Coimisiún na Scrúduithe Stáit
State Examinations Commission

Junior Certificate Examination 2014
Sample Paper

Mathematics
(Project Maths – Phase 2)

Paper 1

Higher Level

Time: 2 hours, 30 minutes

300 marks

Examination number

Centre stamp

Running total	

For examiner			
Question	Mark	Question	Mark
1		11	
2		12	
3		13	
4			
5			
6			
7			
8			
9			
10		Total	

Grade

Instructions

There are 13 questions on this examination paper. Answer **all** questions.

Questions do not necessarily carry equal marks. To help you manage your time during this examination, a maximum time for each question is suggested. If you remain within these times you should have about 10 minutes left to review your work.

Question 13 carries a total of 50 marks.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

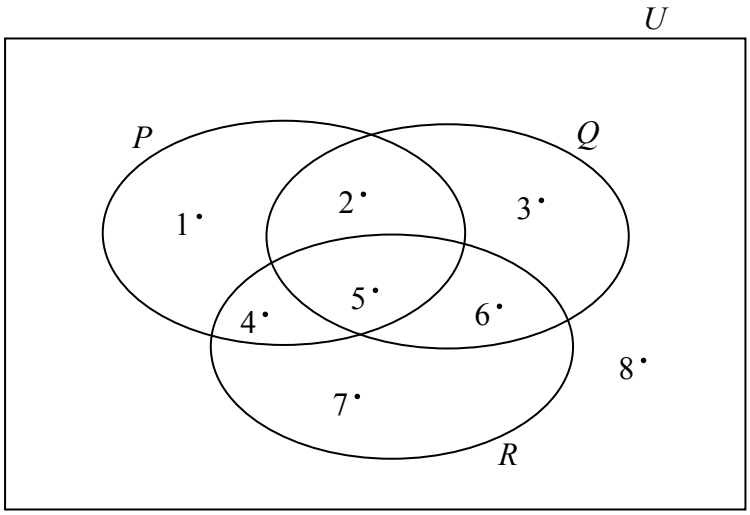
You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Question 1 **(Suggested maximum time: 5 minutes)**



(i) From the Venn diagram above list the elements of:

$P \cup Q$ _____

$Q \cap R$ _____

$P \cup (Q \cap R)$ _____

(ii) Miriam says: "For all sets, union is distributive over intersection." **Name** a set that you would use along with $P \cup (Q \cap R)$ to show that Miriam's claim is true for the sets P, Q and R in the Venn diagram above.

Question 2 **(Suggested maximum time: 5 minutes)**

$U = \{2,\ 3,\ 4,\ 5,\ \ldots,\ 30\}$, A = {multiples of 2}, B = {multiples of 3}, C = {multiples of 5}.

(i) Find $\#\left[\left(A \cup B \cup C\right)'\right]$, the number of elements in the complement of the set $A \cup B \cup C$.

(ii) How many divisors does each of the numbers in $\left(A \cup B \cup C\right)'$ have? _____

(iii) What name is given to numbers that have this many divisors? _____

Question 3 **(Suggested maximum time: 10 minutes)**

A group of 100 students were surveyed to find whether they drank tea (T), coffee (C) or a soft drink (D) at any time in the previous week.

24 had not drunk any of the three.

51 drank tea or coffee but not a soft drink.

41 drank tea.

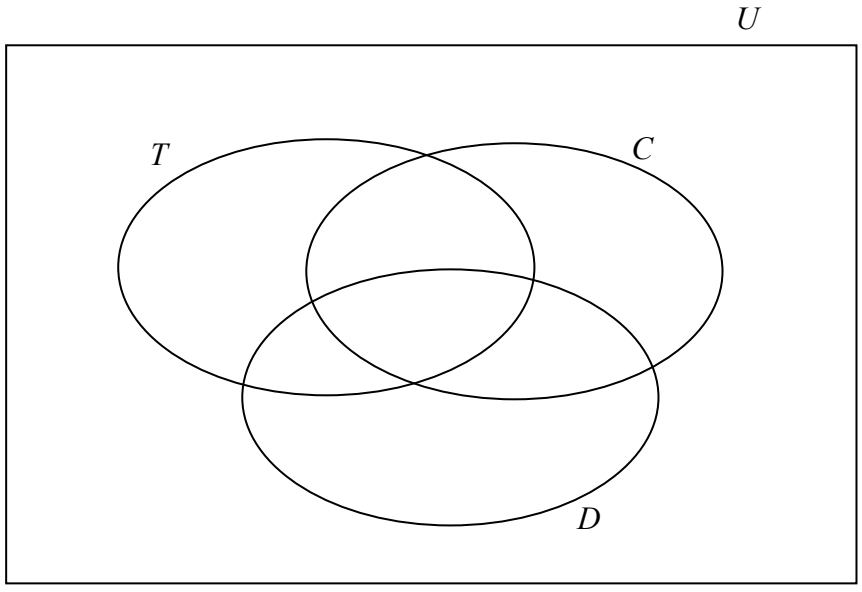
20 drank at least two of the three.

