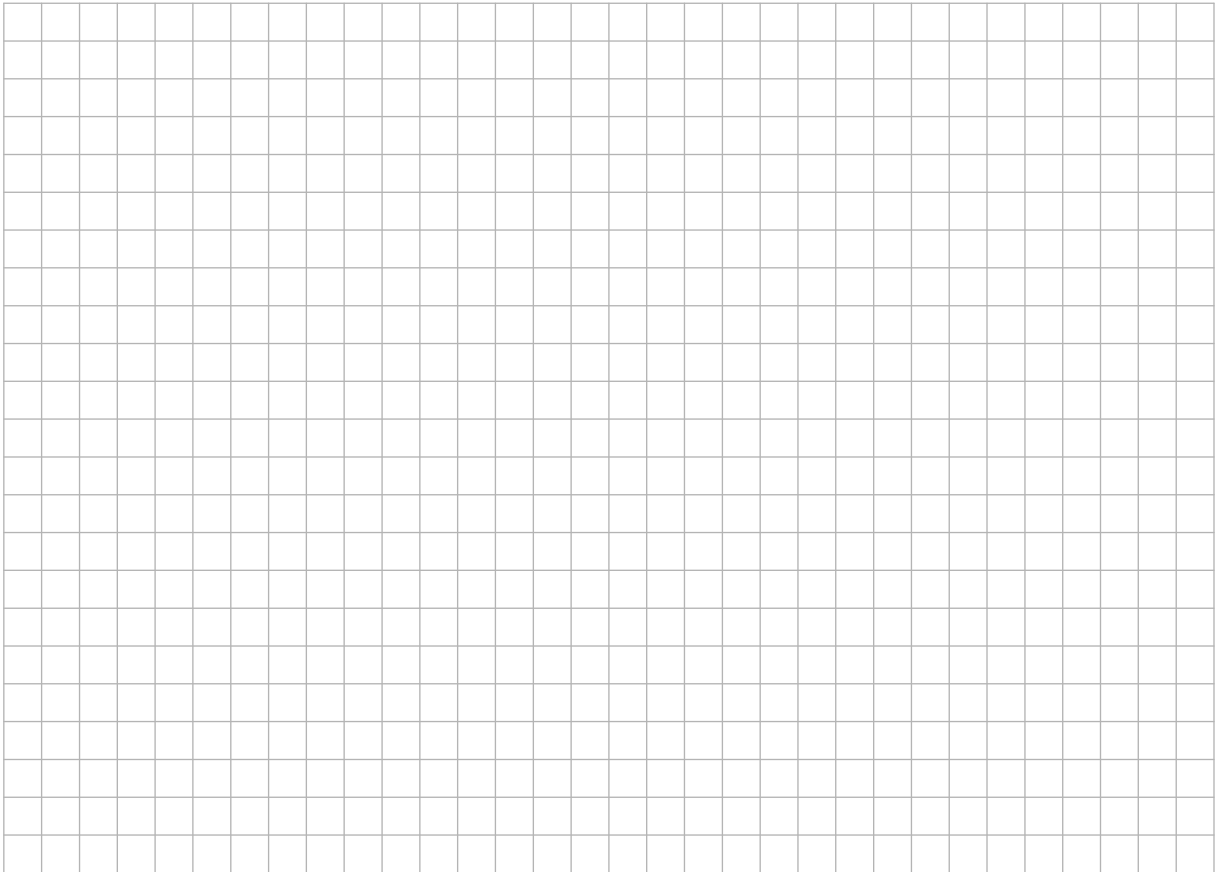
Photo: William Warby. Wikimedia Commons. CC BY 2.0

Note that the function in part (a) is relevant to $v(t)$ above.

