

# Coimisiún na Scrúduithe Stáit State Examinations Commission 

## Leaving Certificate 2015

## Marking Scheme

## Mathematics

## Ordinary Level

## Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

## Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.
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## Coimisiún na Scrúduithe Stáit

State Examinations Commission

## Leaving Certificate Examination 2015

## Mathematics

## Paper 1

## Ordinary Level

Friday 5 June
Afternoon 2:00-4:30

300 marks

## Model Solutions - Paper 1

Note: The model solutions for each question are not intended to be exhaustive - there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.

## Instructions

There are two sections in this examination paper.

| Section A | Concepts and Skills | 150 marks | 6 questions |
| :--- | :--- | :--- | :--- |
| Section B | Contexts and Applications | 150 marks | 3 questions |

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the Formulae and Tables booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here: $\square$

Answer all six questions from this section.

## Question 1

(25 marks)
Padraic works in America and travels between Ireland and America.
(a) In Ireland, he exchanged $€ 2000$ for US dollars when the exchange rate was $€ 1=\$ 1 \cdot 29$. Find how many US dollars he received.

$$
€ 2000 \times 1 \cdot 29=\$ 2580
$$

(b) Padraic returned to Ireland and exchanged $\$ 21000$ for euro. He received $€ 15000$. Write the exchange rate for this transaction in the form $€ 1=\$ \square \cdot \square \square$.

$$
\begin{aligned}
& € 15000=\$ 21000 \\
& € 1=\frac{21000}{15000}=\$ 1 \cdot 40 \\
& € 1=\$ 1 \cdot 40
\end{aligned}
$$

(c) Padraic wants to exchange some dollars for sterling. On a day when the euro to dollar exchange rate is $€ 1=\$ 1 \cdot 24$ and the euro to sterling exchange rate is $€ 1=£ 0 \cdot 83$, find the dollar to sterling exchange rate. Write your answer in the form $\$ 1=£ \square \cdot \square \square$.

$$
\begin{array}{ll}
€ 1=\$ 1 \cdot 24 & \\
€ 1=£ 0 \cdot 83 & \$ 1 \cdot 24=€ 1 \\
\$ 1 \cdot 24=£ 0 \cdot 83 & \$ 1=€ \frac{1}{1 \cdot 24}=0 \cdot 80645 \\
\$ 1=\frac{0 \cdot 83}{1 \cdot 24}=£ 0 \cdot 67 & \\
& \text { or } \\
& € 0 \cdot 80645=£ 0 \cdot 80645 \times 0 \cdot 83=£ 0 \cdot 67 \\
& \$ 1=£ 0 \cdot 67
\end{array}
$$

$$
\$ 1=£ 0 \cdot 67
$$

## Question 2

(a) John, Mary and Eileen bought a ticket in a draw. The ticket cost $€ 50$. John paid $€ 25$, Mary paid $€ 15$ and Eileen paid $€ 10$. The ticket won a prize of $€ 20000$. The prize is divided in proportion to how much each paid. How much prize money does each person receive?

John: $\quad \frac{25}{50}(20000)=€ 10000$
Mary: $\quad \frac{15}{50}(20000)=€ 6000$
Eileen: $\frac{10}{50}(20000)=€ 4000$
(b) Assuming that the Earth is a sphere of radius 6378 km :
(i) Find the length of the equator, correct to the nearest km.

$$
\begin{aligned}
2 \pi r & =2 \pi(6378) \\
& =40074 \cdot 15 \\
& =40074 \mathrm{~km}
\end{aligned}
$$

Equator

(ii) Find the volume of the Earth in the form $a \times 10^{n}$, where $1 \leq a<10$ and $n \in \mathbb{N}$.

Give the value of $a$ correct to three decimal places.

$$
\left.\begin{array}{rlrl}
\frac{4}{3} \pi r^{3} & =\frac{4}{3} \pi(6378)^{3} & & \frac{4}{3} \pi r^{3}
\end{array}=\frac{4}{3}(3.14)(6378)^{3}\right)
$$

(c) The mass of the Earth is $5.97 \times 10^{24} \mathrm{~kg}$. The mass of the Sun is $1.99 \times 10^{30} \mathrm{~kg}$. How many times greater than the mass of the Earth is the mass of Sun? Give your answer correct to the nearest whole number.

$$
\frac{1 \cdot 99 \times 10^{30}}{5 \cdot 97 \times 10^{24}}=333333
$$

## Question 3

(a) Simplify $3(4-5 x)-2(5-6 x)$.

$$
\begin{aligned}
3(4-5 x)-2(5-6 x) & =12-15 x-10+12 x \\
& =2-3 x
\end{aligned}
$$

(b) List all the values of $x$ that satisfy the inequality $2-3 x \geq-6, x \in \mathrm{~N}$.

$$
\begin{aligned}
& 2-3 x \geq-6 \\
& \Rightarrow-3 x \geq-8 \\
& \Rightarrow x \leq \frac{8}{3} \\
& x \in\{1,2\}
\end{aligned}
$$

(c) $g(x)$ is a function and $(2-3 x) \times g(x)=15 x^{2}-22 x+8$, for all $x \in \mathbb{R}$. Find $g(x)$.

$$
\begin{gathered}
- 3 x + 2 \longdiv { 1 5 x ^ { 2 } - 2 2 x + 8 } \\
\frac{15 x^{2}-10 x}{-12 x+8} \\
-12 x+8 \\
g(x)=
\end{gathered}
$$

or

$$
\begin{aligned}
& (2-3 x)(A x+B)=15 x^{2}-22 x+8 \\
& \Rightarrow 2 B=8 \text { and }-3 A=15 \\
& \Rightarrow B=4 \text { and } A=-5 \\
& g(x)=-5 x+4
\end{aligned}
$$

(a) Solve the equation $-x^{2}+6 x-4=0$. Give each solution correct to one decimal place.

$$
\begin{aligned}
-x^{2}+6 x-4=0 \Rightarrow \quad x & =\frac{-6 \pm \sqrt{6^{2}-4(-1)(-4)}}{2(-1)} \\
& =\frac{-6 \pm \sqrt{20}}{-2} \\
& =\frac{-6 \pm 2 \sqrt{5}}{-2} \\
x & =3+\sqrt{5} ; \quad x=3-\sqrt{5} \\
x & =5 \cdot 236 ; \quad x=0 \cdot 763 \\
x & =5 \cdot 2 ; \quad x=0 \cdot 8
\end{aligned}
$$

(b) Find the co-ordinates of the turning point of the function $f(x)=-x^{2}+6 x-4, x \in \mathbb{R}$.

$$
\begin{aligned}
& f(x)=-x^{2}+6 x-4 \\
& f^{\prime}(x)=-2 x+6=0 \\
& \Rightarrow-2 x=-6 \\
& \Rightarrow x=3 \\
& f(3)=-3^{2}+6(3)-4=5 \\
& (3,5)
\end{aligned}
$$

(c) Use your answers to parts (a) and (b) above to sketch the curve $y=f(x)$. Show your scale on both axes.