8 drank tea and a soft drink but not coffee.

9 drank a soft drink and coffee.

4 drank all three.

(i) Represent the above information on the Venn diagram.



(ii) Find the probability that a student chosen at random from the group had drunk tea or coffee.

(iii) Find the probability that a student chosen at random from the group had drunk tea and coffee but not a soft drink.

Question 4 **(Suggested maximum time: 10 minutes)**

Dermot has €5,000 and would like to invest it for two years. A special savings account is offering a rate of 3% for the first year and a higher rate for the second year, if the money is retained in the account. Tax of 33% will be deducted each year from the interest earned.

(i) How much will the investment be worth at the end of one year, after tax is deducted?

(ii) Dermot calculates that, after tax has been deducted, his investment will be worth about €5,268 at the end of the second year. Calculate the rate of interest for the second year.

Question 5 **(Suggested maximum time: 10 minutes)**

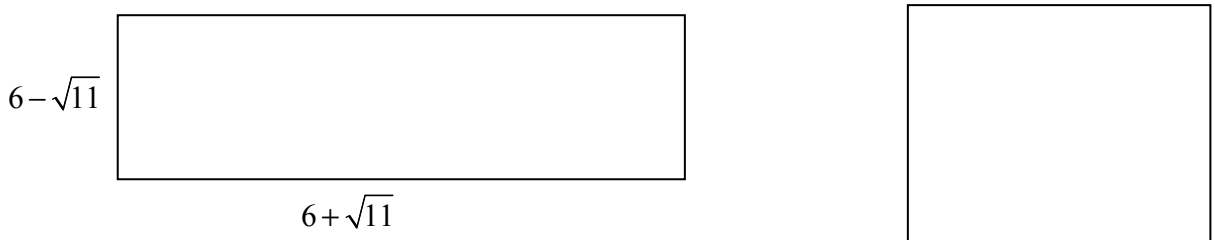
A meal in a restaurant cost Jerry €136·20. The price included VAT at 13·5%. Jerry wished to know the price of the meal before the VAT was included. He calculated 13·5% of €136·20 and subtracted it from the cost of the meal.

(i) Explain why Jerry will not get the correct answer using this method.

(ii) From July 1, 2011, the VAT rate on food, in restaurants, was reduced to 9%. How much would Jerry have paid for the meal after this date if the VAT reduction was correctly applied?

Question 6 **(Suggested maximum time: 5 minutes)**

The rectangle and square below have the same area. The dimensions of both are in cm. The diagrams are not drawn to scale.



(i) Find the area of the rectangle.

(ii) Find the length of one side of the square. _____

Question 7 **(Suggested maximum time: 15 minutes)**

Given any two positive integers m and n ($n>m$), it is possible to form three numbers a, b and c where:

$$a = n^2 - m^2, \qquad b = 2nm, \qquad c = n^2 + m^2$$

These three numbers a, b and c are then known as a "Pythagorean triple".

(i) For $m = 3$ and $n = 5$ calculate a, b and c.

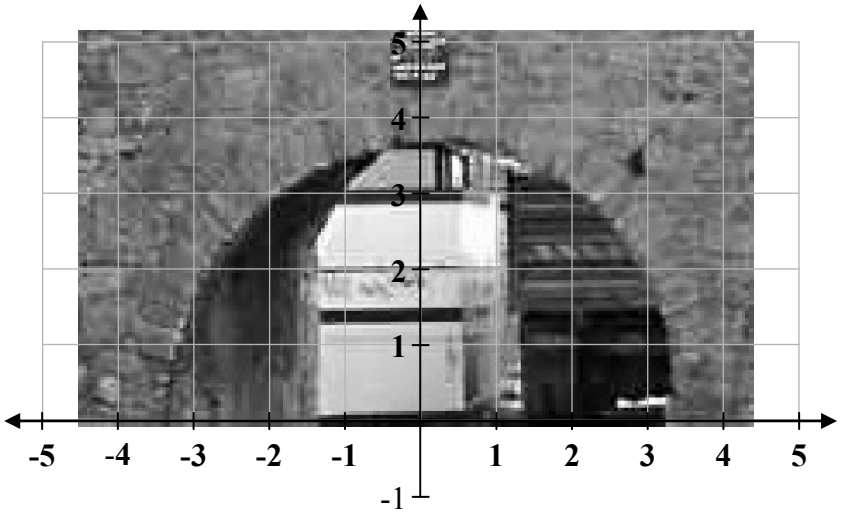
(ii) If the values of a, b, and c from part **(i)** are the lengths of the sides of a triangle, show that the triangle is right-angled.

