2011. M227



Coimisiún na Scrúduithe Stáit State Examinations Commission

Leaving Certificate Examination 2011

Mathematics (Project Maths – Phase 2)

Paper 1

Ordinary Level

Friday 10 June Afternoon 2:00 – 4:30

300 marks

Examination number

Centre stamp

For ex	aminer
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Running	total	
Rummg	ioiui	

Instructions

There are **three** sections in this examination paper:

Section A	Concepts and Skills	100 marks	4 questions
Section B	Contexts and Applications	100 marks	2 questions
Section C	Functions and Calculus (old syllabus)	100 marks	3 questions

Answer questions as follows:

In Section A, answer all four questions

In Section B, answer **both** Question 5 **and** Question 6

In Section C, answer **any two** of the three questions.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A

Answer **all four** questions from this section.

Question 1

(25 marks)

(a) Explain what a *prime number* is.

(b) Express 2652 as a product of prime numbers.

-	 	 			 	 	 	 		 	 	 	 	 		
-																

(c) The number $2^{61}-1$ is a prime number. Using your calculator, or otherwise, express its value, correct to two significant figures, in the form $a \times 10^n$, where $1 \le a < 10$ and $n \in \mathbb{N}$.

					 									 	_

(d) Use your answer to part (c) to state how many digits there are in the exact value of $2^{61}-1$.

		_			 											
	_	_	_							 		 				

page running

(a) A certain deposit account will earn 3% interest in the first year and 6% interest in the second year. The interest is added to the account at the end of each year. If a person invests €20 000 in this account, how much will they have in the account at the end of the two years?

-						 										
<u> </u>		 	 			 										
-																
<u> </u>						 										

(b) Show that, to the nearest euro, the same amount of interest is earned by investing the money for two years in an account that pays compound interest at 4.49% (AER).



(25 marks)

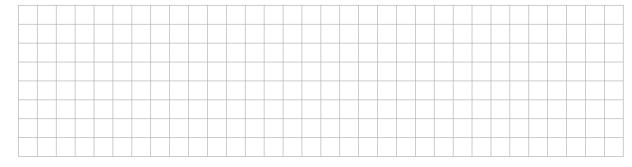
Question 3

The terms in an arithmetic sequence are given by the formula

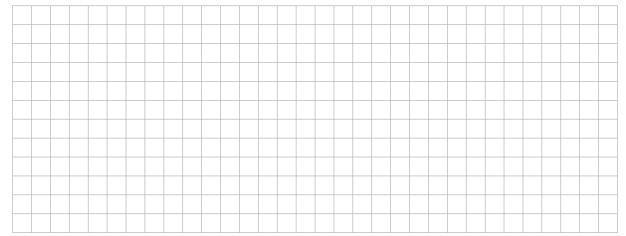
 $T_n = 38 - 4n$, for n = 1, 2, 3, 4, ...

(a) Write out the first three terms in the sequence.

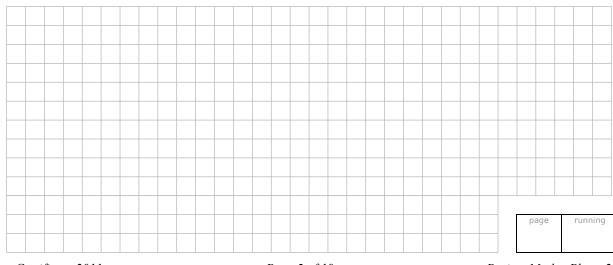
(b) What is the first negative term in the sequence?



(c) Find the sum of the first 15 terms of the sequence.



(d) Find the value of *n* for which the sum of the first *n* terms of the sequence is 0.

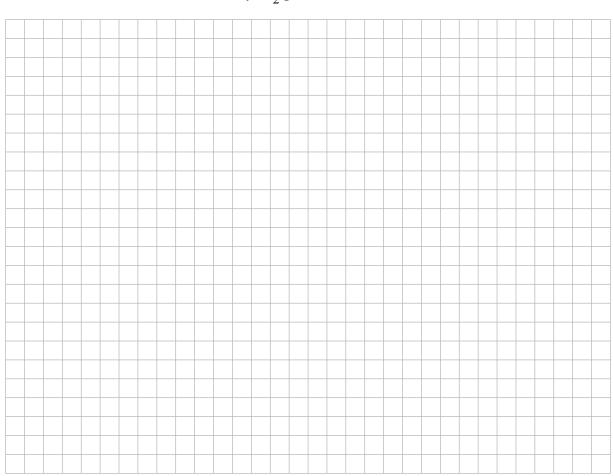


Project Maths, Phase 2 Paper1 – Ordinary Level

(25 marks)