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|  |  |  |  |  |  | $y^{4}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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The diagram shows the graph of the function $f(x)=5 x-x^{2}$ in the domain $0 \leq x \leq 5, x \in \mathbb{R}$.
(a) The function $g$ is $g(x)=x+3, x \in \mathbb{R}$. The points $A(1, k)$ and $B$ are the points of intersection of $f$ and $g$.
Find the co-ordinates of $A$ and of $B$.

$$
\begin{aligned}
& g(x)=f(x) \\
& \Rightarrow x+3=5 x-x^{2} \\
& \Rightarrow x^{2}-4 x+3=0 \\
& \Rightarrow(x-1)(x+3)=0 \\
& \Rightarrow x=1, x=3 \\
& g(1)=1+3=4 \Rightarrow A(1,4) \\
& g(3)=3+3=6 \Rightarrow B(3,6)
\end{aligned}
$$

(b) The points $O(0,0)$ and $C(5,0)$ are on the graph of $f$.
(i) Draw the quadrilateral $O C B A$ on the diagram above.
(ii) Find the area of the quadrilateral $O C B A$.

$$
\text { Area } \begin{aligned}
O C B A & =|\triangle O D A|+|A D E B|+|\triangle B E C| \\
& =\frac{1}{2}(1)(4)+\frac{1}{2}(4+6)(2)+\frac{1}{2}(2)(6) \\
& =18 \text { square units }
\end{aligned}
$$

## Question 6

(a) The complex number $z_{1}=a+b i$, where $i^{2}=-1$, is shown on the Argand diagram below.
(i) Write down the value of $a$ and the value of $b$.

$$
a=3 ; \quad b=1
$$

(ii) The image of $z_{1}$ under reflection in the real axis is $z_{2}=c+d i$. Write down the value of $c$ and the value of $d$.

$$
c=3 ; \quad d=-1
$$


(b) (i) The angle $\theta$ is formed by joining $z_{1}$ to $0+0 i$ to $z_{2}$.

Find $\cos \theta$, correct to one decimal place.

$$
\begin{aligned}
& \left|z_{1}\right|=\sqrt{3^{2}+1^{2}}=\sqrt{10} \\
& \cos \theta=\frac{10+10-4}{2 \sqrt{10} \sqrt{10}} \\
& \\
& =\frac{16}{20}=0 \cdot 8
\end{aligned}
$$

or

$\tan \frac{1}{2} \theta=\frac{1}{3} \Rightarrow \frac{1}{2} \theta=18.4349^{\circ} \Rightarrow \theta=36.87 \Rightarrow \cos \theta=0.8$
(ii) Show that $\left|z_{1}\right| \times\left|z_{2}\right| \times \cos \theta=a c+b d$, where $a, b, c$, and $d$ are the values from part (a) above
$\left|z_{1}\right| \times\left|z_{2}\right| \times \cos \theta=\sqrt{10} \times \sqrt{10} \times 0 \cdot 8=8$
$a c+b d=3 \times 3+1 \times(-1)=9-1=8$
$\left|z_{1}\right| \times\left|z_{2}\right| \times \cos \theta=a c+b d$

Answer all three questions from this section.

## Question 7

(70 marks)
The first three patterns in a sequence of patterns are shown below.


1


2


3
(a) Draw the fourth pattern in the sequence.

(b) Complete the table below.

|  | Number of Black <br> Triangles | Number of White <br> Triangles | Total Number of <br> Small Triangles |  |
| :---: | :---: | :---: | :---: | :---: |
| Pattern 1 | 3 | 1 | $T_{1}$ | $=4$ |
| Pattern 2 | 6 | 3 | $T_{2}$ | $=9$ |
| Pattern 3 | 10 | 6 | $T_{3}$ | $=16$ |
| Pattern 4 | 15 | 10 | $T_{4}$ | $=25$ |
| Pattern 5 | 21 | 15 | $T_{5}=36$ |  |

(c) Show that the numbers of black triangles form a quadratic sequence.

| 3 |  | 6 |  | 10 |  | 15 |  | 21 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 3 |  | 4 |  | 5 |  | 6 |  |

The second difference is constant - therefore quadratic
(d) (i) How many black triangles are in the $9^{\text {th }}$ pattern?
(ii) How many white triangles are in the $9^{\text {th }}$ pattern?

Answer: 55
Answer: 45

$$
T_{9}=100
$$

(e) Write an expression in $n$ for the total number of triangles in the $n^{\text {th }}$ pattern.

> Answer:
$T_{n}=(n+1)^{2}$
(f) The number of black triangles in the $n^{\text {th }}$ pattern is given by the formula $B_{n}=\frac{1}{2} n^{2}+\frac{3}{2} n+c$. Find the value of $c$.

$$
\begin{aligned}
& B_{n}=\frac{1}{2} n^{2}+\frac{3}{2} n+c \\
& B_{1}=\frac{1}{2}(1)^{2}+\frac{3}{2}(1)+c=3 \Rightarrow 2+c=3 \Rightarrow c=1
\end{aligned}
$$

(g) Use your answers to parts (e) and (f) above to find a formula for the number of white triangles in the $n^{\text {th }}$ pattern.

$$
\begin{aligned}
W_{n}+B_{n}=(n+1)^{2} \Rightarrow W_{n} & =n^{2}+2 n+1-\left(\frac{1}{2} n^{2}+\frac{3}{2} n+1\right) \\
& =\frac{1}{2} n^{2}+\frac{1}{2} n
\end{aligned}
$$

(h) One particular pattern has a total of 625 triangles. Find the number of black triangles and the number of white triangles in that pattern.

$$
\begin{aligned}
& (n+1)^{2}=625=25^{2} \Rightarrow n+1=25 \Rightarrow n=24 \\
& B_{24}=\frac{1}{2}(24)^{2}+\frac{3}{2}(24)+1=288+36+1=325 \\
& W_{24}=625-325=300
\end{aligned}
$$

## Question 8

The daily profit of an oil trader is given by the profit function $p=96 x-0 \cdot 03 x^{2}$, where $p$ is the daily profit, in euro, and $x$ is the number of barrels of oil traded in a day.
(a) Complete the table below.

| Number of barrels <br> traded in a day | $x$ | 500 | 1000 | 1500 | 2000 | 2500 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Daily profit ( $€)$ | $p$ | 40500 | 66000 | 76500 | 72000 | 52500 |

(b) Draw the graph of the trader's profit function on the axes below for $500 \leq x \leq 2500, x \in \mathbb{R}$.

(c) Use your graph to estimate:
(i) The daily profit when 1750 barrels are traded.

Answer: €76 125
(ii) The numbers of barrels traded when the daily profit is $€ 60000$.

```
Answer: 850 or 2350
```

(d) (i) Use calculus to find the number of barrels of oil traded that will earn the maximum daily profit.

$$
\begin{aligned}
& p=96 x-0 \cdot 03 x^{2} \\
& \frac{d p}{d x}=96-0 \cdot 06 x=0 \\
& \Rightarrow x=1600 \text { barrels }
\end{aligned}
$$

(ii) Find this maximum profit.

$$
\begin{aligned}
& p=96 x-0 \cdot 03 x^{2} \\
& x=1600 \Rightarrow p=96(1600)-0 \cdot 03(1600)^{2} \\
&
\end{aligned}
$$

(e) The trader will not make a profit if he trades more than $k$ barrels of oil in a day. Calculate the value of $k$.

$$
\begin{aligned}
& p=96 x-0 \cdot 03 x^{2}=0 \\
& \Rightarrow x(96-0 \cdot 03 x)=0 \\
& \Rightarrow x=0 ; \quad 96-0 \cdot 03 x=0 \\
& \Rightarrow x=0 ; \quad x=3200 \\
& k=3200 \text { barrels }
\end{aligned}
$$

## Question 9

Amanda buys a new car for $€ 25000$. She knows that the value of cars depreciates by a fixed percentage each year. She draws the graph below to represent the value of her car for the next six years

(a) (i) Use Amanda's graph to estimate the length of time it will take for the car to depreciate from $€ 15000$ to $€ 10000$.