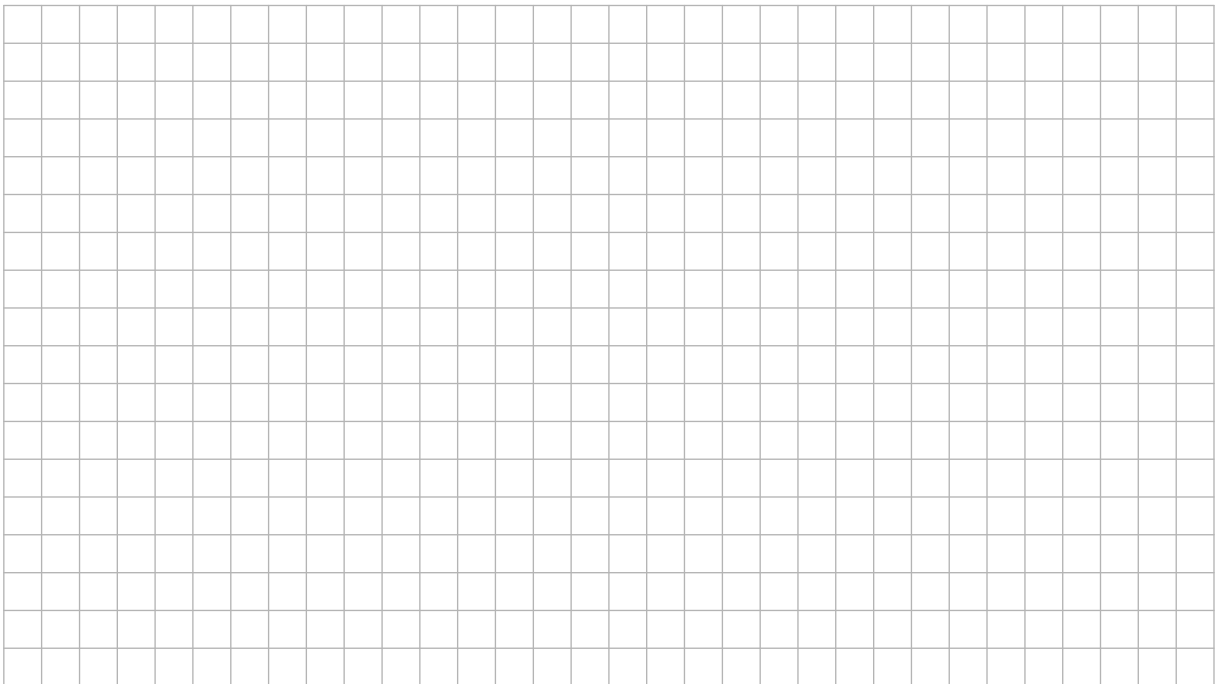
(i) Sketch the graph of v as a function of t for the first 7 seconds of the race.



(ii) Find the distance travelled by the sprinter in the first 5 seconds of the race.