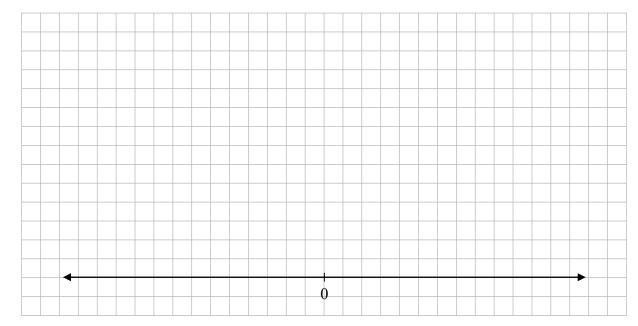
(a) Solve the simultaneous equations:

 $2f + \frac{2}{3}g + 1 = 0$ f + $\frac{1}{2}g + 1 = 0$.



(b) Solve the following inequality, and show the solution set on the number line below.

$5 - \frac{3}{4}x \le \frac{19}{8}$



Section **B**

Answer both Question 5 and Question 6.

Question 5

(50 marks)

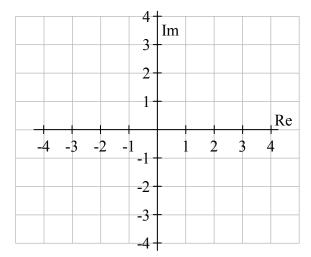
- z is the complex number 1+i, where $i^2 = -1$.
- (a) (i) Find z^2 and z^3 .



(ii) Verify that $z^4 = -4$.

	•														

(iii) Show z, z^2, z^3 and z^4 on the Argand diagram.



(iv) Make one observation about the pattern of points on the diagram.



Leaving Certificate 2011

Project Maths, Phase 2 Paper1 – Ordinary Level (b) Using the value of z^4 , or otherwise, find the values of z^8 , z^{12} and z^{16} , and insert their values in the table below.

z^4	z^{8}	z^{12}	z^{16}
-4			

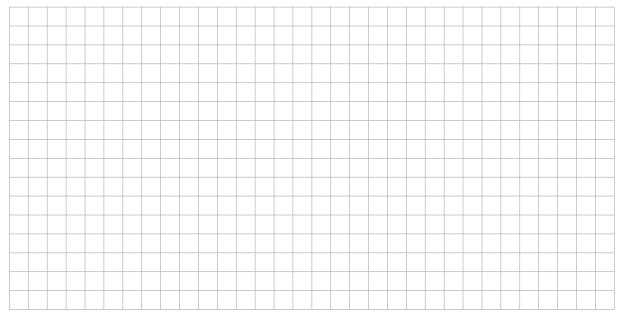
(c) Based on the pattern of values in part (b), or otherwise, state whether z^{40} is positive or negative. Explain how you got your answer.

(d) Write z^{40} as a power of 2.

(e) Find z^{41} .

 	 	 		 			 _		 	 	 			 	

(f) On an Argand diagram, how far from the origin is z^{41} ?



page	running

At a certain point during the flight of a space shuttle, the booster rockets separate from the shuttle and fall back to earth. The altitude of these booster rockets (their height above sea level) is given by the following formula:

$$h = 45 + \frac{7}{10}t - \frac{1}{200}t^2$$

where h is the altitude in kilometres, and t is the time in seconds after separation from the shuttle.



Image: NASA

(a) Complete the table below, showing the altitude of the rockets at the indicated times.

altitude in km, h	time in seconds, <i>t</i>	0	20	40	60	80	100
	altitude in km, h						

- (b) On the opposite page, draw a graph of the altitude of the rockets for the first 100 seconds after separation from the shuttle.
- (c) Use your graph to estimate the greatest altitude reached by the rockets.

Answer:

(d) Use the graph to estimate **one** time at which the altitude is 60 km. Show your work clearly on the graph.

Answer:

(50 marks)

Graph of altitude over time.

						┼┼┼┼╄┼╄
0	 20	40		60	80	100
			time /s			

page	running

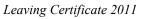
(e) Check your answer to part (d) using the formula for the altitude.



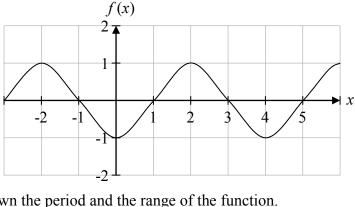
(f) By solving an equation, find the value of t at which the altitude of the rockets is 9 km.

 		 		 	 _	 									

(g) By finding the change in altitude in one second, or otherwise, find an estimate for the speed at which the rockets are falling when their altitude is 9 km.

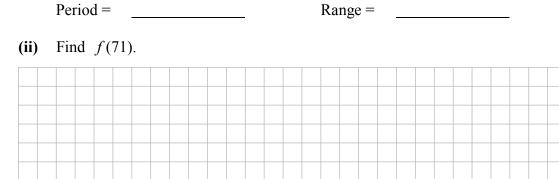


 $f: x \to f(x)$ is a periodic function defined for $x \in \mathbb{R}$. **(a)** The period is as indicated in the diagram.