Answer: $4 \cdot 1-2 \cdot 3=1 \cdot 8$ years
(ii) Continue Amanda's graph to find the approximate value of the car after 7.5 years.

Answer: 4700
(iii) What name is given to the type of curve Amanda has drawn above?

Answer: Exponential
(b) On the day that Amanda bought her car, Cathal also bought a new car. His car cost $€ 22500$. Amanda's car will travel 15 km on a litre of fuel. Cathal's car will travel 10 km on a litre of fuel. How many kilometres will the cars have to travel until the cost price of the car plus the cost of fuel are equal, for the two cars? Assume that fuel costs $€ 1 \cdot 50$ per litre over the period.

Amanda: Cost per km $=\frac{150}{15}=10$ cent per km
Cathal: Cost per $\mathrm{km}=\frac{150}{10}=15$ cent per km
Cost difference per km is 5 cent
Number of km: $\frac{€ 2500}{€ 0 \cdot 05}=50000 \mathrm{~km}$

## or

Let k be the number of kilometres.

$$
\begin{aligned}
& 25000+0 \cdot 1 k=22500+0 \cdot 15 k \\
& 0 \cdot 05 k=2500 \Rightarrow k=50000 \mathrm{~km}
\end{aligned}
$$

## Marking Scheme - Paper 1, Section A and Section B

## Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

| Scale label | A | B | C | D | E |
| :--- | :---: | :---: | :---: | :---: | :---: |
| No of categories | 2 | 3 | 4 | 5 | 6 |
| 5 mark scales | 0,5 | $0,3,5$ | $0,3,4,5$ | $0,2,3,4,5$ |  |
| 10 mark scales |  | $0,3,10$ | $0,3,7,10$ | $0,2,5,9,10$ |  |
| 15 mark scales |  | $0,3,15$ | $0,3,10,15$ | $0,3,7,11,15$ |  |
| 20 mark scales |  |  |  |  |  |
| 25 mark scales |  |  |  |  |  |

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

## Marking scales - level descriptors

## A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response


## C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response


## E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response


## Detailed marking notes

NOTE: In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded.
Rounding and units penalty to be applied only once in each section (a), (b), (c) etc.

- Throughout the scheme indicate by use of * where an arithmetic error occurs.

Summary of mark allocations and scales to be applied

## Section A

## Section B

Question 1
(a) 10 B
(b) 10 B
(c) 5 C

Question 2
(a) 15 B
(b)(i)(ii) 5 C
(c) 5 B

Question 3
(a) 15 B
(b) 5 C
(c) 5 C

## Question 4

(a) 15 C
(b) 5 C
(c) 5 B

Question 5
(a) 15 C
(b)(i)(ii) 10D

Question 6
(a) 15 D
(b)(i) 5B
(b)(ii) 5 C

## Question 7

(a) 15B
(b) 15B
(c) 5 D
(d)(i)(ii)(iii) $\quad 15 \mathrm{C}$
(e) 5B
(f) 5B
(g) $\quad 5 \mathrm{C}$
(h) 5 C

Question 8

| (a) | 15 D |
| :--- | :--- |
| (b) | 15 D |
| (c)(i)(ii) | 5 C |
| (d)(i) | 5 C |
| (d)(ii) | 5 A |
| (e) | 5 C |

## Question 9

(a)(i) 15 B
(a)(ii) 5 A
(a)(iii) 5 A
(b) 5B

## Section A

## Question 1

(a) Scale 10B $(0,3,10)$

Partial credit:

- Incorrect use of 1.29
(b) Scale 10B (0, 3, 10)

Partial credit:

- Equates
- Indicates division
(c) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Work on Dollar Euro rate or Sterling Euro rate

High partial credit:

- Correct Dollar Euro rate or Sterling Euro rate


## Question 2

(a) Scale 15B (0, 3, 15)

Partial credit:

- Any correct ratio established or one correct amount
(b)(i)(ii) Scale 5C (0, 3, 4, 5)

Low Partial credit:

- Correct formula or work of relevance in either part
- Finds diameter ( 12756 km )


## High Partial Credit

- One part correct
- Substitutes correctly into both formulae
(c) Scale 5B (0, 3, 5)

Partial credit:

- Formulates division


## Question 3

(a) Scale 15B (0, 3, 15)

Partial credit:

- Some correct attempt at simplifying
(b) Scale 5C (0, 3, 4, 5)

Note: Accept $\{0,1,2\}$
Low Partial credit:

- Any correct transposition


## High Partial Credit

- Not more than two errors in solving
- $x \leq 8 / 3$
(c) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Formulate division
- Attempt at factorising

High partial credit:

- One full correct step in division
- $(3 x-2)(5 x-4)$ and stops


## Question 4

(a) Scale 15C (0, 3, 10, 15)

Low partial credit:

- Correct quadratic formula written
- Attempt at factorising


## High partial credit:

- Correct substitution into quadratic formula
- Finds one correct solution
- Solutions found with not more than 3 errors

Note: $x=3+\sqrt{5}, x=3-\sqrt{5}$ merits 14 marks
(b) Scale 5C (0, 3, 4, 5)

Low Partial credit:

- Differentiates at least one term correctly
- States $f^{\prime}(x)=0$ or writes $f^{\prime}(x)$ or writes $\frac{d y}{d x}$
- Correct answer without work
- Some work towards a graphical solution


## High Partial Credit

- Finds correct $x$ value
- Correct structure to $f^{\prime}(x)=0$, but with errors and finishes correctly

Note: Accept correct turning point found graphically for full marks.
(c) Scale 5B $(0,3,5)$

Partial credit:

- Any correct information shown on graph, e.g. indicates scale
- General shape of curve drawn


## Question 5

(a) Scale 15C (0, 3, 10, 15)

Low partial credit:

- $g(x)=f(x)$
- Any work towards a graphical solution
- Finds A and stops
- Finds A and B without work

High partial credit:

- Solve for $x$ values
- Show points on the graph
- Not more than one error working out points algebraically
(b)(i)(ii) Scale 10D (0, 2, 5, 9, 10)

Low Partial credit:

- One or more points plotted correctly
- Any incorrect quadrilateral drawn
- Any relevant formula written


## Middle Partial credit:

- Correct quadrilateral drawn
- Correct area of any section, without a quadrilateral established (e.g. OAC)


## High Partial Credit

- Correct area of any section, provided quadrilateral established
- Area formula established, with correct substitutions, for total area


## Question 6

(a) Scale 15D (0, 3, 7, 11, 15)

Low partial credit:

- One correct value
- Plots $z_{2}$ correctly and stops

Middle Partial credit:

- Two correct values

High partial credit:

- Three correct values
(b)(i) Scale 5B (0, 3, 5)

Partial credit:

- Cosine formula written
- Finds $\left|z_{1}\right|$
- Any work of merit
- Joins $o Z_{1}$ or $o Z_{2}$ or $Z_{1} Z_{2}$
(b)(ii) Scale 5C (0, 3, 4, 5)

Low Partial credit:

- $a c+b d$ correct
- Any work of merit
- Finds $\left|z_{1}\right|$ and/or $\left|z_{2}\right|$

High partial credit:

- one error only in substitution, with conclusion


## Section B

## Question 7

(a) Scale 15B ( $0,3,15$ )

Partial credit:

- Outline triangle drawn
- Division consistent with pattern, but not 9 triangles in fifth row
(b) Scale 15B (0, 3, 15)

Partial credit:

- One correct entry
(c) Scale 5D (0, 2, 3, 4, 5)

Low partial credit:

- Correct sequence of numbers

Middle Partial credit:

- First difference established

High partial credit:

- Second difference established
(d)(i)(ii)(iii) Scale 15C ( $0,3,10,15$ )

Low partial credit:

- One correct answer

High partial credit:

- Two correct answers
(e) Scale 5B $(0,3,5)$

Partial credit:

- $n^{2}$ as answer
- Writes $a n^{2}+b n+c$
(f) Scale 5B ( $0,3,5$ )

Partial credit:

- Any relevant substitution into $B_{n}$
(g) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Formula for the number of black and/or $T_{n}$ from (e) written down

High partial credit:

- $W_{n}+B_{n}=(n+1)^{2}$
(h) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Black $=625$ - White or White $=625-$ Black
- Pattern extended
- Correct answer without work


## High partial credit:

- Finds $n$, or number of black, or number of white


## Question 8

(a) Scale 15D (0, 3, 7, 11, 15)

Low partial credit:

- Relevant work towards finding one $p$ value

Middle Partial credit:

- One correct $p$

High partial credit:

- Two correct $p$ values
(b) Scale 15D (0, 3, 7, 11, 15)

Low partial credit:

- One point plotted correctly
- Correct shape of graph.


## Middle Partial credit:

- Three or four points plotted incorrectly, with a correct shape

High partial credit:

- Fails to join the five points or joins with a straight line
- One or two points plotted incorrectly, with a correct shape
(c)(i)(ii) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Some indication from graph that 1750 or 60000 or both are used

High partial credit:

- One correct answer
(d)(i) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Indicates that differentiation is being used

High partial credit:

- Correct differentiation
(d) (ii) Scale 5A $(0,5)$
(e) Scale 5C (0, 3, 4, 5)

Low partial credit:

- $p=0$
- Evidence of more than one substitution
- Graph extended to $x$-axis

High partial credit:

- Not more than two errors in solving


## Question 9

(a)(i) Scale 15B $(0,3,15)$

Partial credit:

- 10000 or 15000 indicated on graph, with no further work
- One value read correctly
- Answer between $1 \cdot 55$ and $2 \cdot 05$ (inclusive), without work
(a)(ii) Scale 5A $(0,5)$
(a)(iii) Scale 5A $(0,5)$
(b) Scale 5B $(0,3,5)$

Partial credit:

- 2500 written
- 1 or 2 costs per km found (Amanda and/or Cathal)


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Coimisiún na Scrúduithe Stáit<br>State Examinations Commission

## Leaving Certificate Examination 2015

## Mathematics

## Paper 2

## Ordinary Level

Monday 8 June
Morning 9:30-12:00

300 marks

## Model Solutions - Paper 2

Note: The model solutions for each question are not intended to be exhaustive - there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.

## Instructions

There are two sections in this examination paper.

| Section A | Concepts and Skills | 150 marks | 6 questions |
| :--- | :--- | :--- | :--- |
| Section B | Contexts and Applications | 150 marks | 3 questions |

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the Formulae and Tables booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here: $\square$

Answer all six questions from this section.

## Question 1

(25 marks)
A bank issues a unique six-digit password to each of its online customers. The password may contain any of the numbers 0 to 9 in any position and numbers may be repeated. For example, the following is a valid password.

(a) How many different passwords are possible?

$$
10^{6}=1000000
$$

(b) (i) How many different passwords do not contain any zero?

$$
9^{6}=531441
$$

(ii) One password is selected at random from all the possible passwords. What is the probability that this password contains at least one zero?

Number of passwords with at least one zero: 1000 000-531441=468559
$\mathrm{P}($ at least one zero $)=\frac{468559}{1000000}=0 \cdot 468559$
or
$1-\mathrm{P}($ no zero $)=1-\frac{531441}{1000000}=0 \cdot 468559$
(c) John is issued with one such password from the bank. Each time John wants to access his account online, the bank's website requires him to input three of his password digits into the boxes provided. For example, he may be asked for the $2^{\text {nd }}, 4^{\text {th }}$ and $5^{\text {th }}$ digits, as shown below.


In how many different ways can the bank select the three required boxes?
List as:

| $123,124,125,126,134,135,136,145,146,156$, |
| :--- |
| $234,235,236,245,246,256,345,346,356,456$, |$\quad=20 \quad$ or $\binom{6}{3}=20$

## Question 2

The line $p$ makes equal intercepts on the axes at $A$ and at $B$, as shown.
(a) (i) Write down the slope of $p$.

Slope of $p=1$
(ii) The point $(1,5)$ is on $p$. Find the equation of $p$. Write your answer in the form $a x+b y+c=0$,
 where $a, b$, and $c \in Z$

$$
\begin{aligned}
& y-y_{1}=m\left(x-x_{1}\right) \\
& y-5=1(x-1) \Rightarrow x-y+4=0
\end{aligned}
$$

(b) The line $q$ is perpendicular to $p$ and contains the point $O(0,0)$. Find the equation of $q$.

$$
\begin{aligned}
& q \perp p \Rightarrow \quad \text { slope of } q=-1 \\
& y-y_{1}=m\left(x-x_{1}\right) \\
& y-0=-1(x-0) \Rightarrow x+y=0
\end{aligned}
$$

(c) The lines $p$ and $q$ intersect at the point $C$. Explain why the triangles $O C A$ and $O B C$ are congruent.

In $\triangle O C A$ and $\triangle O B C$,
$|O A|=|O B| \quad$... equal intercepts
$|O C|=|O C| \ldots$ common line segment
$|\angle O C A|=|\angle B C O| \quad \ldots$ right angle
Giving R.H.S.
Hence, the triangles $O C A$ and $O B C$ are congruent.

## Question 3

(a) Draw the circle $c: x^{2}+y^{2}=25$. Show your scale on both axes.

(b) Verify, using algebra, that $A(-4,3)$ is on $c$.

$$
\begin{aligned}
& x^{2}+y^{2}=25 \\
& (-4)^{2}+3^{2}=16+9=25=\text { RHS }
\end{aligned}
$$

Centre of $c: O(0,0)$

$$
\begin{aligned}
|O A| & =\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}} \\
& =\sqrt{(-4-0)^{2}+(3-0)^{2}} \\
& =\sqrt{25}=5=\text { radius of } c
\end{aligned}
$$

(c) Find the equation of the circle with centre $(-4,3)$ that passes through the point $(3,4)$.

$$
\begin{aligned}
& r=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}} \\
&=\sqrt{(3+4)^{2}+(4-3)^{2}}=\sqrt{49+1}=\sqrt{50} \\
&(x-h)^{2}+(y-k)^{2}=r^{2} \Rightarrow \quad(x+4)^{2}+(y-3)^{2}=(\sqrt{50})^{2}=50
\end{aligned}
$$

(a) The diagram shows a parallelogram, with one side produced.
Use the data on the diagram to find the value of $x$, of $y$, and of $z$.