(iii) If $n^2 - m^2$, $2nm$, and $n^2 + m^2$ are the lengths of the sides of a triangle, show that the triangle is right-angled.

Question 8 **(Suggested maximum time: 5 minutes)**

The picture below shows the top section of the Spanish Arch in Galway city. George wants to see if the arch can be described by a function. He puts a co-ordinate grid over the arch as shown.

(i) Complete the table below to show the value of y for each of the given values of x.



x	y
–3	
–2	
–1	
0	
1	
2	
3	

(ii) Is it possible to represent this section of the Spanish Arch by a quadratic function? Give a reason for your answer.

Answer:

Reason:

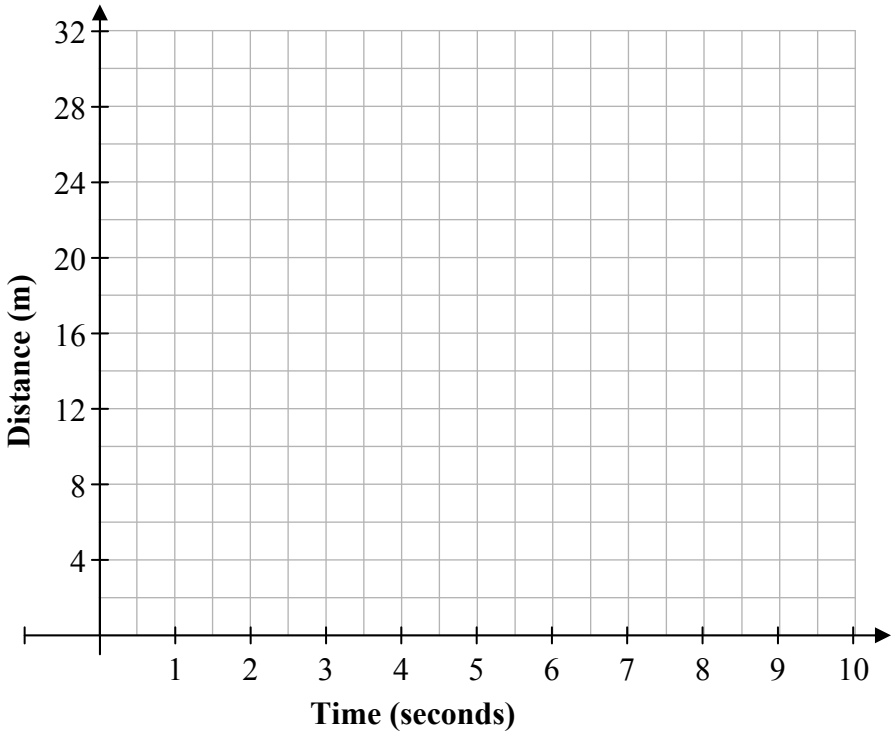
Question 9 **(Suggested maximum time: 10 minutes)**

Bill and Jenny are two athletes running in the same direction at steady speeds on a race-track. Tina is standing beside the track. At a particular time, Bill has gone 7 m beyond Tina and his speed is 2 m/s. At the same instant Jenny has gone 2 m beyond Tina and her speed is 3 m/s.

(i) Complete the table below to show the distance between the two runners and Tina over the next 10 seconds.

Time	Bill Distance (m)	Jenny Distance (m)
0	7	2
1	9	
2		
3		
4		
5		
6		
7		
8		
9		
10		

(ii) On the grid below draw graphs for the distance between Bill and Tina and the distance between Jenny and Tina over the 10 seconds.



(iii) After how many seconds will both runners be the same distance from Tina? _____

(iv) After 9 seconds, which runner is furthest from Tina and what is the distance between the runners?

Furthest from Tina =	
Distance between Runners =	

(v) Write down a formula to represent the distance between Bill and Tina for any given time. State clearly the meaning of any letters used in your formula.

(vi) Write down a formula to represent the distance between Jenny and Tina for any given time.

_____

(vii) Use your formulas from **(v)** and **(vi)** to verify the answer that you gave to part **(iii)** above.

