

## Coimisiún na Scrúduithe Stáit State Examinations Commission

## Leaving Certificate Examination 2011 Sample Paper

## Mathematics (Project Maths – Phase 2)

Paper 1

Ordinary Level

Time: 2 hours, 30 minutes

300 marks

Examination number	For ex	aminer
	Question	Mark
	1	
	2	
Centre stamp	3	
	4	
	5	
	6	
	7	
	8	
	9	
Running total	Total	

Grade

## **Instructions**

There are **three** sections in this examination paper:

Section A	Concepts and Skills	100 marks	4 questions
Section B	Contexts and Applications	100 marks	2 questions
Section C	Functions and Calculus (old syllabus)	100 marks	3 questions

Answer questions as follows:

In Section A, answer all four questions

In Section B, answer both Question 5 and Question 6

In Section C, answer **any two** of the three questions.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

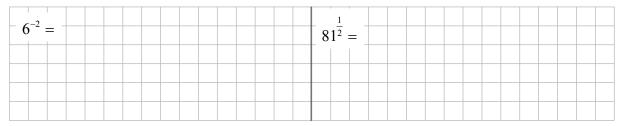
Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

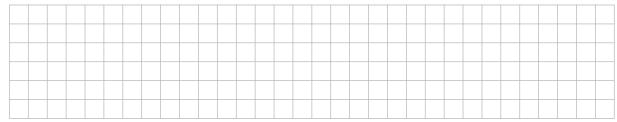
Answer all four questions from this section.

Question 1 (25 marks)

(a) Write  $6^{-2}$  and  $81^{\frac{1}{2}}$  without using indices.



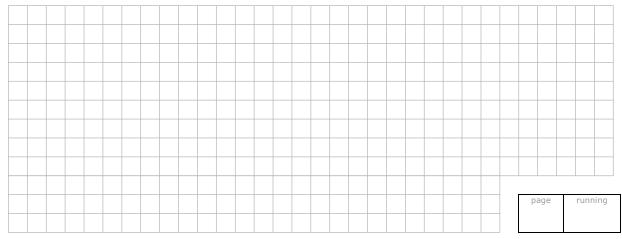
**(b)** Express  $2^{24}$  in the form  $a \times 10^n$ , where  $1 \le a < 10$  and  $n \in \mathbb{Z}$ , correct to three significant figures.



(c) Show that  $\frac{\left(a\sqrt{a}\right)^3}{a^4}$  simplifies to  $\sqrt{a}$ .



(d) Solve the equation  $49^x = 7^{2+x}$  and verify your answer.



Question 2 (25 marks)

(a) A sum of €5000 is invested in an eight-year government bond with an annual equivalent rate (AER) of 6%. Find the value of the investment when it matures in eight years' time.



**(b)** A different investment bond gives 20% interest after 8 years. Calculate the AER for this bond.



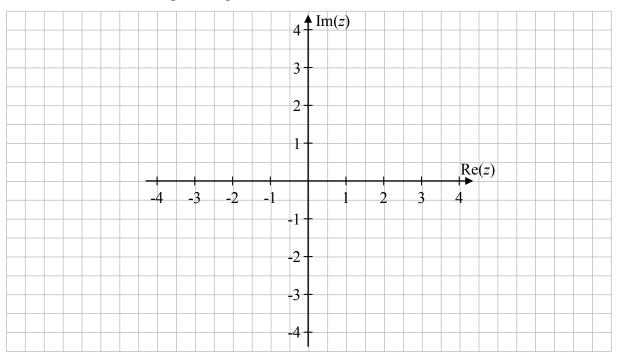
Question 3 (25 marks)

Two complex numbers are u = 3 + 2i and v = -1 + i, where  $i^2 = -1$ .

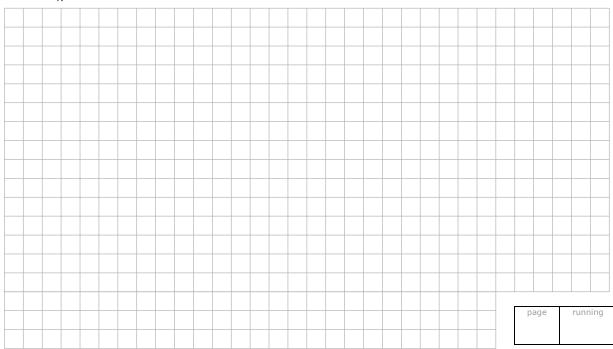
(a) Given that w = u - v - 2, evaluate w.