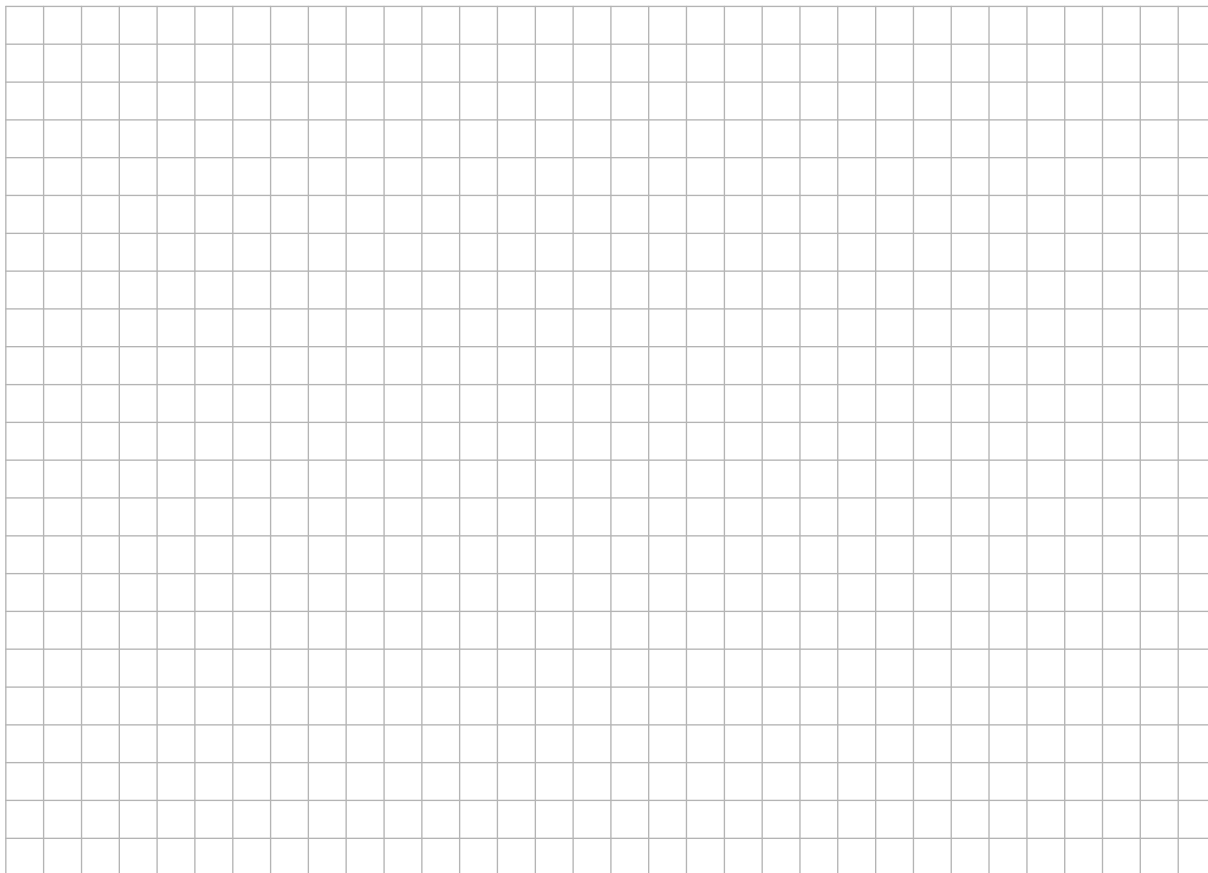
(iii) Find the sprinter's finishing time for the race. Give your answer correct to two decimal places.



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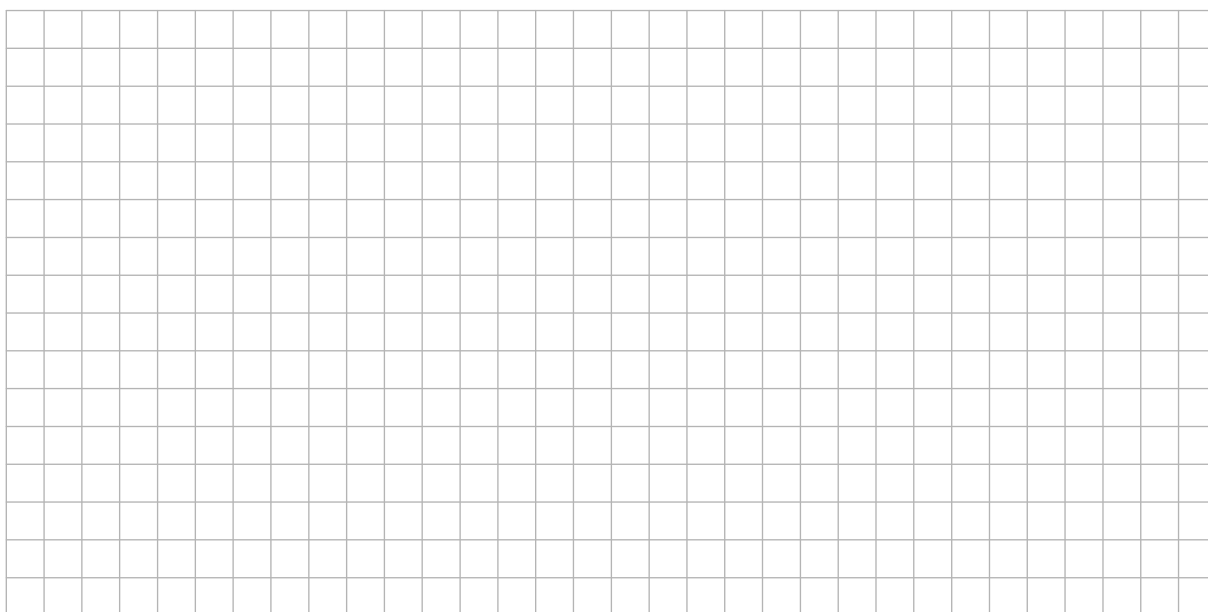
(c) A spherical snowball is melting at a rate proportional to its surface area. That is, the rate at which its volume is decreasing at any instant is proportional to its surface area at that instant.