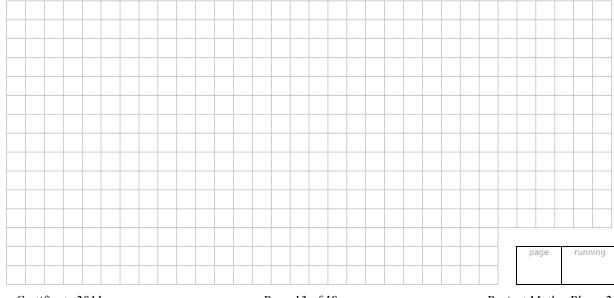


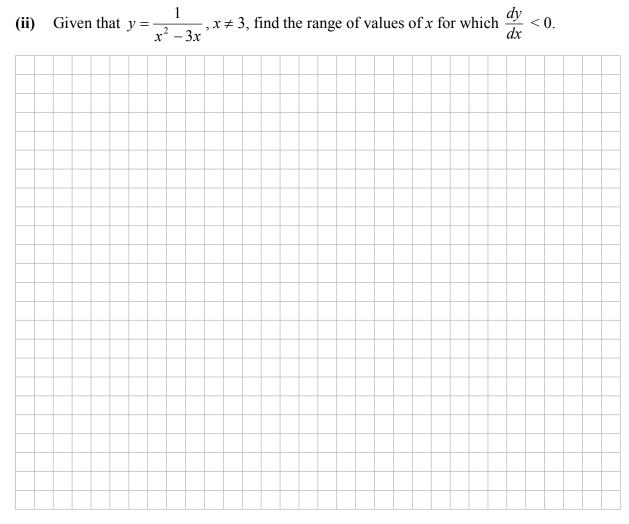
Functions and Calculus (old syllabus)

Write down the period and the range of the function. (i)

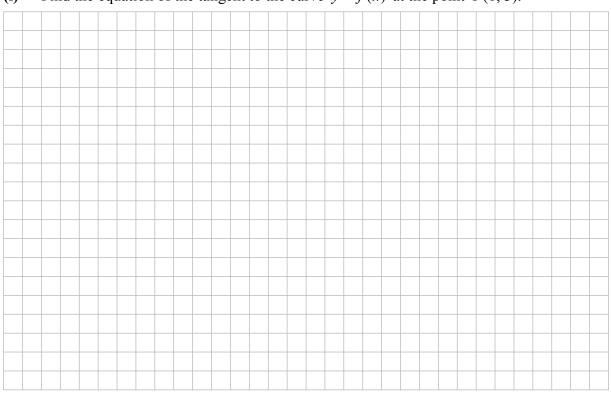


Differentiate $(4x-1)(3-2x^2)$ with respect to x and simplify your answer. **(b)** (i)



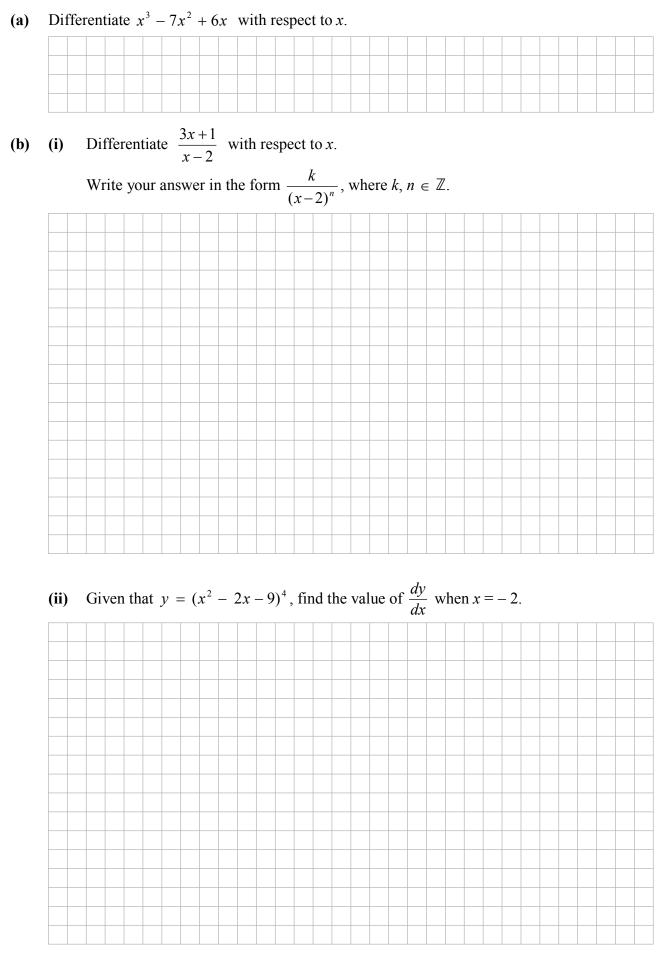


- (c) Let $f(x) = 2x + \frac{1}{x}$, where $x \in \mathbb{R}$ and $x \neq 0$.
 - (i) Find the equation of the tangent to the curve y = f(x) at the point P(1, 3).