**(b)** Plot u, v, and w on the Argand diagram below.



(c) Find  $\frac{2u+v}{w}$ .



Question 4 (25 marks)

(a) Solve the equation  $x^2 - 6x - 23 = 0$ , giving your answer in the form  $a \pm b\sqrt{2}$ , where  $a, b \in \mathbb{Z}$ .



**(b)** Solve the simultaneous equations

$$2r-s=10$$

$$rs - s^2 = 12$$

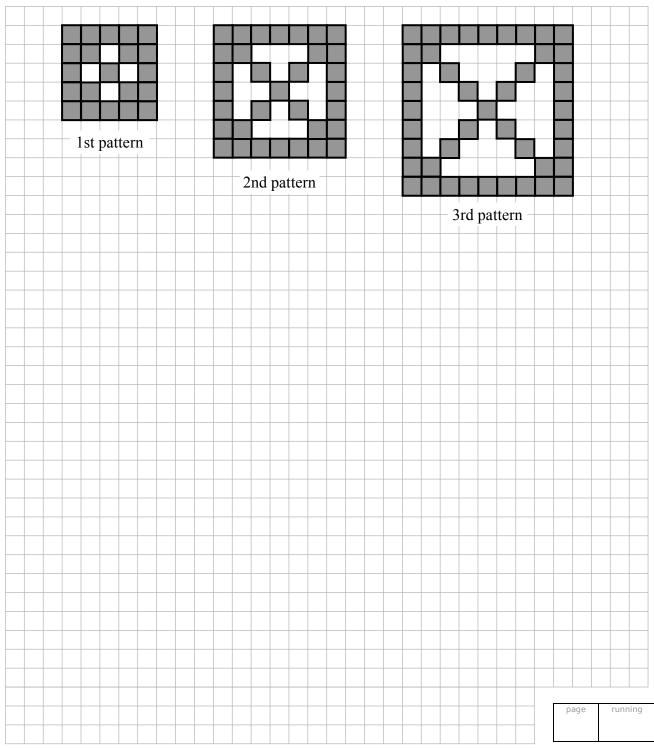


Answer both Question 5 and Question 6.

Question 5 (50 marks)

Síle is investigating the number of square grey tiles needed to make patterns in a sequence. The first three patterns are shown below, and the sequence continues in the same way. In each pattern, the tiles form a square and its two diagonals. There are no tiles in the white areas in the patterns – there are only the grey tiles.

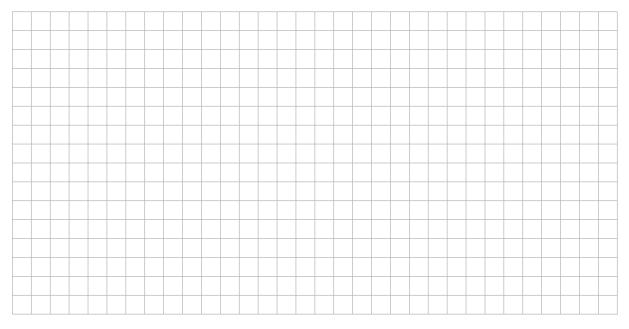
(Questions start overleaf.)



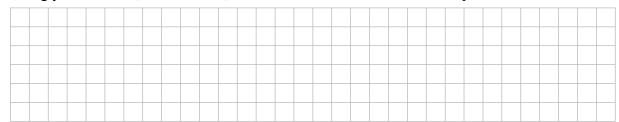
(a) In the table below, write the number of tiles needed for each of the first five patterns.

Pattern	1	2	3	4	5
No. of tiles	21	33			

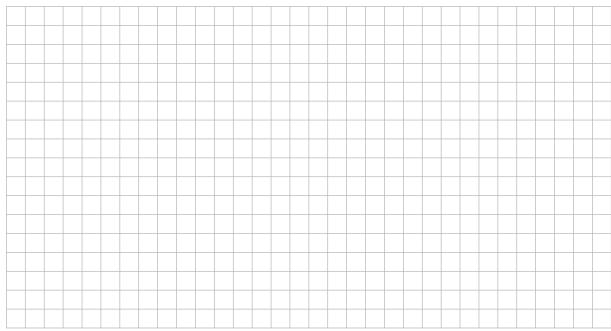
(b) Find, in terms of n, a formula that gives the number of tiles needed to make the nth pattern.