(i) Prove that the radius of the snowball is decreasing at a constant rate.

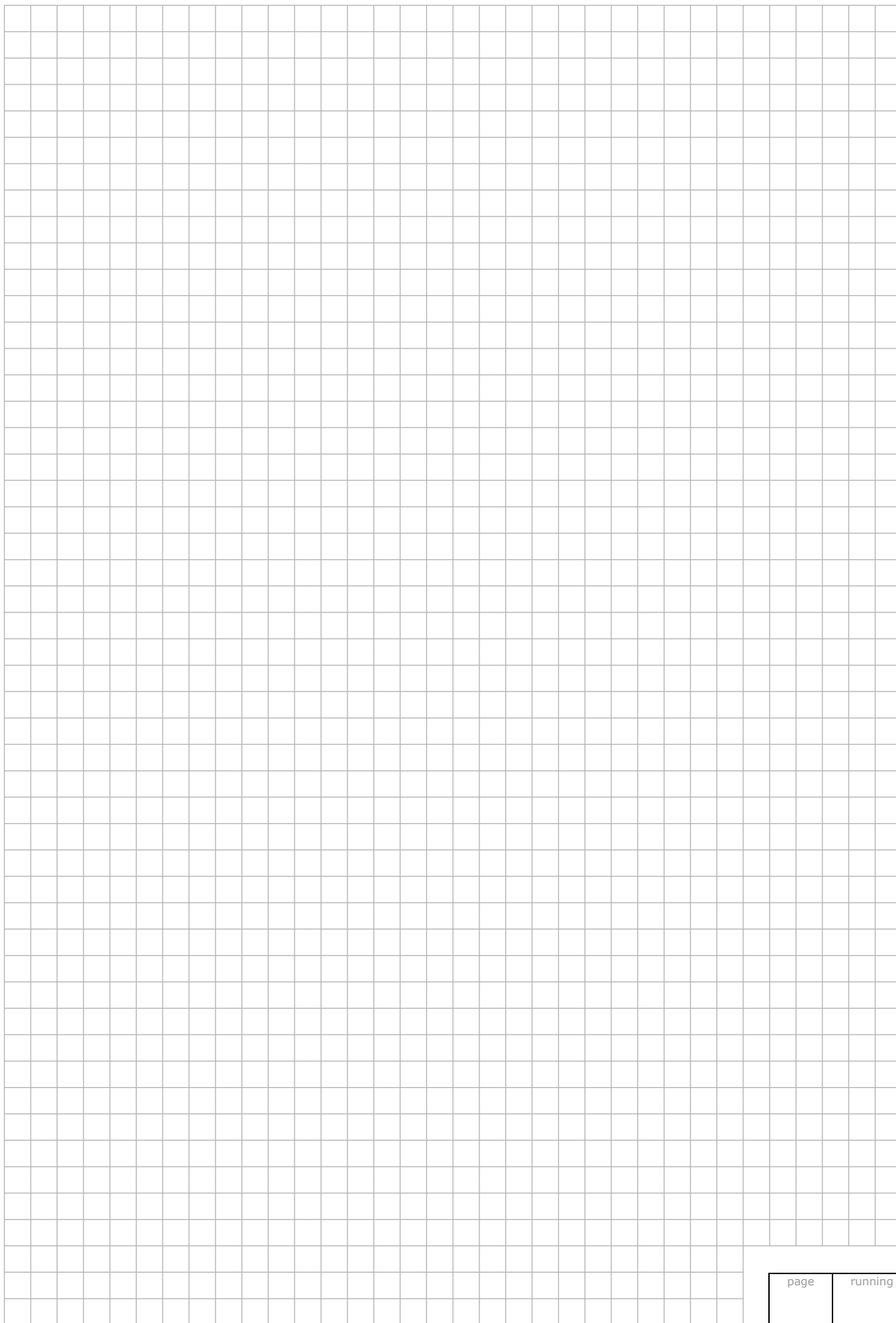


(ii) If the snowball loses half of its volume in an hour, how long more will it take for it to melt completely?