Give a reason for your answer in each case.


$$
x=30^{\circ}
$$

Reason: Alternate angles

$$
\begin{aligned}
& y=40^{\circ} \\
& {\left[180^{\circ}-\left(100^{\circ}+30^{\circ}\right)\right]}
\end{aligned}
$$

Reason: Angle sum in a triangle is $180^{\circ}$.

$$
\begin{aligned}
& z=70^{\circ} \\
& {\left[x+y=30^{\circ}+40^{\circ}\right]}
\end{aligned}
$$

Reason: Corresponding angles/ External angle equals sum of opposite interior angles.
(b) The area of the parallelogram $A B C D$ is $480 \mathrm{~m}^{2}$.
(i) Find the area of the triangle $A B D$.

$$
|\triangle A B D|=\frac{1}{2}(480)=240 \mathrm{~m}^{2}
$$


(ii) $E$ is the midpoint of $[C D]$. Find the area of the triangle $B C E$.

$$
|\triangle B C E|=\frac{1}{2}|\triangle B C D|=120 \mathrm{~m}^{2}
$$

The diagram shows the triangles $B C D$ and $A B D$, with some measurements given.
(a) (i) Find $|B C|$, correct to two decimal places.

$$
\begin{aligned}
& \frac{|B C|}{\sin 42^{\circ}}=\frac{16}{\sin 110^{\circ}} \\
& \begin{aligned}
\Rightarrow|B C| & =\frac{16 \sin 42^{\circ}}{\sin 110^{\circ}} \\
& =11.393=11.39 \mathrm{~m}
\end{aligned}
\end{aligned}
$$


(ii) Find the area of the triangle $B C D$, correct to two decimal places.
$|\angle D B C|=180^{\circ}-\left(42^{\circ}+110^{\circ}\right)=28^{\circ}$
Area $\Delta=\frac{1}{2} a b \sin C$

$$
\begin{aligned}
|\Delta B C D| & =\frac{1}{2}(16)(11.39) \sin 28^{\circ} \\
& =42.778=42.78 \mathrm{~m}^{2}
\end{aligned}
$$

(b) Find $|A B|$, correct to two decimal places.

$$
\begin{aligned}
& |\angle B D A|=180^{\circ}-\left(63^{\circ}+42^{\circ}\right)=75^{\circ} \\
& \begin{aligned}
& a^{2}=b^{2}+c^{2}-2 b c \cos A \\
&|A B|^{2}=10^{2}+16^{2}-2(10)(16) \cos 75^{\circ} \\
&=273.178 \\
&|A B|=16.528=16.53 \mathrm{~m}
\end{aligned}
\end{aligned}
$$

(a) Mark ten or more points on each of the scatter graphs below to show an example of the type of correlation named under each graph.



(iii) No correlation
(b) A few days before the Scottish Independence Referendum in September 2014 a YouGov poll estimated the support for the 'No' campaign to be $54 \%$.
(i) If YouGov sampled 1000 people, find the margin of error.

Write your answer as a percentage, correct to one decimal place.

$$
\frac{1}{\sqrt{n}}=\frac{1}{\sqrt{1000}}=0 \cdot 03162=3 \cdot 2 \%
$$

(ii) Create a $95 \%$ confidence interval for the level of support for the 'No' campaign in the population.

$$
\begin{aligned}
\hat{p} \pm \frac{1}{\sqrt{n}} & =54 \pm 3 \cdot 2 \\
& =50 \cdot 8 \leq p \leq 57 \cdot 2
\end{aligned}
$$

Answer all three questions from this section.

## Question 7

(35 marks)
The diagram below shows the right-angled triangle $A B C$, which is used in the logo for a company called Deane Construction Limited (DCL). The triangle $P Q R$ is the image of $A B C$ under an enlargement.

(a) (i) Construct the centre of enlargement and label it $O$.
(ii) Measure, in centimetres, $|O B|$ and $|O Q|$.

$$
|O B|=15 \mathrm{~cm} \quad|O Q|=22 \cdot 5 \mathrm{~cm}
$$

(iii) Use your measurements to find the scale factor of the enlargement, correct to one decimal place.

$$
\frac{|O Q|}{|O B|}=\frac{22 \cdot 5}{15}=1 \cdot 5
$$

(b) The area of the triangle $A B C$ is $7.5 \mathrm{~cm}^{2}$.

Use the scale factor to find the area of the image triangle $P Q R$ under the enlargement.

$$
7 \cdot 5 \times 1 \cdot 5^{2}=16 \cdot 875 \mathrm{~cm}^{2}
$$

(c) (i) Given that $|A B|=5 \mathrm{~cm}$, use the scale factor to find $|P Q|$.

$$
|P Q|=|A B| \times 1 \cdot 5=5 \times 1 \cdot 5=7 \cdot 5 \mathrm{~cm}
$$

(ii) Given that $|Q R|=8.7 \mathrm{~cm}$, use the scale factor to find $|B C|$.

$$
|Q R|=|B C| \times 1 \cdot 5 \Rightarrow|B C|=\frac{|Q R|}{1 \cdot 5}=\frac{8 \cdot 7}{1 \cdot 5}=5 \cdot 8
$$

(iii) Hence, show that $|\angle A B C|=|\angle P Q R|$.

$$
\begin{array}{ll}
\cos |\angle A B C|=\frac{5}{5 \cdot 8} & \cos |\angle P Q R|=\frac{7 \cdot 5}{8 \cdot 7}=\frac{5}{5 \cdot 8} \\
\cos |\angle A B C|=\cos |\angle P Q R| \Rightarrow & |\angle A B C|=|\angle P Q R| \text { since angles are acute }
\end{array}
$$

## Question 8

(a) A company has a spherical storage tank. The diameter of the tank is 12 m .
(i) Write down the radius of the tank.

$$
\text { Radius }=6 \mathrm{~m}
$$


(ii) Find the volume of the tank, correct to the nearest $\mathrm{m}^{3}$.

$$
\frac{4}{3} \pi r^{3}=\frac{4}{3} \pi(6)^{3}=904 \cdot 779 \approx 905 \mathrm{~m}^{3}
$$

(b) The company paints the outside curved surface of the spherical tank.
(i) Find the curved surface area of the tank, correct to one decimal place.

$$
4 \pi r^{2}=4 \pi(6)^{2}=452 \cdot 389 \approx 452 \cdot 4 \mathrm{~m}^{2}
$$

(ii) The curved surface is painted with a special paint. One litre of paint will cover $3.5 \mathrm{~m}^{2}$. Find how many litres of paint are used, correct to the nearest litre.
$\frac{452 \cdot 4}{3 \cdot 5}=129 \cdot 2 \approx 129$ or 130 litres
(iii) The paint is sold in 25 litre tins. Each tin costs $€ 180$. Find the total cost of the paint.

$$
\begin{aligned}
& \frac{129}{25}=5 \cdot 17 \\
& 6 \times € 180=€ 1080
\end{aligned}
$$

(c) At another site the company has a differentlyshaped tank with the same volume.
This tank has hemispherical ends and a cylindrical mid-section of length $h \mathrm{~m}$, as shown. The radius of each hemispherical end is 4.5 m .