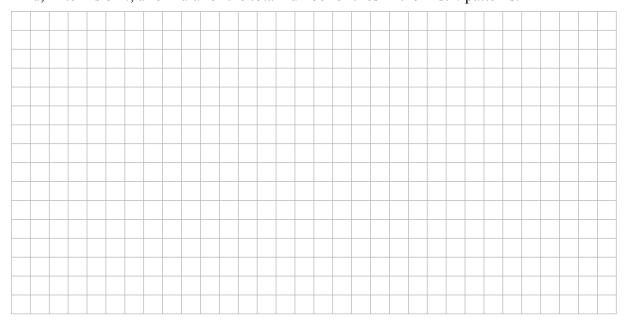
(c) Using your formula, or otherwise, find the number of tiles in the tenth pattern.



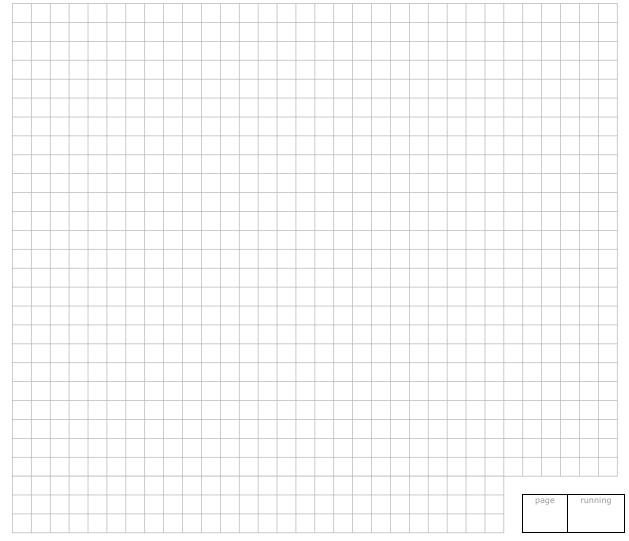
(d) Síle has 399 tiles. What is the biggest pattern in the sequence that she can make?



(e) Find, in terms of n, a formula for the total number of tiles in the first n patterns.



(f) Sile starts at the beginning of the sequence and makes as many of the patterns as she can. She does not break up the earlier patterns to make the new ones. For example, after making the first two patterns, she has used up 54 tiles, (21 + 33). How many patterns can she make in total with her 399 tiles?



Question 6 (50 marks)

John is given two sunflower plants. One plant is 16 cm high and the other is 24 cm high. John measures the height of each plant at the same time every day for a week. He notes that the 16 cm plant grows 4 cm each day, and the 24 cm plant grows 3.5 cm each day.

(a) Draw up a table showing the heights of the two plants each day for the week, starting on the day that John got them.

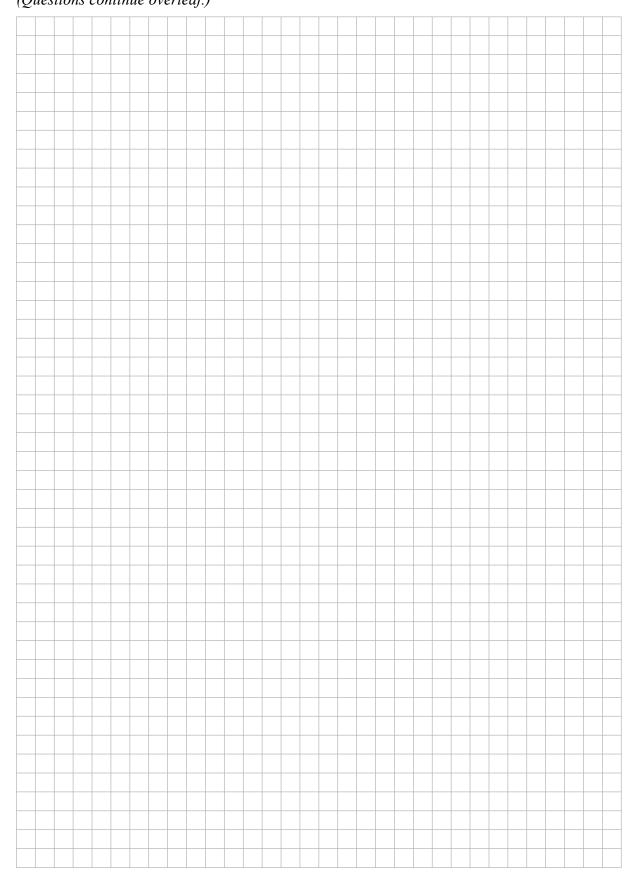


(b) Write down two formulas – one for each plant – to represent the plant's height on any given day. State clearly the meaning of any letters used in your formulas.