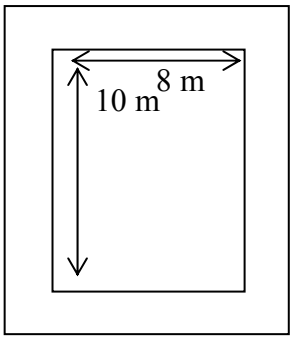
(viii) After 1 minute, Jenny stops suddenly. From the time she stops, how long will it be until Bill is again level with her?

(ix) If Jenny had not stopped, how long in total would it be until the runners are 100 m apart?

Question 10 **(Suggested maximum time: 20 minutes)**

A plot consists of a rectangular garden measuring 8 m by 10 m surrounded by a path of constant width. The total area of the plot is 143 m^2. Three students, Kevin, Elaine and Tony, have been given the problem of trying to find the width of the path. Each of them is using a different method, but all of them are using x to represent the width of the path.

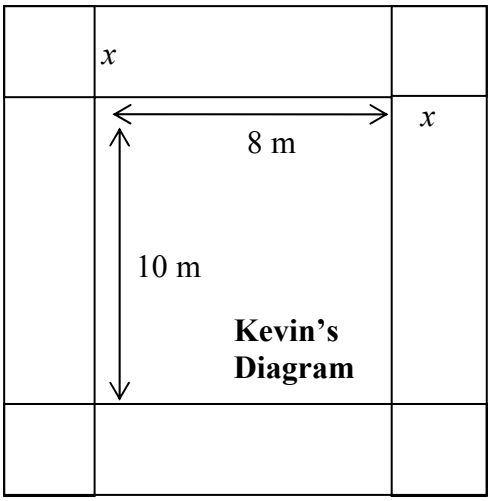
Kevin divides the path into eight pieces. He writes down the area of each piece in terms of x. He then forms an equation by setting the area of the path plus the area of the garden equal to the total area of the plot.



(i) Write, in terms of x, the area of each section into Kevin's diagram below.

(ii) Write down and simplify the equation which you think Kevin got. Give your answer in the form $ax^2 + bx + c = 0$.

Equation:

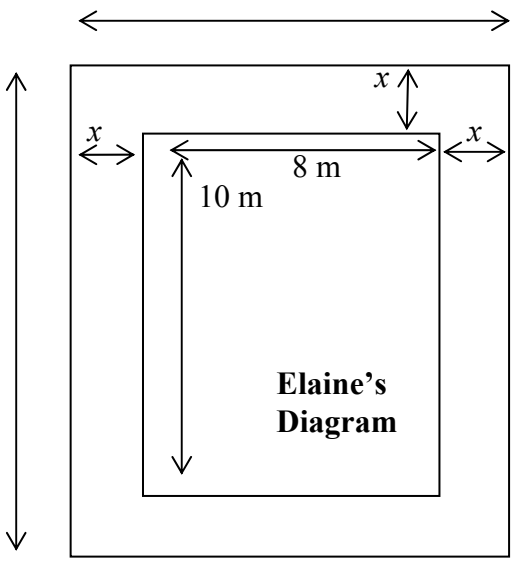


Elaine writes down the length and width of the plot in terms of x. She multiplies these and sets the answer equal to the total area of the plot.

(iii) Write, in terms of x, the length and width of the plot on Elaine's diagram.

(iv) Write down and simplify the equation which you think Elaine got. Give your answer in the form $ax^2 + bx + c = 0$.

Equation:



(v) Solve an equation to find the width of the path.