(i) Find the volume of one hemispherical end, correct to the nearest $\mathrm{m}^{3}$.

$$
V=\frac{2}{3} \pi r^{3}=\frac{2}{3} \pi(4 \cdot 5)^{3}=190 \cdot 85 \approx 191 \mathrm{~m}^{3}
$$

(ii) Find the length, $h$, of the cylindrical section, correct to one decimal place.

$$
\begin{aligned}
& 2(191)+\pi r^{2} h=905 \\
& \Rightarrow \pi(4 \cdot 5)^{2} h=905-382=523 \\
& \Rightarrow h=\frac{523}{\pi(4 \cdot 5)^{2}}=8 \cdot 22 \approx 8 \cdot 2 \mathrm{~m}
\end{aligned}
$$

## Question 9

The heights of a random sample of 1000 students were collected and recorded.
(a) Tick one box from the table below to indicate how you would categorise the type of data collected. Explain your choice.

| Categorical Nominal |  |
| :--- | :---: |
| Categorical Ordinal |  |
| Numerical Discrete |  |
| Numerical Continuous | $\checkmark$ |

Height is a number and can take an infinity of values between any two consecutive whole numbers.
(b) The sample of 1000 students was made up of 500 boys and 500 girls. The data from the 500 girls was used to create the information shown in Table 1.

| Table 1 (Girls) |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Height <br> (cm) | $145-150$ | $150-155$ | $155-160$ | $160-165$ | $165-170$ | $170-175$ | $175-180$ | $180-185$ |
| Number of <br> girls | 15 | 48 | 80 | 112 | 125 | 81 | 29 | 10 |

(i) Use the information in Table 1 to estimate the mean height of the girls, using midinterval values.
$\mu=\frac{\Sigma f x}{\Sigma f}=\frac{82215}{500}=164 \cdot 43 \mathrm{~cm}$
(ii) What is the largest possible value for the range of the heights of the girls in this sample?
$185-145=40 \mathrm{~cm}$
(iii) The median height of the girls in the sample is 164.5 cm . Explain what this means in the context of the heights of the 500 girls.

250 girls are $164 \cdot 5 \mathrm{~cm}$ tall or taller and 250 girls are $164 \cdot 5 \mathrm{~cm}$ tall or shorter
(c) (i) Use the data in Table 1 to complete Table 2 by finding the percentage of girls in each of the height categories.

| Table 2 (Girls, \%) |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Height <br> (cm) | $145-150$ | $150-155$ | $155-160$ | $160-165$ | $165-170$ | $170-175$ | $175-180$ | $180-185$ |
| Percentage <br> of girls | 3 | $9 \cdot 6$ | 16 | $22 \cdot 4$ | 25 | $16 \cdot 2$ | $5 \cdot 8$ | 2 |

(ii) Use the data in Table 2 to draw a histogram showing the percentage of girls in each height category.

(iii) A histogram showing the percentage of boys in each height category is given above. John examines both histograms and comments that "There are roughly twice as many boys as girls in the 175 to 180 cm category". Do the histograms support his claim? Explain your answer.

Answer: Yes
Reason: The area of the bar for the category 175-180 in the histogram for boys is approximately twice the area of the corresponding category in the histogram for the girls.
(iv) Mary examines both histograms and comments that "I see that there are more tall girls than tall boys". Do the two histograms support her claim? Explain your answer.

Answer: No
Reason: The combined areas of the categories from 165-185 is greater in the histogram for boys indicating that there are more boys than girls with a height greater than 165 cm .
[Similarly, for the combined areas of the three categories in the range 170-185.]
(d) (i) The mean height of the boys in the sample is $166 \cdot 7 \mathrm{~cm}$ and the standard deviation of their height is 8.9 cm . Assuming that boys' heights are normally distributed, use the Empirical Rule to find an interval that will contain the heights of approximately $95 \%$ of all boys.

Interval: $166 \cdot 7 \pm 2(8 \cdot 9)=[148 \cdot 9,184 \cdot 5]$
(ii) The standard deviation of the heights of the girls in the sample is 7.7 cm while the standard deviation of the heights of the boys is 8.9 cm . Interpret this difference in the context of the data.

There is a greater spread about the mean in the heights of the 500 boys than there is in the spread of the girls about the mean height for girls.

## Scheme - Paper 2, Section A and Section B

## Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

| Scale label | A | B | C | D | E |
| :--- | :---: | :---: | :---: | :---: | :---: |
| No of categories | 2 | 3 | 4 | 5 | 6 |
| 5 mark scales | 0,5 | $0,3,5$ | $0,3,4,5$ | $0,2,3,4,5$ |  |
| 10 mark scales |  | $0,4,10$ | $0,4,9,10$ | $0,2,5,8,10$ |  |
| 15 mark scales |  |  | $0,4,11,15$ | $0,4,7,11,15$ |  |
| 20 mark scales |  |  | $0,10,17,20$ | $0,5,10,15,20$ |  |
| 25 mark scales |  |  |  |  |  |

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

## Marking scales - level descriptors

## A-scales (two categories)

- incorrect response
- correct response


## B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response


## C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response


## E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, in scale 10C, 9 marks may be awarded.
Throughout the scheme indicate by use of * where an arithmetic error occurs.

## Summary of mark allocations and scales to be applied

## Section A

## Question 1

$\begin{array}{lr}\text { (a) } & 10 \mathrm{~B} \\ \text { (b)(i)(ii) } & 10 \mathrm{C} \\ \text { (c) } & 5 \mathrm{~B}\end{array}$
(c)

5B

## Question 2

(a)(i)(ii) 10C
(b)

10B
(c) 5 C

Question 3
(a) 10 D
(b) 10B
(c) 5B

## Question 4

(a) 20D
(b)(i) (ii) 5C

Question 5
(a)(i) 15D
(a)(ii) 5C
(b) 5 D

Question 6
(a)(i)(ii)(iii) 20D
(b)(i) (ii) 5C

## Section B

## Question 7

(a)(i) 15 C
(a)(ii)(iii) 10 C
(b) 5B
(c)(i)(ii) (iii) 5D

Question 8

| (a)(i) | 5 A |
| :--- | ---: |
| (a)(ii) | 10 B |
| (b)(i) | 5 C |
| (b)(ii) | 5 A |
| (b)(iii) | 5 C |
| (c)(i) | 5 B |
| (c)(ii) | 10 D |

## Question 9

(a) 5 C
(b)(i) 20 C
(b)(ii) 5C
(b)(iii) 5B
(c)(i) 5C
(c)(ii) 10B
(c)(iii) 5B
(c)(iv) 5B
(d)(i) 5C
(d)(ii) 5B

## Section A

## Question 1

(a) Scale 10B $(0,4,10)$

Partial credit:

- Any work of merit e.g. Mention of 10 or 6 , use of !.
- Works with relevant ${ }^{n} \mathrm{P}_{\mathrm{r}}$.e.g. ${ }^{10} \mathrm{P}_{6}$.
- Partial list of possibilities, ignoring repetition. e.g. 10.9.8.7.6....
- Answer almost correct e.g. $10^{6}$ or 10.9.8.7.6.5 $=151200$ or 9.9.9.9.9.9 $=531441$.