-						 	 		 	 		 				

(ii) Q is another point on the curve y = f(x) such that the tangent at Q is parallel to the tangent at P. Find the co-ordinates of Q.



(c) A ball is rolled in a straight line along a surface. The distance, *s* metres, the ball travels is given by

$$s = 18t - 2t^2$$

where *t* is the time in seconds from the instant the ball begins to move.

(i) Find the speed of the ball after 3 seconds.

(ii) How far is the ball from the starting point when it stops moving?

L	 	 	 			 					 		 	 	 	

(iii) Show that the speed of the ball decreases at a constant rate while it is moving.

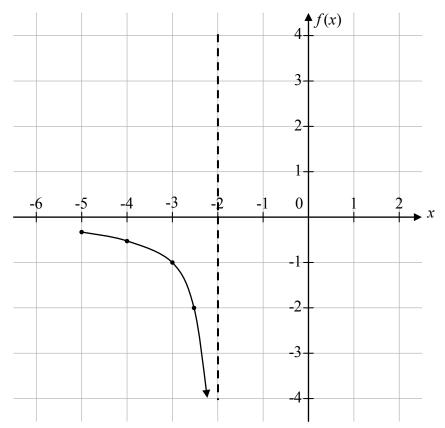
page running

Let
$$f(x) = \frac{1}{x+2}$$
, where $x \in \mathbb{R}$ and $x \neq -2$.

(i) Complete the following table:

x	-5 -4		-3	-2.5	-1.5	-1	0	1
f(x)	f(x) -0.5		-1	-2				

(ii) The diagram shows part of the graph of the function f. Complete the graph from x = -5 to x = 1.



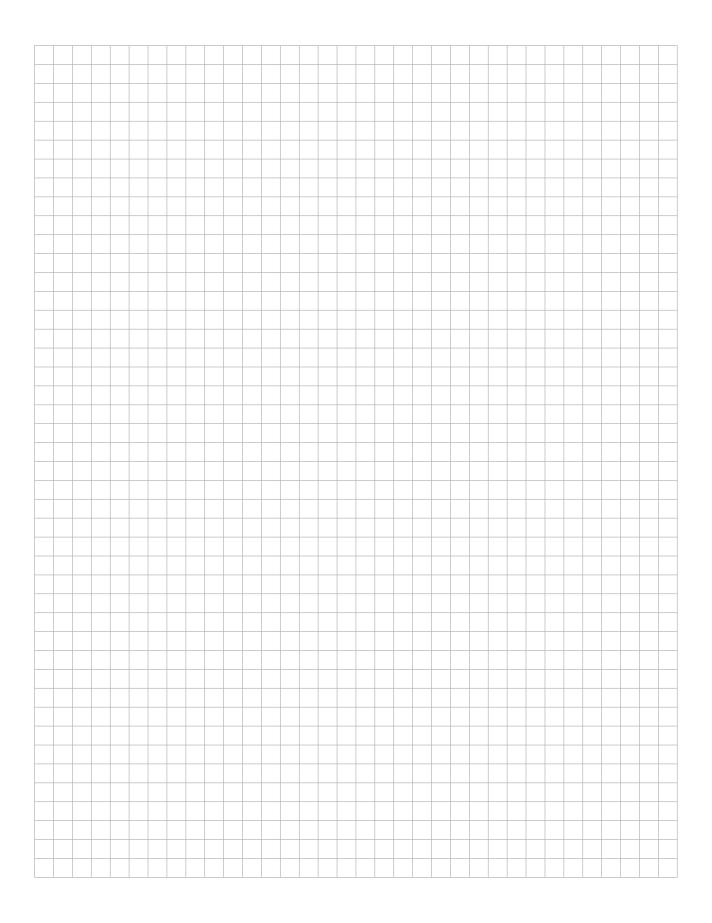
(iii) On the diagram above, draw the graph of the function g(x) = x + 2 in the domain $-5 \le x \le 1$, where $x \in \mathbb{R}$.

															Γ

(iv) Use your graphs to estimate the range of values of x for which $f(x) \le g(x)$.

(v) Prove that the curve y = f(x) has no turning points.

page	running



Leaving Certificate 2011 – Ordinary Level

Mathematics (Project Maths – Phase 2) – Paper 1

Friday 10 June Afternoon 2:00 – 4:30