Give your answer correct to the nearest minute.

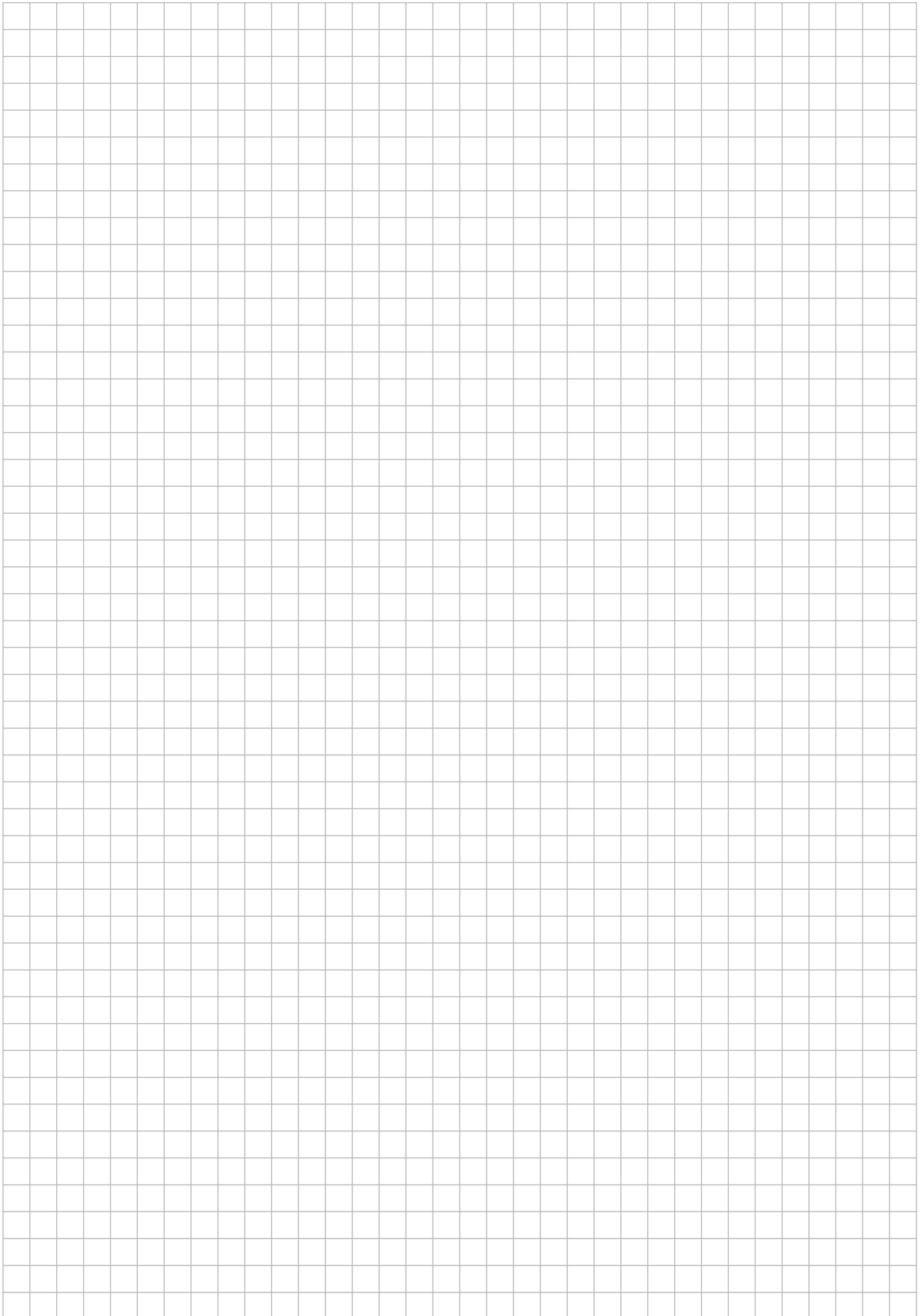