Full credit:

- Accept correct answers without work.
(b)(i)(ii) Scale 10C (0, 4, 9, 10)

Note: $\quad$ Accept correct or consistent work from (a).
Low partial credit:

- Any work of merit e.g. Works with value from (a) reduced to exclude zero.
- Works with relevant ${ }^{n} P_{r}$ e.g. ${ }^{9} \mathrm{P}_{6}$.

High partial credit:

- Any one correct answer.
- Two almost correct answers e.g. 9.8.7.6.5.4 $=60480 \rightarrow \mathrm{P}(\mathrm{b})(\mathrm{ii})=\frac{60480}{\operatorname{Ans}(a)}$.

Full credit:

- Accept correct answers without work.
(c) Scale 5B $(0,3,5)$

Partial credit:

- Any work of merit.


## Full credit

- Correct answer without work.


## Question 2

(a)(i)(ii) Scale 10C (0, 4, 9, 10)

Low partial credit:

- Writes a correct relevant formula e.g. [Slope or equation of line].
- Identifies slope and stops.


## High partial credit:

- One incorrect or omitted substitution into equation of line formula

$$
\text { e.g. }[y-5=\#(x-1)] .
$$

(b) Scale 10B $(0,4,10)$

Partial credit:

- Writes equation of line formula.
- Indicates $m_{1} \times m_{2}=-1$.
(c) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Correctly identifies any one pair of corresponding sides or angles.
- States SSS, SAS, RHS, ASA.


## High partial credit:

- Correctly identifies either two or three of the corresponding sides or angle.
- Fails to finish.
- Any 2 elements identified.
- Diagram marked with 3 elements identified.

Full credit:

- Accept correct explanation on basis of diagram information.


## Question 3

(a) Scale 10D (0, 2, 5, 8, 10)

Low partial credit:

- Any relevant work e.g. Identifies correct centre of circle or works to $r^{2}=25 \mathrm{and} /$ or $\sqrt{25}$ and stops.
- Attempt to scale an axis.


## Middle partial credit:

- Any two of the four components Centre, Radius, Scale or Drawing correct.


## High partial credit.

- Any three of the four components Centre, Radius, Scale or Drawing correct.
(b) Scale 10B $(0,4,10)$

Partial credit:

- Identifies correct relevant formula, e.g. writes $\left[\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}, x^{2}+y^{2}=r^{2}\right]$.
- Any correct or consistent substitution.
- Radius $=5$ (relevant work).
- No conclusion.
(c) $\quad$ Scale $5 \mathrm{~B}(0,3,5)$


## Partial credit:

- Identifies a correct relevant formula.
e.g. $\left[\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}},(x-h)^{2}+(y-k)^{2}=r^{2}, x^{2}+y^{2}+2 g x+2 f y+c=0\right]$.
- Any correct substitution and stops.
- Radius calculated.
- Fully correct or consistent substitution without calculation.
- Correct calculation with one incorrect substitution.


## Question 4

(a) Scale 20D $(0,5,10,15,20)$

Low partial credit:

- Any correct relevant work or statement for $x, y$, or $z$.
- One correct value without reason.

Middle partial credit:

- One correct value and reason.
- Two correct values without reasons.
- Two correct with one reason.

High partial credit:

- Two correct values, both with reasons.
- All correct values with one reason.

Note: Answers may be on the diagram.
(b)(i)(ii) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Any correct relevant work on the area of triangle in either part.

High partial credit:

- One correct area.


## Full credit:

- Accept correct answer without work.


## Question 5

(a)(i) Scale 15D (0, 4, 7, 11, 15)

Low partial credit:

- Some correct substitution into a correct relevant formula. [ $\sin 42^{\circ},|B C|=\frac{16}{\sin 110^{\circ}}$ ].
- Writes Sine Rule.


## Middle partial credit:

- Fully correct substitution without calculation e.g. $\frac{|B C|}{\sin 42^{\circ}}=\frac{16}{\sin 110^{\circ}}$.


## High partial credit:

- Calculates $|B C|$ with one incorrect substitution.
- Incorrect calculator mode. (Once only). [Radian $=331 \cdot 452 \ldots$, Gradian $=9 \cdot 928 \ldots]$.
- A correct answer without work.
(a)(ii) $\quad$ Scale $5 \mathrm{C}(0,3,4,5)$

Note: Accept consistent answer from previous work.
Low partial credit:

- Writes the correct formula for the area of a triangle.
- Any correct or consistent substitution into formula.
- Any work of merit e.g. Indicates total angle measure in a triangle $=180^{\circ}, 28^{\circ}$.


## High partial credit:

- Formula substituted correctly without calculation [Area $\left.=\frac{1}{2}(16)(11 \cdot 39) \sin 28^{\circ}\right]$.
- Calculates area of the triangle with one incorrect substitution.
- Incorrect calculator mode. (Once only). [Radian $=24 \cdot 684 \ldots$,Gradian $=38 \cdot 797 \ldots]$.
- A correct or consistent answer without work.
(b) Scale 5D (0, 2, 3, 4, 5)

Low partial credit:

- Some correct substitution into a correct relevant formula [a=10, b=16].
- Any work of merit such as identification of any correct information from the diagram. [ $C=75^{\circ}$ ].
- Writes Cosine Rule correctly.


## Middle partial credit:

- An almost correct formula substitution and stops.

$$
\text { e.g. }\left[|A B|^{2}=(10)^{2}+(16)^{2}-2(10)(16) \operatorname{Cos} \#\right] \text {. }
$$

## High partial credit:

- Formula substituted correctly. $\left[|A B|^{2}=(10)^{2}+(16)^{2}-2(10)(16) \cos 75^{\circ}\right]$.
- Calculates $|A B|$ with one incorrect substitution.
- Leaves answer as $|A B|^{2}$.
- Incorrect calculator mode. (Once only). [Radian $=7 \cdot 81 \ldots$, Gradian $=15 \cdot 28 \ldots]$.
- A correct or consistent answer without work.


## Question 6

(a)(i)(ii)\& (iii) Scale 20D ( $0,5,10,15,20$ )

Low partial credit:

- Plots some points to illustrate a scatter graph but shows no understanding of correlation in any of the cases.
- Plots one correlation with less than ten points.

Middle partial credit:

- Plots one correct type of correlation.
- Plots two types of correlation only with less than ten points.

High partial credit:

- Plots two correct types of correlation.
- Plots three types of correlation some with less than ten points.
(b)(i)(ii) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Writes one/both formulae and stops e.g. [ $\left.\hat{p}-E \leq p \leq \hat{p}+E, \hat{p} \pm 1 \cdot 96 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$.
- Works with $\sqrt{1000}$.

High partial credit:

- One correct solution.
- Substantive work in both parts.


## Section B

## Question 7

(a)(i) $\quad$ Scale $15 \mathrm{C}(0,4,11,15)$

Low partial credit:

- An attempt to draw any construction ray e.g. [Join B to Q].

High partial credit:

- Two construction rays drawn.
- Construction rays drawn but centre not identified correctly.
(a)(ii)(iii) Scale 10C (0, 4, 9, 10)

Note: Accept tolerance of 5 mm
Accept correct or consistent work from previous section.
Low partial credit:

- One measurement correct.
- Works with two sides of given triangles to get $k$ e.g. $\left[k=\frac{|P Q|}{|A B|}=\frac{7 \cdot 5}{5}=1 \cdot 5\right]$.