(c) John assumes that the plants will continue to grow at the same rates. Draw graphs to represent the heights of the two plants over the first *four weeks*.

(Questions continue overleaf.)



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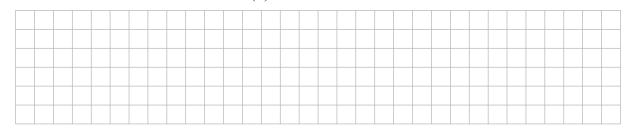
	Ans	swer:																						
(ii)	Explain what the point of intersection means, with respect to the two plants. Your answer should refer to the meaning of <i>both</i> co-ordinates.																							
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Answer any two of the three questions from this section.

Question 7 (50 marks)

(a) Let  $h(x) = x^2 + 1$ , where  $x \in \mathbb{R}$ .

Write down a value of x for which h(x) = 50.



- **(b)** Let  $g(x) = \frac{1}{x-2}$ , where  $x \in \mathbb{R}$  and  $x \neq 2$ .
  - (i) Complete the following table:

x	0	1	1.5	1.75	2.25	2.5	3	4
g(x)		-1		-4		2		

(ii) Draw the graph of the function g in the domain  $0 \le x \le 4$ .



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- (c) Let  $f(x) = x \frac{5}{x}$ , where  $x \in \mathbb{R}$  and  $x \neq 0$ .
  - (i) Find f'(x), the derivative of f(x).

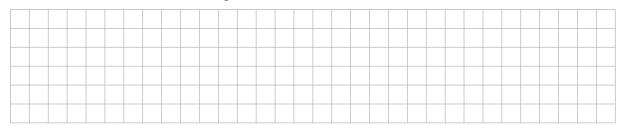


(ii) Find the co-ordinates of the two points at which the tangent to the curve y = f(x) is parallel to the line y = 6x.



Question 8 (50 marks)

(a) Differentiate  $x^2 - 6x + 1$  with respect to x.



(b) (i) Differentiate 5 - 3x with respect to x from first principles.



(ii) Given that  $y = (x^2 - 4)(3x - 1)$ , find the value of  $\frac{dy}{dx}$  when x = 2.

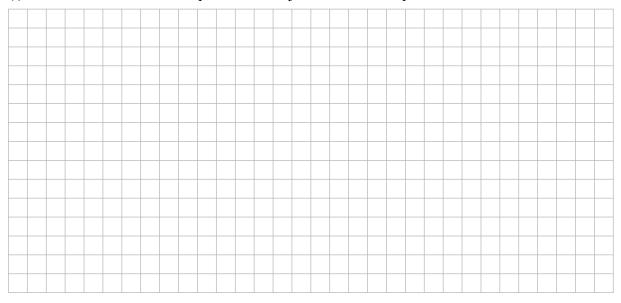


(c) The speed, v, of an object at time t is given by

$$v = 96 + 40t - 4t^2$$

where t is in seconds and v is in metres per second.

(i) At what times will the speed of the object be 96 metres per second?



(ii) What will the acceleration of the object be at t = 2.5 seconds?

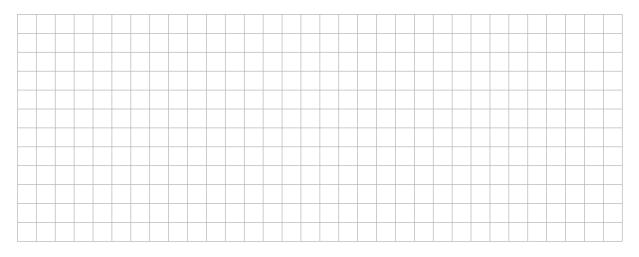


(iii) At what value of t will the acceleration become negative?



Let  $f(x) = x^3 - 3x + 1$ , where  $x \in \mathbb{R}$ .

(a) Find f(-3), f(-2), f(0), f(2) and f(3).