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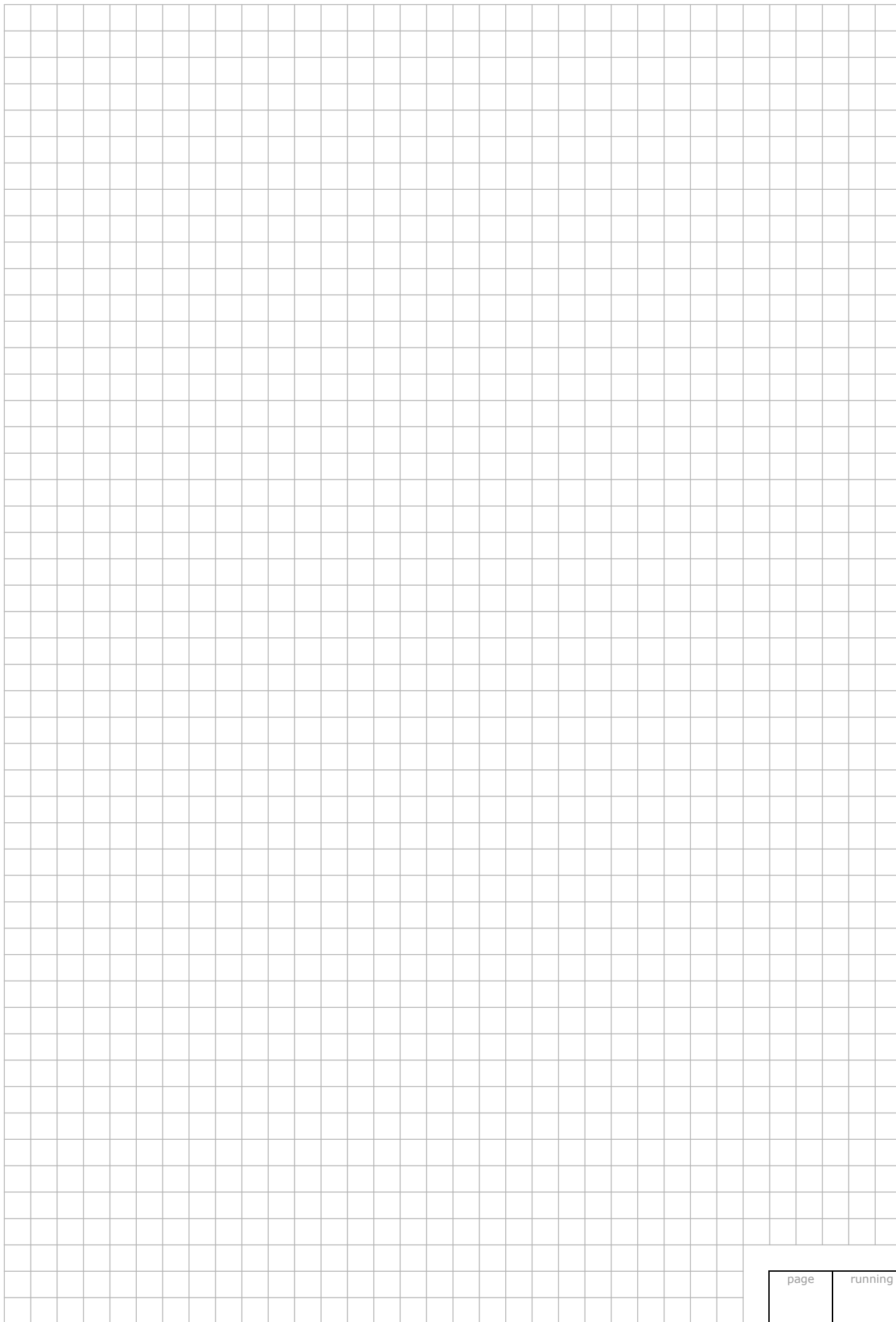