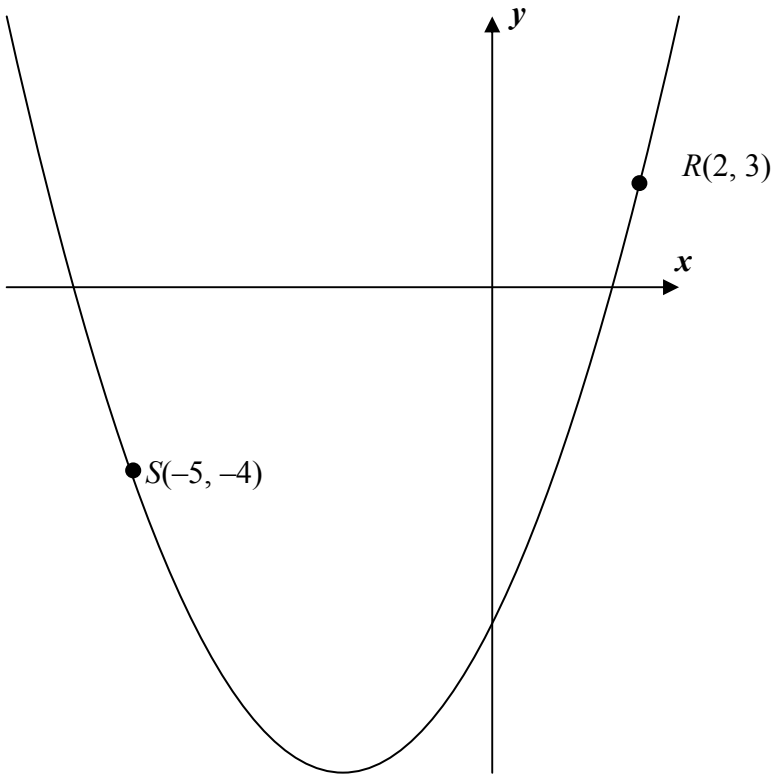
(vi) Tony does not answer the problem by solving an equation. Instead, he does it by trying out different values for x. Show some calculations that Tony might have used to solve the problem.

(vii) Which of the three methods do you think is best? Give a reason for your answer.

Answer:
Reason:

Question 11 **(Suggested maximum time: 20 minutes)**

Part of the graph of the function $y = x^2 + ax + b$ where $a, b \in \mathbb{Z}$ is shown below.



The points $R(2, 3)$ and $S(-5, -4)$ are on the curve.

(i) Use the given points to form two equations in a and b.

(ii) Solve your equations to find the value of a and the value of b.

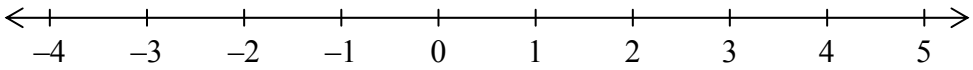
(iii) Write down the co-ordinates of the point where the curve crosses the y-axis. (,)

(iv) Find the points where the curve crosses the x-axis. Give your answers correct to one place of decimals.

Question 12 **(Suggested maximum time: 5 minutes)**

(a) **(i)** Solve the inequality $-2 < 5x + 3 \leq 18,\ x \in \mathbb{R}$.

(ii) Graph your solution on the number line below.



(b) Niamh is in a clothes shop and has a voucher which she **must** use. The voucher gives a €10 reduction when more than €35 is spent. She also has €50 cash. Write down an inequality in x to show the range of money she could spend in the shop.

$\square < x \leq \square$

Write down an inequality in y to show the price range of articles she could buy.

$\square < y \leq \square$

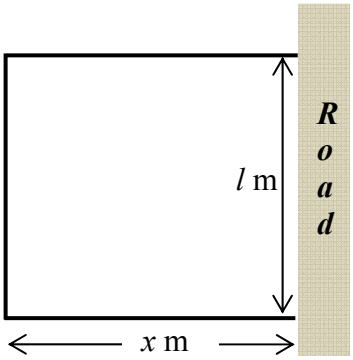
Question 13 **(Suggested maximum time: 20 minutes)**

A rectangular site, with one side facing a road, is to be fenced off.

The side facing the road, which does not require fencing, is l m in length.

The sides perpendicular to the road are x m in length.

The length of fencing that will be used to enclose the rest of the site is 140 m.



(i) Write an expression, in terms of x, for the length (l) of the side facing the road.

(ii) Show that the area of the site, in m^2, is $-2x^2 + 140x$.

(iii) Let f be the function $f : x \mapsto -2x^2 + 140x$.

Evaluate $f(x)$ when $x = 0, 10, 20, 30, 40, 50, 60, 70$.

Hence, draw the graph of f for $0 \leq x \leq 70$, $x \in \mathbb{R}$.

Use your graph from part **(iii)** to estimate:

(iv) the maximum possible area of the site

(v) the area of the site when the road frontage (l) is 30 m long.

You may use this page for extra work.

page	running

Note to readers of this document:

This sample paper is intended to help teachers and candidates prepare for the June 2014 examination in *Mathematics* under Phase 2 of *Project Maths*. The content and structure do not necessarily reflect the 2015 or subsequent examinations.

In the 2014 examination, one question will be similar in content and style to those that have appeared as Questions 5 and 6 on the examination in previous years. On this sample paper, Question 6 from the 2013 examination has been inserted, as Question 13, to illustrate.

Junior Certificate 2014 – Higher Level

Mathematics (Project Maths – Phase 2) – Paper 1

Sample Paper
Time: 2 hours, 30 minutes