High partial credit:

- Two measurements correct.
- Error in scale factor.
(b) Scale 5B $(0,3,5)$

Partial credit:

- Use of an incorrect relevant formula [Use of $k$ ].
- Leaves as $7 \cdot 5|1 \cdot 5|^{2}$.
- Correct answer without use of scale factor. $\left[|P Q|=\frac{|A B|}{k}\right]$.
- Correct answer without work.
(c)(i) (ii) \& (iii) Scale 5D (0, 2, 3, 4, 5)

Note: Accept correct or consistent from previous work.
Low partial credit:

- Any use of scale factor e.g. $|P Q|=\frac{|A B|}{k}$.


## Middle partial credit:

- $|P Q|$ or $|B C|$ correct.


## High partial credit:

- $|P Q|$ and $|B C|$ correct.

No Credit:

- Correct answer without work.


## Question 8

(a)(i) $\quad$ Scale 5A $(0,5)$

Full credit:

- Accept a correct answer without work. (6 m).
(a)(ii) $\quad$ Scale 10B (0, 4, 10)

Note: Accept correct /consistent answer from (a)(i).
Partial credit:

- Any work of merit e.g. writes the correct formula, identifies correct/consistent radius.
- Treats as $k \pi(r)^{3}$ with correct/consistent radius substituted and stops.
- Correct answer given in terms of $\pi \ldots 288 \pi$.

Note: $\quad\left[905 \cdot 14\left(\frac{22}{7}\right), 904 \cdot 32(3 \cdot 14)\right]$.
Full credit:

- Accept a correct/consistent answer without work.
(b)(i) $\quad$ Scale 5C $(0,3,4,5)$

Note: Accept correct/consistent answer from (a)(i).
Low partial credit:

- Any work of merit e.g. writes the correct formula, identifies correct/consistent radius.
- Treats as $k \pi(r)^{2}$ with correct/consistent radius substituted and stops.


## High partial credit.

- Correct formula with correct/consistent substitution and stops.
- Correct answer given in terms of ... $144 \pi$.
- Finishes with only one error...e.g. $k \pi(r)^{2}$ calculated.

Note: $\left[452 \cdot 57\left(\frac{22}{7}\right), 452 \cdot 16(3 \cdot 14)\right]$.
Full credit:

- Accept a correct/consistent answer without work.
(b)(ii) $\quad$ Scale 5A (0, 5)

Note: Accept correct /consistent answer from (b)(i).
Full credit:

- Accept a correct/consistent answer without work.
(b)(iii) $\quad$ Scale 5C (0, 3, 4, 5)

Note: Accept correct /consistent answer from (b)(ii).
Low partial credit:

- Any correct relevant work, e.g. Writes answer from (b) (ii), sets up division.
- Calculates the required number of tins [Accept (5•17)/(5•2)].


## High partial credit:

- Finishes with only one error...e.g. incorrect number of tins.
- Sets up correct cost multiplication and stops.
(c)(i) $\quad$ Scale 5B $(0,3,5)$

Partial credit:

- Writes correct formula.
- Treats as $k \pi(r)^{3}$ with correct/consistent radius substituted.
- Correct formula with correct/consistent substitution and stops.
- Correct answer given in terms of $\pi \ldots \frac{243}{4} \pi$ or $60 \cdot 75 \pi$.

Full credit:

- Accept a correct/consistent answer without work.
(c)(ii) $\quad$ Scale 10D (0, 2, 5, 8, 10)

Note: Accept correct /consistent answer from (a)(ii) and (c)(i).
Low partial credit:

- Any correct or relevant work e.g. writes answers from (a)(ii) or (c)(i).
- Work towards setting up relevant equation with or without substitution.


## Middle partial credit:

- Calculates the volume of the cylinder (523).
- Finishes with two errors... e.g. one incorrect volume and a transposition error.


## High partial credit:

- Isolates $h$ correctly and stops e.g. $\left[h=\frac{523}{\pi(4 \cdot 5)^{2}}\right]$.
- Finishes with only one error...e.g. index error, transposition error.


## Question 9

(a) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Tick in Numerical Continuous with no explanation.


## High partial credit:

- A tick and an incomplete explanation for Numerical Continuous.

Full credit:

- A tick and a correct explanation for Numerical Continuous.
- A tick and a correct explanation for Numerical Discrete.
(b) (i) Scale 20C (0,10, 17, 20)

Low partial credit:

- Work of merit.

High partial credit:

- Correctly calculates $\sum f x$ and fails to finish e.g. [ 82215].
- Finishes $\frac{\sum f x}{\sum f}=\frac{\Delta}{500}$ (with only one error).

Note: $\Delta=$ Candidate's $\sum f x$ worked.

## Full credit:

- Correct answer without work.
(b) (ii) $\quad$ Scale 5C (0, 3, 4, 5)

Low partial credit:

- One end of range identified correctly.
- Subtraction of two boundaries indicated e.g. 170-155


## High partial credit

- Both ends of range identified correctly.


## Full credit:

- Correct answer without work.
(b) (iii) Scale 5B (0, 3, 5)

Partial credit:

- Presents an incomplete explanation e.g. "Half are tall".

Full Credit:

- Indicates "middle" of the sample.
(c)(i) $\quad$ Scale 5C (0, 3, 4, 5)

Low partial credit:

- One/two correct percentage.
- Attempt to find \% e.g. Writes 100.


## High partial credit:

- Any three correct percentages.
- Decimal format used.
(c)(ii) Scale 10B (0, 4, 10)

Partial credit:

- Any work of merit e.g. One correct block.
(c)(iii) $\quad$ Scale 5B $(0,3,5)$

Partial credit:

- Indicates correct answer only.
- Incomplete explanation.
(c)(iv) Scale 5B (0, 3, 5)

Partial credit:

- Indicates correct answer.
- Indicates correct answer and an incomplete explanation.
d(i) Scale 5C (0, 3, 4, 5)
Low partial credit:
- Writes a correct formula and stops.
- Some work using $\sigma$ [Interval as $157 \cdot 8$, or $175 \cdot 6$ ].

High partial credit:

- One end of range identified correctly [ $148 \cdot 9$, or 184•5].
- Incorrect consistent error [Works with $\bar{x} \pm \sigma$ ].
(d)(ii) Scale 5B (0, 3, 5)

Partial credit:

- An incomplete interpretation e.g. Says different values/Subtracts values, with no interpretation.


## Marcanna breise as ucht freagairt trí Ghaeilge

## (Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná $75 \%$ d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú síos.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.
Is é $5 \%$ an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta $5 \%$ i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. 198 marc $\times 5 \%=9 \cdot 9 \Rightarrow$ bónas $=9$ marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle [ 300 - bunmharc] $\times 15 \%$, agus an marc bónais sin a shlánú síos. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

| Bunmharc | Marc Bónais |
| :---: | :---: |
| 226 | 11 |
| $227-233$ | 10 |
| $234-240$ | 9 |
| $241-246$ | 8 |
| $247-253$ | 7 |
| $254-260$ | 6 |
| $261-266$ | 5 |
| $267-273$ | 4 |
| $274-280$ | 3 |
| $281-286$ | 2 |
| $287-293$ | 1 |
| $294-300$ | 0 |

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