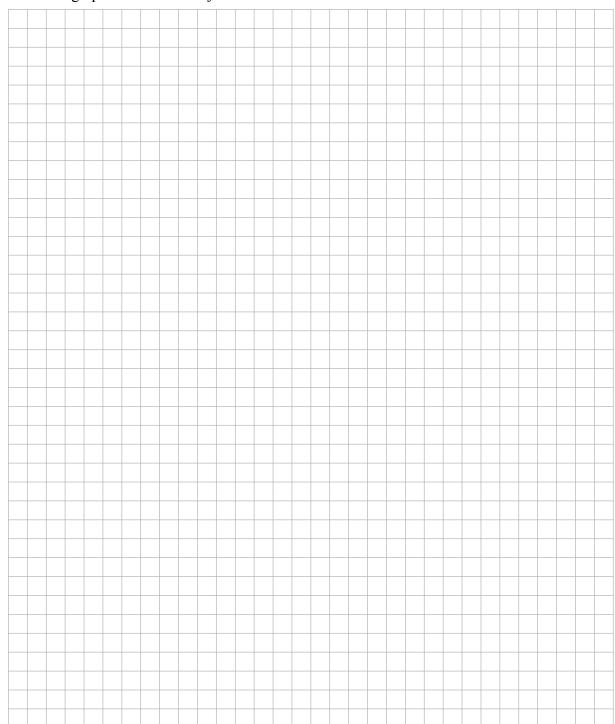
**(b)** Find f'(x), the derivative of f(x).



(c) Find the co-ordinates of the local maximum point and of the local minimum point of the curve y = f(x).

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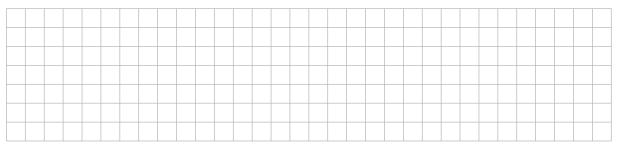
(d) Draw the graph of the function f in the domain  $-3 \le x \le 3$ .



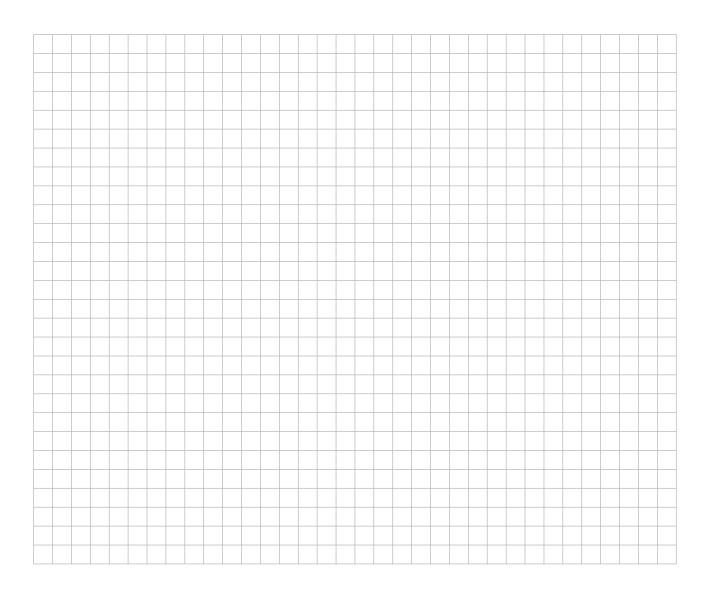
(e) Find the range of values of k for which the equation

$$x^3 - 3x + 1 = k$$

has three real solutions (roots).



You may use this page for extra work page running



*Note to readers of this document:* 

This sample paper is intended to help teachers and candidates prepare for the June 2011 examination in the *Project Maths* initial schools. The content and structure do not necessarily reflect the 2012 or subsequent examinations in the initial schools or in all other schools.

In the 2011 examination, questions 7, 8, and 9 in Section C on paper 1 will be the same questions as those that appear as 6, 7, and 8 on the examination for candidates who are not in the initial schools. On this sample paper, the corresponding questions from the 2010 examination have been inserted to illustrate.

Leaving Certificate 2011 – Ordinary Level

Mathematics (Project Maths – Phase 2) – Paper 1

Sample Paper

Time: 2 hours 30 minutes