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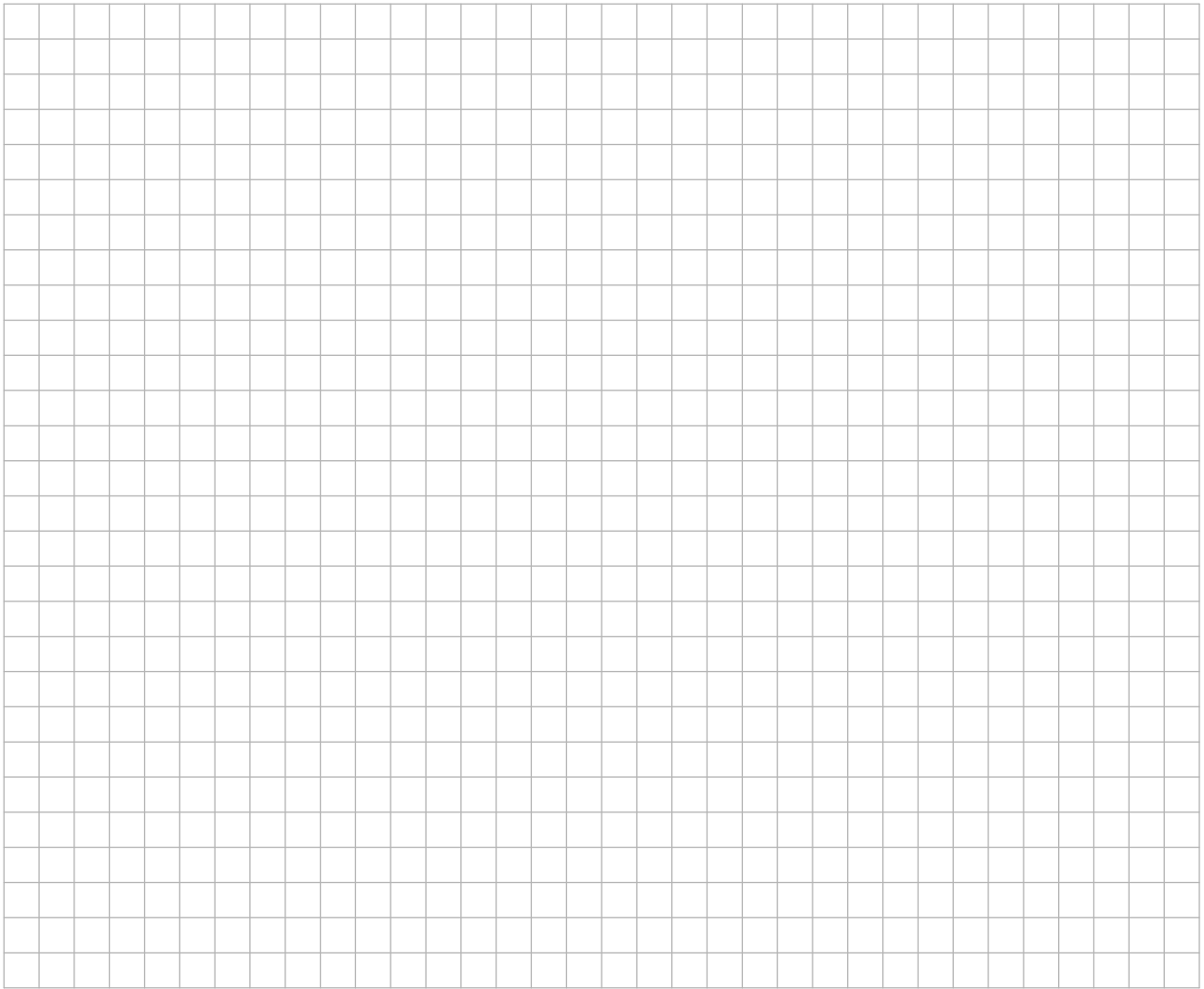
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Note to readers of this document:

This sample paper is intended to help teachers and candidates prepare for the June 2014 examination in *Mathematics* under Phase 3 of *Project Maths*. The content and structure do not necessarily reflect the 2015 or subsequent examinations.

Section A of the examination paper will consist of six questions, each carrying 25 marks.

Section B will consist of two, three, or four questions. These questions will not necessarily carry equal marks. The number of marks for each will be stated on the examination paper. The total number of marks for Section B will be 150.

Leaving Certificate 2014 – Higher Level

Mathematics (Project Maths – Phase 3) – Paper 1

Sample Paper

Time: 2 hours 30 minutes